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Multiple Polarized MIMO with Antenna Selection

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***Abstract*—Antenna subset selection in combination with dual-polarized antennas offers a better solution for compact and low-complexity devices using higher order Multiple-Input Multiple-Output (MIMO). In this paper we provide another degree of freedom to the existing dual-polarized MIMO and analyze the performance of antenna selection for triple-polarized MIMO systems with maximum ratio combining receivers. We theoretically analyze the impact of cross-polar discrimination on the achieved antenna selection gain for both dual and triple-polarized MIMO for non line of sight channels. We proceed to derive the outage probabilities and observe that these systems achieve significant performance gains for compact configurations with only a nominal increase in complexity. We observe that at higher cross polarization discrimination and lower transmit signal to noise ratio, the outage performance for a dual-polarized system is almost the same as triple-polarized system with joint transmit/receive antenna selection. With selection at only one end of the link, the triple-polarized system performs better than dual-polarized counterpart at higher values of transmit SNRs. We also found out that the available degrees of freedom significantly increase with lower values of cross polarization discrimination.**
Keywords-Antenna Selection, XPD, dual-polarized, triple-polarized, Antenna selection gain, Outage probability.

I. Introduction

Multiple-Input Multiple-Output (MIMO) systems have the prospective to significantly improve the performance of wireless systems. For realizing practical MIMO systems, most of the research has been carried out on uni-polarized vertically arranged spatial array configurations where antenna elements are separated in space. Such systems require an inter-element spacing of the order of a wavelength to achieve significant gains in Non Line of Sight (NLOS) channels; even larger spacing is required for Line of Sight (LOS) channels [1][2]. In this regard, dual-polarized antennas have received much attention as a smart option for realizing MIMO architectures in compact devices [3]. Recently, considerations are even carried out using triple-polarized antenna systems to exploit the additional degree of freedom for wireless communications [4][5]. The main weakness of MIMO systems is that the gain in performance comes at a cost of increased hardware complexity in terms of multiple RF chains at the transmitter and receiver. Antenna selection is a technique which can alleviate these costs but still exploits the diversity benefits offered by the MIMO architecture. This technique has been extensively studied in the context of spatial array configurations and channels (see [6][7][8] and the references within). We give prominence to this strategy that it is all the more significant for compact portable devices, which are often constrained by complexity, power and cost.

Antenna selection, when combined with multiple-polarized antennas, may be an answer that could enable compact systems to exploit the benefits of the MIMO architecture with only a minimal increase in complexity. Compact antenna configurations with antenna selection for MIMO communications have been studied in [9][10][11]. However, MIMO channels with polarization diversity cannot be modeled like pure spatial channels, because such subchannels of the MIMO channel matrix are not identically distributed [12]. They differ in terms of average received power, Ricean K-factor, cross-polar discrimination (XPD) and correlation properties [13]. As a result, the performance of antenna selection for these channels needs to be calculated. The main objective of this paper is to analyze the performance of antenna selection for MIMO channels in the presence of polarization diversity. We provide a theoretical treatment for the 2×2 dual-polarized and 3×3 triple-polarized Rayleigh MIMO channel [14]. For the mathematical analysis in this work, we proceed on similar lines as in [10].

The paper is organized as follows: Section II discusses the model for dual-polarized and triple-polarized MIMO channels. Also we present the average squared Frobenius norm of such channels. In Section III we analytically study the effect of XPD on the performance of joint transmit/receive antenna selection in dual-polarized and triple-polarized MIMO channel. In Section IV we present outage probability analysis for joint transmit/receive antenna selection systems. In Section V we present the performance of transmit selection and the effect of XPD on the selection gain. In Section VI the outage probability for transmit antenna selection is presented. In Section VII we show methods to calculate the performance by simulations for obtaining channel gains and compare them with the theoretical results. We conclude with our findings in Section VIII.

II. Dual and Triple-Polarized MIMO

Dual and triple-polarized antennas can be envisaged as an array of two and three co-located antennas with orthogonal polarizations respectively. By using a dual or triple-polarized feed, an antenna can transmit two or three orthogonally polarized waves on the same frequency [4][5][15]. Another such set of antennas can then receive the two or three orthogonally polarized waves and separate them by means of an electrically identical dual or triple-polarized feed. Consider a system with n_t transmit and n_r receive antennas. When all the antennas are vertically polarized, the subchannels of the MIMO channel

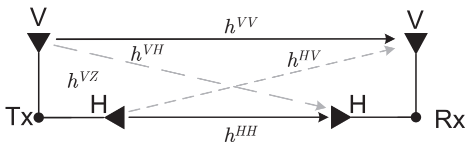

Fig. 1. Configuration of dual-polarized system.

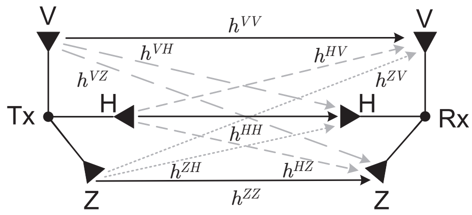

Fig. 2. Configuration of triple-polarized system.

matrix $\mathbf{H}$ are usually assumed to be identically distributed. However, when antennas with different polarizations are employed at either ends of the link, the properties of the co-polar subchannels differ significantly from those of the cross-polar subchannels. Hence for dual-polarized configurations, the channel matrix can be conveniently written as

$$\mathbf{H}_{DP} = \begin{bmatrix} \mathbf{H}^{VV}_{(n_r^V \times n_t^V)} & \mathbf{H}^{VH}_{(n_r^V \times n_t^H)} \\ \mathbf{H}^{HV}_{(n_r^H \times n_t^V)} & \mathbf{H}^{HH}_{(n_r^H \times n_t^H)} \end{bmatrix}_{n_r \times n_t}, \quad (1)$$

The configuration is shown in the Figure 1. Similarly the channel matrix for triple-polarized configuration can be written as

$$\mathbf{H}_{TP} = \begin{bmatrix} \mathbf{H}^{VV}_{(n_r^V \times n_t^V)} & \mathbf{H}^{VH}_{(n_r^V \times n_t^H)} & \mathbf{H}^{VZ}_{(n_r^V \times n_t^Z)} \\ \mathbf{H}^{HV}_{(n_r^H \times n_t^V)} & \mathbf{H}^{HH}_{(n_r^H \times n_t^H)} & \mathbf{H}^{HZ}_{(n_r^H \times n_t^Z)} \\ \mathbf{H}^{ZV}_{(n_r^Z \times n_t^V)} & \mathbf{H}^{ZH}_{(n_r^Z \times n_t^H)} & \mathbf{H}^{ZZ}_{(n_r^Z \times n_t^Z)} \end{bmatrix}_{n_r \times n_t}, \quad (2)$$

where n_t^V and n_t^H are the number of vertically and horizontally polarized elements at the transmitter, respectively. Similarly n_r^V, n_r^H are the number of vertically and horizontally polarized elements at the receiver, respectively for dual-polarized system. For triple-polarized channels we have an additional horizontally polarized antenna in Z direction both at the transmitting and the receiving end. This antenna is orthogonal to both dual-polarized antennas in the X and Y direction. Their number is represented by n_t^Z and n_r^Z, at the transmitter and receiver respectively. The elements of the submatrices $\mathbf{H}^{VV}$ and $\mathbf{H}^{HH}$ correspond to the co-polar subchannels in $\mathbf{H}_{DP}$ and $\mathbf{H}^{ZZ}$ an additional co-polar subchannel in $\mathbf{H}_{TP}$. Similarly $\mathbf{H}^{VH}$ and $\mathbf{H}^{HV}$ correspond to the cross-polar subchannels in $\mathbf{H}_{DP}$. For a triple-polarized structure we have additional cross-polar subchannels as $\mathbf{H}^{VZ}$, $\mathbf{H}^{ZV}$, $\mathbf{H}^{HZ}$ and $\mathbf{H}^{ZH}$. The configuration for triple-polarized antenna structure is shown in Figure 2. The transmitted radio signal, as it traverses through the wireless medium, experiences multiple reflections and scattering, resulting in a coupling of the orthogonal state of polarization. This phenomenon is referred to as depolarization. XPD is a measure of the extent of depolarization in a wireless channel, and for dual-polarized channel it is defined as,

$$\begin{matrix} X_V = E\left\{\left|h^{VV}\right|^2\right\}/E\left\{\left|h^{HV}\right|^2\right\} \\ X_H = E\left\{\left|h^{HH}\right|^2\right\}/E\left\{\left|h^{VH}\right|^2\right\} \end{matrix}. \quad (3)$$

Similarly for triple-polarized channels we have the following XPD definitions as,

$$\begin{matrix} X_{VH} = E\left\{\left|h^{VV}\right|^2\right\}/E\left\{\left|h^{HV}\right|^2\right\} \\ X_{HV} = E\left\{\left|h^{HH}\right|^2\right\}/E\left\{\left|h^{VH}\right|^2\right\} \\ X_{ZV} = E\left\{\left|h^{ZZ}\right|^2\right\}/E\left\{\left|h^{VZ}\right|^2\right\} \\ X_{VZ} = E\left\{\left|h^{VV}\right|^2\right\}/E\left\{\left|h^{ZV}\right|^2\right\} \\ X_{HZ} = E\left\{\left|h^{HH}\right|^2\right\}/E\left\{\left|h^{ZH}\right|^2\right\} \\ X_{ZH} = E\left\{\left|h^{ZZ}\right|^2\right\}/E\left\{\left|h^{HZ}\right|^2\right\} \end{matrix}, \quad (4)$$

where $h^{IJ} : I, J \in \{V, H, Z\}$ is an element of the sub-matrix $\mathbf{H}^{IJ}$ and $E\{Z\}$ denotes the expectation of Z. Typically XPD values are high in channels with limited scattering such as LOS channels and much lower in NLOS channels. However high XPD values have been observed even in NLOS channels, in some measurement campaigns [13]. Further, owing to the different propagation characteristics of horizontally polarized waves and vertically polarized waves, $\beta = E\left\{\left|h^{HH}\right|^2\right\} \leq E\left\{\left|h^{VV}\right|^2\right\} = 1$ and $\gamma = E\left\{\left|h^{ZZ}\right|^2\right\} \leq E\left\{\left|h^{VV}\right|^2\right\} = 1$. This happens due to the Brewster angle phenomenon for horizontally polarized transmission [16]. These subchannel power losses translate into a performance loss for dual-polarized MIMO systems when compared to spatial MIMO [13]. This discrepancy could also arise from the differences in the antenna patterns of the orthogonally polarized elements [17]. Under LOS conditions, the co-polar subchannels are Ricean distributed whereas the cross-polar subchannels are Rayleigh distributed. This is expected due to the fact that the cross-polar subchannel gains result from depolarization of the transmitted signal. Correlation between the elements of the MIMO channel is detrimental to its performance. For spatial MIMO, a large inter-element spacing is required to lower the correlation between the subchannels in some environments [13]. However for dual-polarized MIMO, the correlation between the elements from different submatrices is very low even under LOS channel conditions [13]. Thus, there are a significant differences between dual or triple-polarized MIMO channels compared to spatial MIMO channels. Taking into account these subchannel power losses, the average squared Frobenius norm of this channel can be written as [18],

$$\begin{aligned} \bar{W}_{DP} &= n_r^V n_t^V + \beta(n_r^H n_t^H) + \frac{1}{X_V}(n_r^H n_t^V) \\ &\quad + \frac{\beta}{X_H}(n_r^V n_t^H) \leq n_r n_t. \end{aligned} \quad (5)$$

Similarly the average squared Frobenius norm for triple polarized channels is shown below as,

$$\begin{aligned}\bar{W}_{TP} &= n_r^V n_t^V + \beta(n_r^H n_t^H) + \gamma(n_r^Z n_t^Z) + \\ &\frac{1}{X_{VH}}(n_r^H n_t^V) + \frac{1}{X_{VZ}}(n_r^Z n_t^V) + \\ &\frac{1}{X_{HZ}}(n_r^Z n_t^H) + \frac{1}{X_{ZH}}(n_r^H n_t^Z) + \\ &\frac{\beta}{X_{HV}}(n_r^V n_t^H) + \frac{\gamma}{X_{ZV}}(n_r^V n_t^Z) \le n_r n_t. \quad (6)\end{aligned}$$

From Equations (5) and (6) we note that as the XPD increases or as β decreases, $\bar{W}_{DP}$ and $\bar{W}_{TP}$ diminishes. As a result, the array gain achieved by using dual-polarized or triple-polarized antennas is smaller when compared to pure spatially separated antennas. Due to these power losses, the diversity gain of a dual-polarized MIMO channels is diminished for high XPD and low β values. For example, the available degree of diversity for a $(n_r \times n_t)$ i.i.d. Rayleigh MIMO channel is $n_s = n_r \times n_t$ [18]. But for dual-polarized NLOS channels with $\beta = 1$ and $X_V = X_H = X \to \infty$, the number of diversity order offered by the channel is

$$\eta \approx n_r^V n_t^V + n_r^H n_t^H < n_r n_t. \quad (7)$$

Similarly for triple-polarized NLOS channels with $\beta = 1$, $\gamma = 1$ and $X_{VH} = X_{HV} = X_{VZ} = X_{ZV} = X_{HZ} = X_{ZH} = X \to \infty$, the number of diversity order offered by the channel is reduced to

$$\eta \approx n_r^V n_t^V + n_r^H n_t^H + n_r^Z n_t^Z < n_r n_t. \quad (8)$$

Thus MIMO systems employing polarization diversity suffer SNR and diversity penalties, when compared to their spatial counterparts.

III. Effect of XPD on joint Transmit/Receive Selection Gain

Antenna selection refers to the process of selecting the "optimal" l_t out of the n_t available transmit antennas and/or the "optimal" l_r out of the n_r receive antennas. Symbolically we denote this process as $(n_r, l_r)/(n_t, l_t)$ selection. We assume here the availability of a perfect low bandwidth feedback channel for implementing selection at the transmitter. We also assume that the delay of this feedback signal is minimal. In this section we study the influence of XPD on selection gain achieved by using antenna selection for both transmit and receive side. To make the analysis as simple as possible, we first consider a 2×2 dual-polarized MIMO channel. In this case the channel matrix in (1) reduces to

$$\mathbf{H}_{DP} = \begin{bmatrix} h^{VV} & h^{VH} \\ h^{HV} & h^{HH} \end{bmatrix}. \quad (9)$$

and that for 3×3 triple-polarized channel in (2) reduces to

$$\mathbf{H}_{TP} = \begin{bmatrix} h^{VV} & h^{VH} & h^{VZ} \\ h^{HV} & h^{HH} & h^{HZ} \\ h^{ZV} & h^{ZH} & h^{ZZ} \end{bmatrix}. \quad (10)$$

All the subchannels are assumed to be independent complex circularly symmetric Gaussian random variables. This is an appropriate assumption for the typical NLOS indoor channel. Further, we make the simplifying assumptions that all the XPD values given in Section II are equal to X. Also $1 \le X \le \infty$ and $\gamma = \beta = 1$. We start our analysis with joint antenna selection at the transmitter and receiver i.e., $(2,1)/(2,1)$, $(3,1)/(3,1)$ and $(3,2)/(3,2)$ arrangements. We then move to the analysis of transmit antenna selection i.e., $(2,2)/(2,1)$, $(3,3)/(3,1)$ and $(3,3)/(3,2)$. We do this because we analyze transmit antenna selection in a different way as would be shown subsequently in the separate section. For $(2,1)/(2,1)$ and $(3,1)/(3,1)$ selection, the strategy would be to select the SISO subchannel which has the maximum instantaneous power. The instantaneous post processing SNR for the selected SISO channel $(\tilde{h})$ is given by YE_s/N_o where the random variable, $Y = |\tilde{h}|^2$. For a circularly symmetric complex Gaussian random variable Z with zero mean and variance σ^2, the cumulative distribution function (CDF) of $Z = |h|^2$ is given by, $F_Z(z) = (1 - e^{-z/\sigma^2})$. Since all the elements of $\mathbf{H}$ are assumed to be mutually independent, the CDF of Y can be derived as follows.

A. Dual Polarized $(2,1)/(2,1)$

From (3) we have $E\left\{\left|h^{HV}\right|^2\right\} = 1/X_V = 1/X$, so

$$\begin{aligned}F_Y(y)_{(2,1)/(2,1)} &= Pr(|h^{VV}|^2 < y)^2 Pr(|h^{HV}|^2 < y)^2 \\ &= (1 - e^{-y})^2(1 - e^{-yX})^2. \quad (11)\end{aligned}$$

The probability density function (PDF), $f_Y(y) = \frac{dF_Y(y)}{dy}$. is given by,

$$\begin{aligned}f_Y(y)_{(2,1)/(2,1)} &= 2(e^{-y}(1 - e^{-y})(1 - e^{-yX})^2 \\ &+ Xe^{-yX}(1 - e^{-yX})(1 - e^{-y})^2). \quad (12)\end{aligned}$$

Using the identity, $\int_0^\infty xe^{ax}dx = 1/a^2$, $G(X) = E\{Y\}$, which indicates the effective SNR gain achieved by using antenna selection, can be computed to be,

$$G(X) = \frac{3(1+X)}{2X} + \frac{2}{1+2X} + \frac{2}{2+X} - \frac{9}{2(1+X)}. \quad (13)$$

The average SNR gain is a monotonically decreasing function of X as shown in Figure 3. The selection gain is maximum at 3.2 dB when $X = 1$ and asymptotically diminishes to 1.76 dB. Here we can also calculate the probability that one of the cross-polar subchannels is selected, as follows,

$$\begin{aligned}P(X) &= Pr\left\{(\tilde{h} = h^{VH}) \cup (\tilde{h} = h^{HV})\right\} \\ &= 2Pr\left\{h^{VH} > h^{HV}\right\} \\ &\quad Pr\left\{h^{VH} > h^{HH}\right\} Pr\left\{h^{VH} > h^{VV}\right\} \\ &= 2(1/2)Pr\left\{h^{VH} > h^{HH}\right\}^2 \\ &= \frac{1}{(1+X)^2}. \quad (14)\end{aligned}$$

We observe from the above equation that as the XPD increases the probability of the cross-polar subchannels being selected, decreases and thus the average SNR gain diminishes. Further, $\lim_{X\to\infty} P(X) = 0$, which indicates that in the limiting case, the available degrees of diversity reduces to two when compared to four for $X = 1$. Thus a high XPD results in a diversity loss for dual-polarized MIMO channels when compared to spatial channels.

B. Triple Polarized $(3,1)/(3,1)$

From Equations (4), following the same procedure as in Section III-A we have $X_{VH} = X_{HV} = X_{VZ} = X_{ZV} = X_{HZ} = X_{ZH} = X$,

$$\begin{aligned} F_Y(y)_{(3,1)/(3,1)} &= Pr(|h^{VV}|^2 < y)Pr(|h^{HH}|^2 < y) \\ &\quad Pr(|h^{ZZ}|^2 < y)Pr(|h^{VH}|^2 < y) \\ &\quad Pr(|h^{HV}|^2 < y)Pr(|h^{VZ}|^2 < y) \\ &\quad Pr(|h^{ZV}|^2 < y)Pr(|h^{HZ}|^2 < y) \\ &\quad Pr(|h^{ZH}|^2 < y) \\ &= (1-e^{-y})^3(1-e^{-yX})^6. \end{aligned} \tag{15}$$

The (PDF) then reads

$$\begin{aligned} f_Y(y)_{(3,1)/(3,1)} &= 3(e^{-y}(1-e^{-y})^2(1-e^{-yX})^6 \\ &\quad +2Xe^{-y}(1-e^{-y})^3(1-e^{-yX})^5). \end{aligned} \tag{16}$$

$G(X) = E\{Y\}$ is then calculated as in previous section. Also we can calculate the probability that one of the cross-polar subchannels is selected, as follows,

$$P(X) = \frac{1}{(1+2X)^3}. \tag{17}$$

Also, $\lim_{X\to\infty} P(X) = 0$, which indicates that in the limiting case, we observe that the available degrees of diversity reduces to three when compared to nine for $X = 1$. Thus a high XPD results in a diversity loss for triple-polarized MIMO channels when compared to spatial channels.

C. Triple Polarized $(3,2)/(3,2)$

As we have to select two antennas at each end of the channel, we have to sum the powers of the individual channels or sum of the squares of independent Gaussian random variables. Thus, the resulting CDF becomes a Chi-squared distribution and not simply an exponential. The complete CDF of selecting such channels is then given by

$$F_Y(y)_{(3,2)/(3,2)} = (F_Y(y)_1)(F_Y(y)_2)(F_Y(y)_3), \tag{18}$$

where

$$\begin{aligned} F_Y(y)_1 &= Pr(|h^{VV}|^2 + |h^{HH}|^2 < y)Pr(|h^{VV}|^2 + |h^{ZZ}|^2 < y) \\ &\quad Pr(|h^{HH}|^2 + |h^{ZZ}|^2 < y) \\ &= (1-e^{-y/2})^3, \end{aligned} \tag{19}$$

where each term above is a central Chi-Squared distribution with zero means and $\sigma_1^2 = \sigma_2^2 = 1$.

$$\begin{aligned} F_Y(y)_2 &= Pr(|h^{VV}|^2 + |h^{HZ}|^2 < y)Pr(|h^{VV}|^2 + |h^{ZH}|^2 < y) \\ &\quad Pr(|h^{ZZ}|^2 + |h^{VH}|^2 < y)Pr(|h^{ZZ}|^2 + |h^{HV}|^2 < y) \\ &= (\frac{1}{X-1}(e^{-yX} - Xe^{-y} + X - 1))^4, \end{aligned} \tag{20}$$

where each term above is generalized central Chi-Squared distributed with zero means and $\sigma_1^2 = 1$, $\sigma_2^2 = 1/X$ [19].

$$\begin{aligned} F_Y(y)_3 &= Pr(|h^{VH}|^2 + |h^{HZ}|^2 < y)Pr(|h^{HV}|^2 + |h^{HZ}|^2 < y) \\ &= (1-e^{-2yX}(2yX+1))^2, \end{aligned} \tag{21}$$

where each term is generalized Chi-Squared distributed with zero means and $\sigma_1^2 = 1/X$, $\sigma_2^2 = 1/X$ [19]. This turns out to be an Erlang distribution. The results are shown in Figure 3.

IV. Outage Analysis with Transmit/Receive Selection

In this section, we derive the mutual information for both the antenna structures. We do this for systems with antenna selection and without selection. Later, considering the fact that the mutual information, depending on the channel realizations, is a random variable, we define the outage probability and then derive the same for both configurations. For the given systems, without antenna selection the mutual information can be bounded as follows

$$I \le \log(1 + \frac{\gamma}{n_t}\sum |h^{IJ}|^2), \tag{22}$$

where $\gamma = E_s/N_0$ and E_s is the transmit signal power. We assume here that the power is divided equally among n_t transmit antennas. The information theoretic outage probability defines an event when the channel mutual information cannot satisfy a certain target rate. This target rate may be set by some application such as audio, video, or some multimedia application. Mathematically, the probability of outage can be written as [20]

$$P[R] = P[I < R] \tag{23}$$

where R represents the rate requirement set by some particular application. For our scheme, using the mutual information expression (22) and the outage probability definition in (23), we derive the outage probability for the investigated scheme as follows. Methods to find outage probability for fading channels are given in [21].

$$P[I < R]_{(2,1)/(2,1)} = \int_0^{\epsilon} f_Y(y)_{(2,1)/(2,1)} dy, \tag{24}$$

where ϵ for $(2,1)(2,1)$ system is given by $\frac{(2^R-1)}{\gamma}$. For triple-polarized channels we have the following outage expressions

$$P[I < R]_{(3,1)/(3,1)} = \int_0^{\epsilon} f_Y(y)_{(3,1)/(3,1)} dy \tag{25}$$

$$P[I < R]_{(3,2)/(3,2)} = \int_0^{\epsilon} f_Y(y)_{(3,2)/(3,2)} dy, \tag{26}$$

where ϵ for $(3,1)/(3,1)$ and $(3,2)/(3,2)$ system is given by $\frac{(2^R-1)}{\gamma}$ and $\frac{2(2^R-1)}{\gamma}$, respectively. The results are shown in Figure 4. From Figure 4 we see that the performance of a $(3,1)/(3,1)$ system is effected severely compared to a $(2,1)/(2,1)$ system. This can be explained from Equations (14) and (17). From the equations we see that as XPD increases, the available degrees of freedom for $(3,1)/(3,1)$ system decreases from nine to three compared to four to two

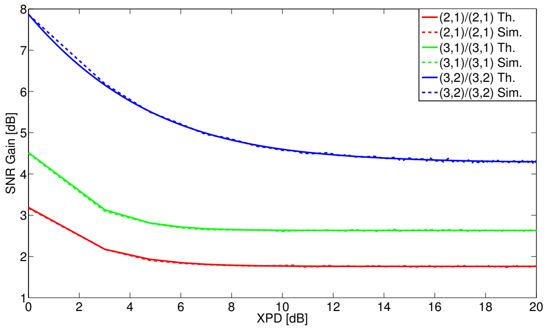

Fig. 3. Selection gains for polarized systems with transmit/receive antenna selection.

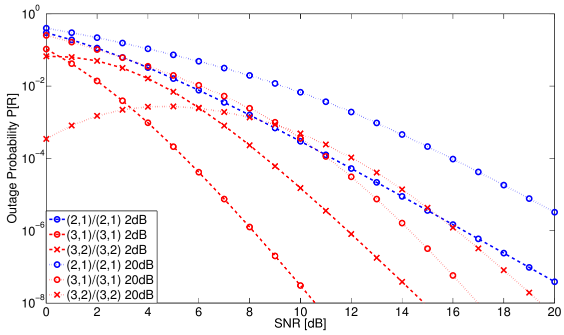

Fig. 4. Outage probabilities for joint transmit/receive antenna selection in multi polarized systems at XPD = 2dB and 20dB.

in a $(2,1)/(2,1)$ system. We have also shown here the trend of outage performance with varying XPD at a given SNR for joint transmit/receive antenna selection in Figure 5. From the figure we observe that the outage performance of $(3,2)/(3,2)$ system improves with increasing XPD values at lower SNRs. All the rest of the systems have a degrading performance for increasing XPD values, both at lower and higher SNRs.

V. Effect of XPD on Transmit Selection Gain

Here we try to understand the effect of XPD on the transmit selection gain for $(2,2)/(3,l_t)$ and $(3,3)/(3,l_t)$ systems. Such configurations could be used in Wireless Local Area Network (WLAN) or cellular systems where one end of the link is allowed to be more complex than the other. The analysis is general and is applicable to any Orthogonal Space Time Block Code (OSTBC) and can be easily adapted for receive antenna selection. The selection strategy outlined below, chooses l_t out of the n_t available transmit antennas to maximize the Frobenius norm of the channel.

$$\tilde{\mathbf{H}} = \underset{S(\overline{\mathbf{H}})}{\operatorname{argmax}} \left\{ \left\| \overline{\mathbf{H}} \right\|_F^2 \right\}, \tag{27}$$

where $\tilde{\mathbf{H}}$ is obtained by eliminating $(n_t - l_t)$ columns from $\mathbf{H}$. The term $S(\overline{\mathbf{H}})$ denotes the set of all possible $\tilde{\mathbf{H}}$. Let $Y_k, k = 1, ..., n_t$ denote the squared Frobenius norm of the n_t columns of $\mathbf{H}$. We derive the performance separately for dual-polarized and triple-polarized systems below.

A. $(2,2)/(2,l_t)$ *Transmit Antenna Selection*

Each column of $\mathbf{H}_{DP}$ has two independent but non-identical zero mean circularly symmetric complex Gaussian random variables with variances 1 and $1/X$ respectively. They have the probability density functions $g_1(y) = e^{-y}$ and $g_2(y) = Xe^{-yX}$ respectively. The random variables $Y_k, k = 1, ..., n_t$ are i.i.d with the probability density function given by

$$\begin{aligned} f_Y(y)_{(2,2)/(2,2)} &= g_1(y) * g_2(y) \\ &= \frac{Xe^{-y}}{X-1}(1 - e^{-(X-1)y}), \end{aligned} \tag{28}$$

where, the operator $(*)$ denotes the convolution operation. The cumulative distribution function, can be derived to be

$$\begin{aligned} F_Y(y)_{(2,2)/(2,2)} &= \int_{-\infty}^{y} f_Y(y)dy \\ &= \left(1 - \frac{e^{-y}}{X-1}(X - e^{-(X-1)y}\right) \end{aligned} \tag{29}$$

Applying the principles of ordered statistics [22], we generate new random variables $Y[k], k = 1, ..., n_t$ from $Y_k, k = 1, ..., n_t$ such that

$$Y_{[n_t]} \geq Y_{[n_t-1]} \geq ... \geq Y_{[k]} \geq ... \geq Y_{[2]} \geq y_{[1]}, \tag{30}$$

where $Y_{[k]}$ is the kth largest of the n_t random variables distributed according to (32). Note that these ordered random variables are no longer statistically independent. The average SNR after selection can then be computed as,

$$E\{\gamma\} = \gamma_0 \left(E\{Y_{[n_t]}\} + E\{Y_{[n_t-1]}\} + ... + E\{Y_{[n_t-l+1]}\}\right), \tag{31}$$

where $\gamma_0 = \frac{E_S}{l_t N_o}$. The probability density function of of the k-th ordered statistic $Y_{[k]}$ can then be evaluated as [23],

$$\begin{aligned} f_k(y) &= \frac{n_t!}{(k-1)!(n_t-k)!} F_Y(y)^{k-1} \\ &\quad (1 - F_Y(y))^{n_t-k} f_Y(y). \end{aligned} \tag{32}$$

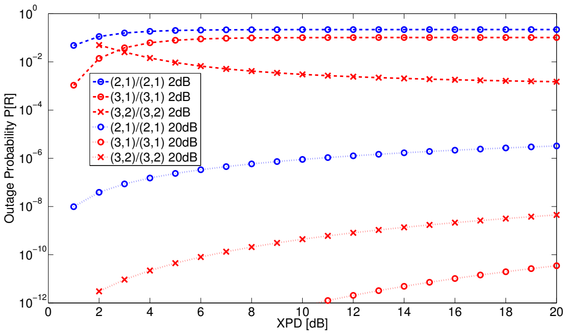

Fig. 5. Outage probabilities for joint transmit/receive antenna selection in multi polarized systems at SNR = 2 and 20dB for varying XPD.

$$
\begin{aligned}
E\left\{Y_{[k]}\right\} &= \frac{n_t!}{(k-1)!(n_t-k)!}\int_0^\infty y F_Y(y)^{k-1}(1-F_Y(y))^{n_t-k} f_Y(y)dy \\
&= \frac{n_t!}{(k-1)!(n_t-k)!}\sum_{r=0}^{k-1}(-1)^r \left(\begin{array}{c} k-1 \\ r \end{array}\right)\int_0^\infty y(1-F_Y(y))^{n_t-k+R} f_Y(y)dy \\
&= \frac{n_t!}{(k-1)!(n_t-k)!}\sum_{r=0}^{k-1}(-1)^r \left(\begin{array}{c} k-1 \\ r \end{array}\right) J_{n_t-k+r},
\end{aligned} \tag{33}
$$

The average value of k-th order statistic $T_{[k]}$ can be computed to be as (33), where

$$
J_m = \int_0^\infty y(1-F_Y(y))^m f_Y(y)dy. \tag{34}
$$

After calculating the average SNRs for $n_t = 2$ and for $k = 1, 2$ from the expressions above, we arrive at the following results,

$$
E\left\{Y_{[1]}\right\} = 2J_1. \tag{35}
$$

$$
E\left\{Y_{[2]}\right\} = 2(J_0 - J_1). \tag{36}
$$

$$
J_0 = \frac{X}{X-1}\left(1 - \frac{1}{X^2}\right). \tag{37}
$$

$$
\begin{aligned}
J_1 &= \left(\frac{X}{X-1}\right)^2\left(\frac{1}{4} - \frac{1}{(X+1)^2}\right) \\
&\quad -\frac{X}{(X-1)^2}\left(\frac{1}{(X+1)^2} - \frac{1}{4X^2}\right).
\end{aligned} \tag{38}
$$

B. $(3,3)/(3,l_t)$ Transmit Antenna Selection

Each column of $\mathbf{H}_{TP}$ has three independent but non-identical zero mean circularly symmetric complex Gaussian random variables with variances 1, $1/X$ and $1/X$ respectively. They have the probability density function given by

$$
\begin{aligned}
f_Y(y) &= g_1(y) * g_2(y) * g_3(y) \\
&= \left(\frac{X}{X-1}\right)^2 e^{-yX} \\
&\quad \left(e^{-(X-1)y} - (X-1)y - 1\right).
\end{aligned} \tag{39}
$$

Calculating the value of J's we have the following,

$$
J_0 = \frac{2 - 3X + X^3}{X(X-1)^2}. \tag{40}
$$

$$
J_1 = \frac{5 + 2X(7 + X(6+X))}{8X(1+X)^2}. \tag{41}
$$

$$
J_2 = \frac{104 + X(836 + X(2606 + 9X(431 + 302X + 92X^2 + 8X^3)))}{81X(2+X)^2(1+2X)^3}. \tag{42}
$$

The average values of ordered SNRs are shown below

$$
E\left\{Y_{[1]}\right\} = 3J_2. \tag{43}
$$

$$
E\left\{Y_{[2]}\right\} = 6(J_1 - J_2). \tag{44}
$$

$$
E\left\{Y_{[3]}\right\} = 3(J_0 - 2J_1 + J_2). \tag{45}
$$

The results are shown in Figure 6.

VI. Outage Analysis with Transmit Selection

In this section we calculate outage probabilities for both dual and triple-polarized MIMO channels with transmit antenna selection. We first calculate the PDFs of the corresponding ordered statistics and then integrate them over the respective range of ϵ. For $(2,2)/(2,2)$ scenario we have from (28),

$$
\begin{aligned}
P[I<R]_{(2,2)/(2,2)} &= \int_0^\epsilon f_Y(y)_{(2,2)/(2,2)}dy \\
&= \int_0^\epsilon (e^{-y}) * (Xe^{-yX})dy \\
&= \int_0^\epsilon e^{-y}dy \int_0^\epsilon Xe^{-yX}dy,
\end{aligned} \tag{46}
$$

where $\epsilon = \frac{2(2^R-1)}{\gamma}$ for dual-polarized systems. The rest of the outages are calculated as follows together with using ϵ values using (32).

$$
\begin{aligned}
P[I<R]_{(2,2)/(2,1)} &= \int_0^\epsilon f_Y(y)_{(2,1)/(2,1)}dy \\
&= \int_0^\epsilon 2! F_Y(y)_{(2,2)/(2,2)} \\
&\quad f_Y(y)_{(2,2)/(2,2)}dy,
\end{aligned} \tag{47}
$$

where $\epsilon = \frac{(2^R-1)}{\gamma}$ for dual-polarized systems with one antenna selected at the transmit side. Now from (39) we have,

$$
\begin{aligned}
P[I<R]_{(3,3)/(3,3)} &= \int_0^\epsilon f_Y(y)_{(3,3)/(3,3)}dy \\
&= \int_0^\epsilon g_1(y) * g_2(y) * g_3(y)dy \\
&= \int_0^\epsilon g_1(y)dy \int_0^\epsilon g_2(y)dy \int_0^\epsilon g_3(y)dy \\
&= \int_0^\epsilon g_1(y)dy \int_0^\epsilon 2g_2(y)dy,
\end{aligned} \tag{48}
$$

where $\epsilon = \frac{3(2^R-1)}{\gamma}$ for triple polarized systems and $g_1(y) = (1 - e^{-y})$, $g_2(y) = (1 - e^{-yX})$ and $g_3(y) = (1 - e^{-yX})$. Again from (32) we have

$$
\begin{aligned}
P[I<R]_{(3,3)/(3,1)} &= \int_0^\epsilon f_Y(y)_{(3,1)/(3,1)}dy \\
&= \int_0^\epsilon 3! F_Y(y)_{(3,3)/(3,3)} \\
&\quad f_Y(y)_{(3,3)/(3,3)}dy,
\end{aligned} \tag{49}
$$

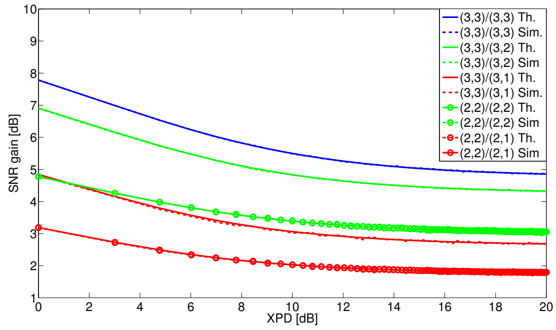

Fig. 6. Selection gains for polarized systems with transmit antenna selection

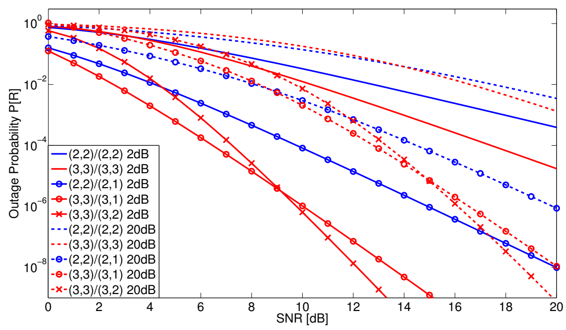

Fig. 7. Outage probabilities for transmit antenna selection in multi polarized systems at XPD = 2dB and 20dB.

where $\epsilon = \frac{(2^R-1)}{\gamma}$ for triple polarized systems with one antenna selected at the transmit side. For the configuration $(3,3)/(3,2)$ we proceed as follows. We convolve the second (highest) order and the first (2nd highest) order statistics. The highest order statistic is calculated from (32) as

$$f_2(y)_{(3,3)/(3,2)} = 3!F_Y(y)(1 - F_Y(y))f_Y(y), \quad (50)$$

and the 2nd highest order statistics is found to be as

$$f_1(y)_{(3,3)/(3,2)} = \frac{3!}{2!}(1 - F_Y(y))^2 f_Y(y). \quad (51)$$

so,

$$P[I < R]_{(3,3)/(3,2)} = \int_0^{\epsilon} f_2(y)_{(3,3)/(3,2)} * f_1(y)_{(3,3)/(3,2)} dy, \quad (52)$$

where $\epsilon = \frac{2(2^R-1)}{\gamma}$ for triple-polarized systems with two antennas selected at the transmit side. The analytical results are shown in Figures 7. Here we have also provided the trends for outage probabilities with respect to varying XPD values for specific SNRs. They are shown in Figures 8. From Figure 7 we see that the performance of $(3,3)/(3,3)$ and $(2,2)/(2,2)$ full complexity system does not improve much while increasing the SNR. The slopes of the curves are almost the same. Compared to these, the systems with antenna selection perform better when SNR is increased.

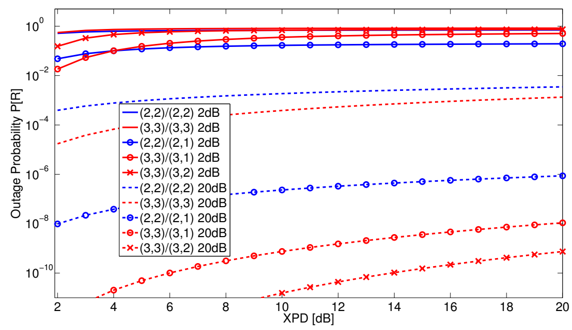

Fig. 8. Outage probabilities for transmit antenna selection in multi polarized systems at 2dB and 20dB SNR for varying XPD.

VII. Simulation Results and Discussion

In Figures 3 and 6 we compare both the analytical and simulation results for selection gains. The simulations completely verify the analytical results presented in the previous sections. Simulations were carried out in the following way. A complex Gaussian matrix with zero mean and unit variance was generated for the given number of antennas. This matrix was multiplied with an XPD matrix to reflect the different variances in the cross-polar components. Selection was performed on the basis of (27). For joint transmit/receive antenna selection, first selection is performed on receive side of the link. The non-selected rows are deleted from the complete matrix. Now the columns are selected from the remaining matrix, deleting the non-selected columns. This gives the selected channel. The process is shown below,

$$H_X = [X]_{n_r \times n_t} \odot [H]_{n_r \times n_t}. \quad (53)$$

where

$$[X]_{2\times 2} = \begin{bmatrix} 1 & 1/X \\ 1/X & 1 \end{bmatrix}. \quad (54)$$

and

$$[X]_{3\times 3} = \begin{bmatrix} 1 & 1/X & 1/X \\ 1/X & 1 & 1/X \\ 1/X & 1/X & 1 \end{bmatrix}. \quad (55)$$

From Figure 3 we see that although the $(3,2)/(3,2)$ system has the maximum SNR gain, its is effected more by the variations in XPD. The difference in the maximum and the minimum SNR gain for this system is 3.6dB compared to 1.87dB and 1.41dB for $(3,1)/(3,1)$ and $(2,1)/(2,1)$ systems respectively. This is because of high probability of any of the selected channels to be cross-polar. A similar behavior can be observed in Figure 6 for transmit antenna selection. Few differences still can be observed. A $(2,1)/(2,1)$ is effected more compared to $(2,2)/(2,1)$ system in the range of XPD values from 0 to 14dB. Similarly a $(3,1)/(3,1)$ system has more performance loss compared to a $(3,3)/(3,1)$ system for a range of XPD values from 0 to 20dB. Comparing a $(3,2)/(3,2)$ and $(3,3)/(3,2)$ systems, the trend is a little different. For low XPD values, a $(3,2)/(3,2)$ system is less effected but this trend changes for larger values of XPD. So the limiting cases can be easily observed from the figures.

VIII. Conclusions

In this paper we have shown analytically and through simulations, the gains of full complexity system and for systems with antenna selection. We found that the selection gain diminishes as the XPD increases. We also found the performance in terms of outage probabilities and observed that the performance is degraded with increasing XPD. While antenna selection with the spatial array configuration performs the best under both LOS and NLOS channel conditions, it requires a larger array length which is not always possible to realize in compact devices. So, antenna selection combined with dual-polarized and triple-polarized antennas presents an attractive solution to the problem of realizing high order MIMO architectures in compact devices.

ACKNOWLEDGMENTS

This work has been funded by the Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility, KATHREIN-Werke KG, and A1 Telekom Austria AG. The financial support by the Federal Ministry of Economy, Family and Youth and the National Foundation for Research, Technology and Development is gratefully acknowledged. The authors would like to thank Christoph F. Mecklenbräuker for his valuable comments and fruitful discussions. This work has also been funded by the Higher Education Commission, Islamabad, Pakistan.

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