

Optimal Pilot Symbol Power Allocation in Multi-Cell Scenarios of LTE

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Abstract—The UMTS Long Term Evolution (LTE) allows the pilot symbol power to be adjusted with respect to that of the data symbols. In this paper, we derive optimal pilot power allocation in multi-cell scenarios under imperfect channel knowledge at the terminals. As cost function, we choose the post-equalization Signal to Interference and Noise Ratio (SINR). We show via simulations that this cost function indeed leads to maximum throughput.

Index Terms—LTE, Channel Estimation, OFDM, MIMO.

I. INTRODUCTION

Current systems for cellular wireless communication are designed for coherent detection. Therefore, the channel estimator is a crucial part of a receiver. UMTS Long Term Evolution (LTE) provides a possibility to change the power radiated at the pilot subcarriers relative to those that of the data subcarriers. Clearly, this additional degree of freedom in the system design provides potential for optimization.

A. Related Work

In order to optimize the pilot symbol power allocation a model that takes into account the pilot power adjusting, receiver structure and channel estimation error at the same time, is needed. It has been shown by simulation that pilot symbol power allocation has a strong impact on the achievable capacity [1]. The authors of [2] showed by simulation the impact of different power allocations on the system's Bit Error Ratio (BER). However, their analysis is based on Signal to Noise Ratio (SNR) so that they only approximate the impact of imperfect channel knowledge on BER for Binary Phase-shift Keying (BPSK) modulation. In [3], optimal pilot symbol allocation is derived analytically for Phase-shift Keying (PSK) modulation of order two and four, using BER as the optimization criterion. In [4] optimal pilot symbol power in Multiple Input Multiple Output (MIMO) systems is derived based on lower bounds for capacity. The authors of [5] investigated power allocation between pilot and data symbols for MIMO systems using post-equalization Signal to Interference and Noise Ratio (SINR) as the optimization function. However, they only approximate the SINR expression and their model is tightly connected with a Linear Minimum Mean Square Error (LMMSE) channel estimator. In [6], we have derived optimal pilot power adjustment in a single cell scenario.

B. Contribution

In this paper, we derive analytical expressions for optimal power allocation in LTE systems with Zero Forcing (ZF) equalizers under imperfect channel state information in multi-cell scenarios. We utilize the post-equalization SINR, as the cost function, which turns to be equivalent to the throughput maximization.

The main contributions of the paper are:

- By maximizing the post-equalization SINR, we deliver optimal values for the pilot symbol power adjustment in cellular MIMO Orthogonal Frequency Division Multiplexing (OFDM) systems.
- Simulation results with an LTE compliant simulator [7, 8] confirm our optimal values for pilot symbol power.
- As with our previous work, all data, tools, as well implementations needed to reproduce the results of this paper can be downloaded from our homepage [9].

The remainder of the paper is organized as follows. In Section II we describe the mathematical system model for Multi User MIMO transmissions. In Section III, we derive the post-equalization SINR expression for ZF under imperfect channel knowledge. The channel estimators of this work are briefly discussed in Section IV and their Mean Square Error (MSE)s are derived. We formulate the optimization problem for optimal pilot symbol power allocation in Section V. Finally, we present LTE simulation results in Section VI and conclude our paper in Section VII.

II. SYSTEM MODEL

In this section, we briefly point out the key aspects of LTE relevant for this paper, and introduce a system model.

In the time domain the LTE signal consists of frames with a duration of 10 ms. Each frame is split into ten equally long subframes and each subframe into two equally long slots with a duration of 0.5 ms. Depending on the cyclic prefix length, being either extended or normal, each slot consists of $N_s = 6$ or $N_s = 7$ OFDM symbols, respectively. In LTE, the subcarrier spacing is fixed to 15 kHz. Twelve adjacent subcarriers of one slot are grouped into a so-called resource block. The number of resource blocks in an LTE slot ranges from 6 up to 100, corresponding to a bandwidth from 1.4 MHz up to

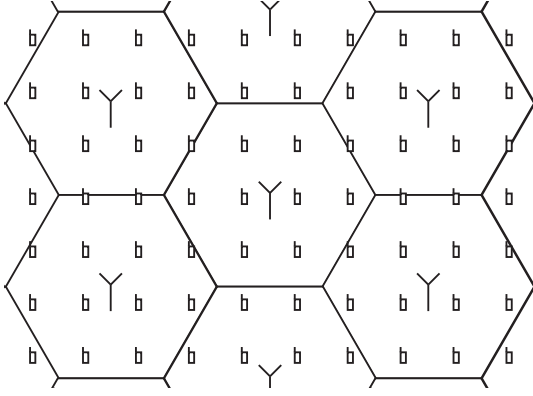


Fig. 1. Example of network layout with hexagonal cells and uniformly distributed users.

20 MHz. LTE utilizes pilot symbols scattered over time and frequency for channel estimation. Pilot symbols of adjacent cells are located at non overlapping positions [10]. In our optimization problem, we utilize the post-equalization SINR as cost function. Similar to [6], we use an MIMO system model, but in this case extended for multiple eNodeBs

$$\mathbf{y}_{0,u} = \mathbf{H}_{0,u} \mathbf{W}_0 \mathbf{s}_0 + \sum_{i=1}^{N_B} \mathbf{H}_{i,u} \mathbf{W}_i \mathbf{s}_i + \mathbf{n}_{0,u}, \quad (1)$$

where the matrix $\mathbf{H}_{i,u}$ is the channel matrix of size $N_r \times N_t$ between the i -th eNodeB and the user u , that is located within the 0-th cell. The precoding matrix of size $N_t \times N_1$ utilized by the i -th eNodeB and the data transmitted by the same eNodeB are denoted by \mathbf{W}_i and \mathbf{s}_i , respectively. The received signal at the u -th user within the cell of interest is disturbed by additive white zero mean Gaussian noise with variance σ_n^2 denoted by $\mathbf{n}_{0,u}$. An example of such system is shown in Figure 1 with uniform user distribution and hexagonal cells. In LTE the precoding matrix can be chosen from a finite set of precoding matrices [10]. The vector \mathbf{s}_i comprises the data symbols of all layers. We denote the effective channel matrix that includes the effect of the channel and precoding by $\mathbf{G}_{i,u}$

$$\mathbf{G}_{i,u} = \mathbf{H}_{i,u} \mathbf{W}_i. \quad (2)$$

Furthermore, let us denote the average data power transmitted at one layer by σ_s^2 , the total data power by σ_x^2 , and the average pilot symbol power by σ_p^2

$$\sigma_s^2 = \mathbb{E} \{ \|s_i\|_2^2 \} = \frac{1}{N_1}, \quad (3)$$

$$\sigma_x^2 = \sum_{n_t=1}^{N_t} \mathbb{E} \{ \|\mathbf{x}_{d,n_t}\|_2^2 \} = 1, \quad (4)$$

$$\sigma_p^2 = \sum_{n_t=1}^{N_t} \mathbb{E} \{ \|\mathbf{x}_{p,n_t}\|_2^2 \} = 1. \quad (5)$$

III. POST-EQUALIZATION SINR

In this section, we derive an analytical expression for the post-equalization SINR of a MIMO system under imperfect channel knowledge and a ZF equalizer given by the system model in Equation (1).

If the equalizer has perfect channel knowledge available, the ZF estimate of the data symbols $\hat{\mathbf{s}}_0$ at the user u is given as

$$\hat{\mathbf{s}}_0 = \underbrace{(\mathbf{G}_{0,u}^H \mathbf{G}_{0,u})^{-1} \mathbf{G}_{0,u}^H}_{F_{ZF}} \mathbf{y}_{0,u}. \quad (6)$$

The data estimate $\hat{\mathbf{s}}_0$ given by Equation (6) results in the post-equalization SINR of the l -th layer given as [11]

$$\gamma_{l,u} = \frac{\sigma_s^2}{\sigma_n^2 \mathbf{e}_l^H (\mathbf{G}_{0,u}^H \mathbf{G}_{0,u})^{-1} \mathbf{e}_l + \mathbf{e}_l^H \mathbf{P}_I \mathbf{e}_l}, \quad (7)$$

where the vector \mathbf{e}_l is an $N_1 \times 1$ zero vector with the l -th element being 1. This vector serves to extract the corresponding layer power after the equalizer. The matrix \mathbf{P}_I represents the interference from adjacent cells and is given as

$$\mathbf{P}_I = \sigma_x^2 \sum_{i=1}^{N_B} F_{ZF} \mathbf{G}_{i,u} \mathbf{G}_{i,u}^H F_{ZF}^H. \quad (8)$$

Let us proceed to the case of imperfect channel knowledge. We define the perfect channel as the channel estimate plus the error matrix due to the imperfect channel estimation

$$\mathbf{H}_{0,u} = \hat{\mathbf{H}}_{0,u} + \mathbf{E}_{0,u}, \quad (9)$$

where the elements of the matrix $\mathbf{E}_{0,u}$ are independent of each other with variance σ_e^2 . Inserting Equation (9) in Equation (1), the input output relation changes to

$$\mathbf{y}_{0,u} = (\hat{\mathbf{H}}_{0,u} + \mathbf{E}_{0,u}) \mathbf{W}_0 \mathbf{s}_0 + \sum_{i=1}^{N_B} \mathbf{H}_{i,u} \mathbf{W}_i \mathbf{s}_i + \mathbf{n}_{0,u}. \quad (10)$$

Since the channel estimation error matrix $\mathbf{E}_{0,u}$ is unknown at the receiver, the ZF solution is given again by Equation (6), but the channel matrix $\mathbf{H}_{0,u}$ is replaced by its estimate $\hat{\mathbf{H}}_{0,u}$, which is known at the receiver

$$\hat{\mathbf{s}}_0 = (\hat{\mathbf{G}}_{0,u}^H \hat{\mathbf{G}}_{0,u})^{-1} \hat{\mathbf{G}}_{0,u}^H \mathbf{y}_{0,u}, \quad (11)$$

with matrix $\hat{\mathbf{G}}_{0,u}$ being equal to $\hat{\mathbf{H}}_{0,u} \mathbf{W}_0$. After some straightforward manipulation the post-equalization SINR under imperfect channel knowledge in the multi-cell scenario can be written as

$$\gamma_{l,u} = \frac{\sigma_s^2}{(\sigma_n^2 + \sigma_e^2 \sigma_x^2) \mathbf{e}_l^H (\hat{\mathbf{G}}_{0,u}^H \hat{\mathbf{G}}_{0,u})^{-1} \mathbf{e}_l + \mathbf{e}_l^H \mathbf{P}_I \mathbf{e}_l}. \quad (12)$$

For simplicity reason, we omit the user and cell indices of the matrix $\hat{\mathbf{G}}$. Note, that in practice the variables σ_s^2 , σ_x^2 and σ_n^2 need to be replaced by their estimates.

IV. CHANNEL ESTIMATION

In this section, we describe channel estimation utilized by LTE terminals and discuss the case of multiple cells.

In [6], we derived the MSE of a Least Squares (LS) channel estimator with linear interpolation and also of an LMMSE channel estimator [12, 13]. However, the subcarriers carrying pilot symbols are disturbed not only by additive noise, but

also by the interference from neighboring eNodeBs. Due to the different position of the pilot symbols of the neighboring eNodeBs, the pilot symbols transmitted by the eNodeB of interest are disturbed by data symbols. Therefore, we can obtain MSE of the mentioned channel estimators in a multi cell scenario by a simple adaption. In case of an LS channel estimator the MSE of the pilot symbols is given as

$$\sigma_{e,p}^2 = \sigma_n^2 + \sigma_x^2 \sum_{i=1}^{N_B} \sigma_{h,i,u}^2, \quad (13)$$

where $\sigma_{h,i,u}^2$ denotes the mean channel power of the channel from the i -th eNodeB to the user u . The overall MSE is given as derived in [6] by

$$\sigma_e^2 = c_e \sigma_{e,p}^2, \quad (14)$$

where c_e is a constant depending on the number of transmit antennas.

V. POWER ALLOCATION

In this section, we describe the problem of optimal pilot power allocation in LTE based on the maximization of the post-equalization SINR under imperfect channel knowledge. Although the shown results are exemplified to LTE, the presented concept can be applied to any MIMO OFDM system.

If we increase the power at the pilot symbol by a factor c_p^2 , the MSE of the channel estimator not only improves by the factor c_p^2 as it was in [6], but also the interference from neighboring eNodeBs is changed. Furthermore to keep the total transmit power constant, the data power has to be adjusted and so also the interference from neighboring cells is changed. Considering all these effects, the new MSE of the channel estimator is given as

$$\tilde{\sigma}_e^2 = \frac{c_e}{c_p^2} \left(\sigma_n^2 + c_d^2 \sigma_x^2 \sum_{i=1}^{N_B} \sigma_{h,i,u}^2 \right), \quad (15)$$

where c_d^2 presents a data power adjustment factor. In order to keep the total transmit power constant, the two factors c_p^2 and c_d^2 are connected. For this purpose, we define a variable p_{off} , expressing the power offset between the mean energy of the pilot symbols and the data symbols, and refer to it as pilot offset. The variables c_p^2 , c_d^2 and p_{off} are interconnected as follows:

$$c_p = \frac{N_p + N_d}{p_{\text{off}} N_d + N_p}, \quad (16)$$

$$c_d = \frac{N_p + N_d}{N_d + \frac{N_p}{p_{\text{off}}}} = p_{\text{off}} c_p. \quad (17)$$

Plugging in the variables c_d^2 and c_p^2 in Equation (12), we obtain the SINR expression with adjusted power of the pilot symbols in a multi-cell scenario

$$\gamma_{l,u} = \frac{\sigma_s^2 c_d^2}{(\sigma_n^2 + \tilde{\sigma}_e^2 \sigma_x^2 c_d^2) \mathbf{e}_l^H (\hat{\mathbf{G}}^H \hat{\mathbf{G}})^{-1} \mathbf{e}_l + c_d^2 \mathbf{e}_l^H \mathbf{P}_I \mathbf{e}_l}. \quad (18)$$

Equation (18) can be further modified

$$\gamma_{l,u} = \frac{\sigma_s^2}{f(p_{\text{off}}) \mathbf{e}_l^H (\hat{\mathbf{G}}^H \hat{\mathbf{G}})^{-1} \mathbf{e}_l + \mathbf{e}_l^H \mathbf{P}_I \mathbf{e}_l}, \quad (19)$$

with a power allocation function $f(p_{\text{off}})$

$$f(p_{\text{off}}) = \frac{\sigma_n^2}{c_d^2} + \tilde{\sigma}_e^2 \sigma_x^2. \quad (20)$$

The target is to find an optimal value of $p_{\text{off,opt}}$ that maximizes the post-equalization SINR of the worst user in the cell while keeping the overall transmit power constant:

$$\begin{aligned} & \underset{p_{\text{off}}}{\text{maximize}} && \min_u (\gamma_{l,u}) \\ & \text{subject to} && N_d \sigma_x^2 + N_p \sigma_p^2 = \text{const} \end{aligned} \quad (21)$$

The lowest SINR is experienced by a user located at the cell edge, where the received signal consists of the desired signal and the interference signal. The most significant part of the interference signal comes from two closest eNodeBs. In Equation (19) the interference term \mathbf{P}_I is greater than zero and independent of p_{off} , therefore by minimizing the power allocation function $f(p_{\text{off}})$, we maximize the post-equalization SINR of a user. In contrast to [6], the power allocation function in a multi eNodeB scenario depends on the noise variance and one should keep in mind, that current wireless systems are interference limited rather than noise limited, therefore the noise variance σ_n^2 should not be chosen too high.

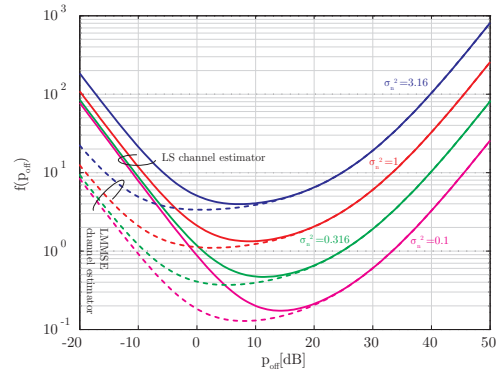


Fig. 2. Power allocation function $f(p_{\text{off}})$ for different channel estimators and different σ_n^2 while using one transmit antenna.

Figure 2 depicts the power allocation function $f(p_{\text{off}})$ from Equation (20) for a single transmit antenna at different values of the noise variance. One can see, that with increasing noise variance the post equalization SINR will be lower. Using an LMMSE channel estimator moves the minimum of an allocation function to the left, which corresponds to the power reduction of the power radiated for the pilot symbols.

Figure 3 shows the power allocation function for a different number of transmit antennas. It can be seen, that using a better channel estimator, the optimal power radiated at the pilot symbols can be decreased compared to a worse channel estimator (compare LMMSE vs. LS).

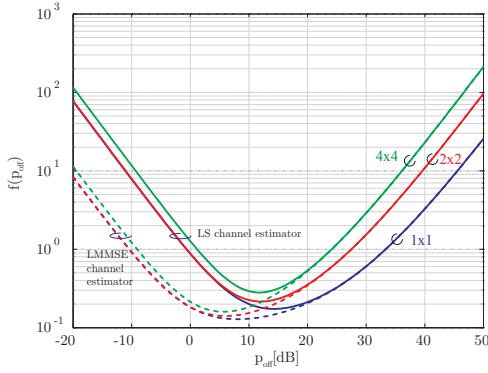


Fig. 3. Power allocation function $f(p_{\text{off}})$ for different channel estimators and a different number of transmit antennas with fixed noise variance $\sigma_n^2 = 0.1$.

TABLE I
VALUES OF THE PARAMETERS OF $f(p_{\text{off}})$ FOR A DIFFERENT NUMBER OF TRANSMIT ANTENNAS AT 1.4 MHz BANDWIDTH, ITU PED-A [14] CHANNEL MODEL, LS AND LMMSE CHANNEL ESTIMATORS

Parameter	Tx = 1	Tx = 2	Tx = 4
N_d	960	912	864
N_p	48	96	144
LS			
c_e	0.3704	0.3704	0.5556
$\sigma_n^2 = 3.16$	≈ 7.2	≈ 5.1	≈ 5.0
$\sigma_n^2 = 1$	≈ 8.9	≈ 6.8	≈ 6.7
$\sigma_n^2 = 0.316$	≈ 11.4	≈ 9.2	≈ 9.1
$\sigma_n^2 = 0.1$	≈ 14.3	≈ 12.0	≈ 11.8
LMMSE			
c_e	0.0394	0.0394	0.0544
$\sigma_n^2 = 3.16$	≈ 0.8	≈ -1.3	≈ -1.6
$\sigma_n^2 = 1$	≈ 2.6	≈ 0.5	≈ 0.2
$\sigma_n^2 = 0.316$	≈ 5.1	≈ 3.1	≈ 2.8
$\sigma_n^2 = 0.1$	≈ 8.1	≈ 6.0	≈ 5.7

Table I show the optimal value of the power offset between powers radiated at the pilots and at the data symbols for a different number of transmit antennas, different noise variances and for our two types of channel estimators.

VI. SIMULATION RESULTS

In this section, we present simulation results and discuss the performance of an LTE Downlink (DL) transmission system using different pilot symbol powers. All results are obtained with the LTE Vienna Link Level Simulator version "r1089" [7, 8], which can be downloaded from www.nt.tuwien.ac.at/ltesimulator. All data, tools and scripts are available online in order to allow other researchers to reproduce the results shown in the paper [9]. The Vienna LTE simulator is a Matlab implementation of all physical layer procedure such as coding, rate matching [15], synchronization [16], channel estimation [17].

Table II shows the most important simulator settings used for the verification of our analytic results shown in the previous section.

TABLE II
SIMULATOR SETTINGS

Parameter	Value
Bandwidth	1.4 MHz
Number of transmit antennas	1, 2, 4
Number of receive antennas	1, 2, 4
Receiver type	ZF
Transmission mode	Open-loop spatial multiplexing
Channel type	ITU PedA [14]
MCS	optimally selected

The simulation results are consistent with the analytical solution presented in Section V. Figure 4 shows the throughput of a Single Input Single Output (SISO) LTE DL transmission with optimally selected CQI and using different channel estimators. The arrows indicate the analytically derived optimal pilot symbol power, that maximize the post-equalization SINR. An excellent match between our analytical solutions with the simulation results is observed. One can observe, that the optimal power assigned to the pilot symbols is lower if using a better channel estimator (compare LS vs. LMMSE).

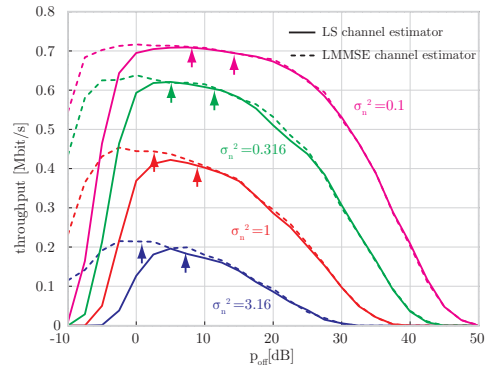


Fig. 4. Throughput for different channel estimators and different σ_n^2 while a using single transmit antenna.

The throughput of an LTE system using various numbers of transmit and receive antennas is depicted in Figure 5. The arrows indicate the optimal value of the variable p_{off} . An excellent match between analytical solutions with the simulation results is observed.

VII. CONCLUSION

In this paper, we extended the problem of pilot power optimization for multi-cell scenarios. As optimization criterion we utilized the post-equalization SINR under imperfect channel state information, that is connected to the system throughput. Furthermore, we derive analytical expression for the post-equalization SINR under imperfect channel knowledge using a ZF equalizer. We confirmed our analytical solution by means of the Vienna LTE simulator.

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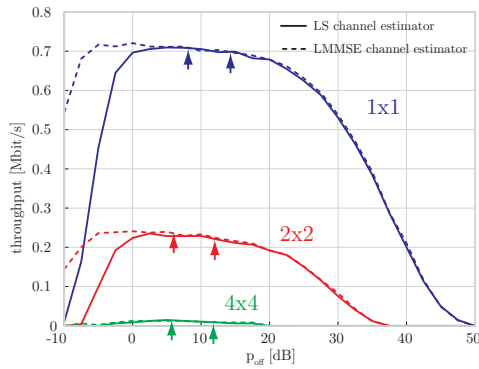


Fig. 5. Throughput for different channel estimators and a different number of transmit antennas with fixed noise variance $\sigma_n^2 = 0.1$.

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