Localization of users in multiuser MB OFDM UWB systems based on TDOA principle

Nikola Gvozdenovic, Miljko Eric

***Abstract*—In this paper, we analyze localization capabilities of multi-band OFDM UWB systems. TDOA (time difference of arrival) method is used for localization. M temporally synchronized sensors are localizing transmitter of unknown location. Space-temporal signal model in AWGN channel and in IEEE 802.15.3a channel model is derived analytically. We investigate the influence of correlation of channels between transmitter and each of sensors in sensor array on localization accuracy. The results are verified through simulation of hypothetical measurement setup where sensor array have circular geometry and the transmitter is placed within the array circle.**

***Index Terms*—MB OFDM UWB, Localization, Sensor Array**

I. Introduction

Recently, the use of Ultra-Wideband Radio for positioning in sensor networks is in focus of research, see e.g. [1]. They propose time method (TOA and TDOA) and/or hybrid solution with RSS to be used for localization in UWB due to ability of UWB systems to resolve multipath components. That makes it feasible to obtain accurate location estimates without the need for complex estimation algorithms.

In particular, positioning in multiband OFDM UWB utilizing received signal strength is investigated in [2]. Enormous frequency diversity of UWB signals minimizes fading effects, and thus allows reliable distance measurement based on RSS. They have proven that RSS based localization can be as accurate as ToA based localization.

Already a few MB OFDM UWB products and platforms are available on the market. Although this products do not have positioning capabilities with minor changes it is feasible to perform measurement in real scenarios. In [3] Java implementation of an active LT (Localization and Tracking) method based on a MB OFDM UWB platform is investigated.

A one-step subspace-based method for direct position estimation in impulse UWB systems based on mathematical model of the signal on sensor array is proposed in [4]. This method can be used in multiuser scenario and in multipath conditions when UWB impulses are partially overlapped in time due to multipath.

This paper is organized as follows: in Section II the overview of MB OFDM UWB standard proposal is given. In Section III we derive mathematical model of the signal on a sensor array taking into account propagation delay. Space-temporal signal model in MB OFDM UWB channel is analyzed in Section IV and localization algorithm in Section V. Before we conclude, simulation results are shown in Section VI.

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II. UWB Multiband approach

MB OFDM UWB was IEEE 802.15 3a WPAN standard proposal and is ECMA-368 standard. Available bandwidth of 7.5 GHz is divided in 14 sub-bands, each occupying 528 MHz. In every sub-band OFDM is used with 128 subcarriers. On subcarriers QPSK is used for modulation. OFDM parameters are summarized in Tab.(I). It supports 10 different bit rates from 53.3 to 480 Mbps. Channelization is based on 4 time-frequency codes. Transmitted signal can be described as:

$$s(t) = \sum_k x_k(t - kT_{\mathrm{SYM}}) e^{j2\pi f_{ck} t} \tag{1}$$

where $f_{ck} = 20904 + n(k) \cdot 528$ [MHz] is carrier frequency of the k-th OFDM symbol defined by time-frequency codes - $n(k)$. Block diagram of MB OFDM UWB transmitter and receiver are given in Fig. 1. Further details about MB OFDM UWB can be found in [5], and [6].

TABLE I
OFDM parameters

Parameters	Value
Number of OFDM subcarriers	128
Number of data subcarriers	100
Number of defined pilot subcarriers	12
Number of guard subcarriers	10
Δf: Subcarrier frequency spacing	4.125 MHz (=528 MHz/128)
T_{FFT}: IFFT/FFT period	242.42 ns (1/Δf)
T_{CP}: Cyclic prefix duration	60.61 ns (=32/528 MHz)
T_{GI}: Guard interval duration	9.47 ns (=5/528 MHz)
T_{SYM}: Symbol duration	312.5 ns (T_{FFT}+T_{CP}+T_{GI})

In this paper we analyze and simulate part of the system between A and B in Fig. 1 (signal in A is generated as random bit sequence). We use time-frequency code $\{1\ 2\ 3\ 1\ 2\ 3\}$.

III. Mathematical model of the signal on a sensor array

Let we assume that M timely synchronized sensors receives Q MB OFDM UWB signals which have the same time-frequency code. Than received signal on the l-th sensor ($l = 1, .., M$) can be expressed as:

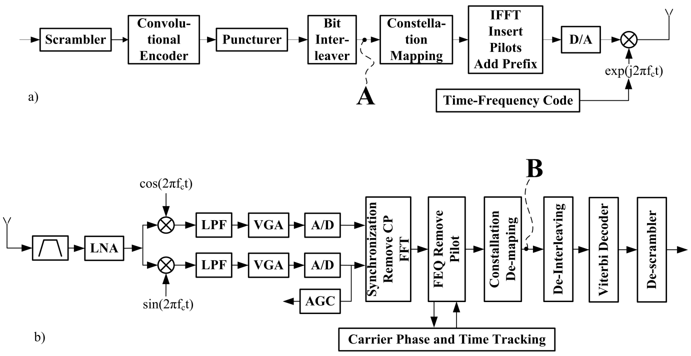

Fig. 1. MB OFDM UWB communications link: a) scheme of transmitter, b)scheme of receiver.

$$f(t) = \sum_{q=1}^{Q} b_{lq} s_q(t - \tau_{lq} - t_q^{(0)}) + n_l(t) \tag{2}$$

where b_{lq} is the channel attenuation between q-th transmitter and l-th sensor, $n(t)$ is AWGN. $\tau_{lq} = \frac{d_{lq}}{v}$ is time delay, where v is signal speed, and d_{lq} is distance from the reference point to the q-th transmitter given with the relation:

$$d_{lq} = \sqrt{(x_l - x_q)^2 + (y_l - y_q)^2 + (z_l - z_q)^2} \tag{3}$$

Substituting Eq.(1) in Eq.(2) we arrive at

$$f(t) = \sum_{q=1}^{Q} \sum_{k} b_{lq} x_{qk}\left(t - kT_{\mathrm{SYM}} - \tau_{lq} - t_q^{(0)}\right) \cdot e^{j2\pi f_{ck}(t-\tau_{lq})} + n_l(t) \tag{4}$$

Received signal is translated from the carrier frequency to the baseband with I-Q mixing, according to the given time-frequency code. Mathematically, translation is performed by multiplying of the signal with the complex sinusoid $e^{-j2\pi f_{ck} t}$ for the k-th period of the signal. Afterwards, signal is sampled with sampling period of $\Delta t = 1/BW$, where $BW = 528\text{MHz}$ is the bandwidth of the RF radio channel. Samples of the received signal on the l-th reference sensor can be expressed as

$$r(n\Delta t) = \sum_{q=1}^{Q} \sum_{k} b_{lq} x_{qk}\left(n\Delta t - kT_{\mathrm{SYM}} - \tau_{lq} - t_q^{(0)}\right) \cdot e^{j2\pi f_{ck}(t-\tau_{lq})} + n_l(n\Delta t) \tag{5}$$

Discrete Fourier transformation of the N signal samples on every sensor is an array with a length $H = N$, given with

$$R(f_h) = \sum_{q=1}^{Q} \sum_{k} b_{lq} X_{qk}(f_h) e^{-j2\pi f_h (kT_{\mathrm{SYM}} + t_q^{(0)})} \cdot e^{-j2\pi(f_h + f_{ck}\tau_{lq})} + N_l(f_h) \tag{6}$$

Where $-BW/2 \leq f_h \leq BW/2$.

Based on the fact that $\tau_{lq} = \frac{d_{lq}}{\lambda_g f_g}$ where f_g is the highest frequency of all three channels, we normalize distances with λ_g, and frequences with f_g

$$R(f_h) = \sum_{q=1}^{Q} \sum_{k} b_{lq} X_{qk}(f_h) e^{-j2\pi f_h (kT_{\mathrm{SYM}} + t_q^{(0)})} \cdot e^{-j2\pi(\frac{f_h}{f_g} + \frac{f_{ck}}{f_g})\frac{d_{lq}}{\lambda_g}} + N_l(f_h) \tag{7}$$

Acording to the $f_h = h\Delta f = hBW/H$,
where $-H/2 \leq h \leq H/2 - 1$ than

$$R(f_h) = \sum_{q=1}^{Q} \sum_{k} b_{lq} X_{qk}(f_h) e^{-j2\pi \frac{h}{H} BW (kT_{\mathrm{SYM}} + t_q^{(0)})} \cdot e^{-j2\pi(q_1 \frac{h}{H} + q_{2k}) d_{lq,norm}} + N_l(f_h) \tag{8}$$

where $q_1 = \frac{BW}{f_g}$ and $q_{2k} = \frac{f_{ck}}{f_g}$

IV. Space-temporal signal model in the MB OFDM UWB channel

A. IEEE 802.15.3a channel model

In the IEEE 802.15.3a channel model multipath components arrive in clusters and their amplitude follow double-exponential decay. Every coefficient which defines amplitude of the multipath component of the frequency selective channel is random variable with log-normal distribution. This model is designed for the baseband signalization, so phase of the impulse response of the channel is 0 or π. Channel impulse response is modeled as

$$h(t) = X \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l} \delta(t - T_l - \tau_{k,l}) \tag{9}$$

where: $\alpha_{k,l}$ - amplitude of the k-th multipath component of the l-th cluster of the impulse response, T_l - delay of the l-th cluster, $\tau_{k,l}$ - delay of the k-th multipath component relatively according to cluster delay (T_l), and X - log-normal shadowing.

This model has four types of channel, with different values of this basic parameters. In this paper we use channel Type 1 (LOS in the range 0-4 m). For further details about this channel model and actual values of the parameters we reference to [5].

B. Space-temporal signal model

Signal on the reception is the sum of delayed and attenuated replicas of the sent signal. Let as assume that transmitted signal is

$$s(t) = x(t) e^{j2\pi f_c t} \tag{10}$$

We also assume that M timely synchronized sensors receive the signal. Then, received signal can be expressed as:

$$f(t) = X \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l} s(t - T_l - \tau_{k,l}) \tag{11}$$

Replacing Eq.(10) into Eq.(11) we get

$$f(t) = X\sum_{l=0}^{L}\sum_{k=0}^{K}\alpha_{k,l}x(t-T_l-\tau_{k,l})e^{j2\pi f_c(t-T_l-\tau_{k,l})} \quad (12)$$

Received signal is translated from the carrier frequency to the baseband with I-Q mixing, according to the given time-frequency code. Mathematically, translation is performed by multiplying of the signal with the complex sinusoid $e^{-j2\pi f_{ck}t}$ for the k-th period of the signal. Afterwards, signal is sampled with sampling period of $\Delta t = 1/BW$, where $BW = 528$MHz is the bandwidth of the RF radio channel. Samples of the received signal on the l-th reference sensor can be expressed as

$$r(n\Delta t) = X\sum_{l=0}^{L}\sum_{k=0}^{K}\alpha_{k,l}x(n\Delta t-T_l-\tau_{k,l})e^{-j2\pi f_c(T_l+\tau_{k,l})} \quad (13)$$

Discrete Fourier transformation of the N signal samples on every sensor is an array with a length $H = N$, given with

$$R(f_h) = X\sum_{l=0}^{L}\sum_{k=0}^{K}\alpha_{k,l}X(f_h)e^{-j2\pi(f_h+f_c)(T_l+\tau_{k,l})} \quad (14)$$

Where $-BW/2 \leq f_h \leq BW/2$.

$T_l + \tau_{k,l} = \Delta t(T_l+\tau_{k,l})_{\text{norm}} = \frac{(T_l+\tau_{k,l})_{\text{norm}}}{BW}$ where f_g is the highest frequency of all three channels, we normalize the frequencies with f_g

$$R(f_h) = X\sum_{l=0}^{L}\sum_{k=0}^{K}\alpha_{k,l}X(f_h)e^{-j2\pi(\frac{f_h}{f_g}+\frac{f_c}{f_g})f_g\frac{(T_l+\tau_{k,l})_{\text{norm}}}{BW}} \quad (15)$$

Acording to the $f_h = h\Delta f = hBW/H$,
where $-H/2 \leq h \leq H/2-1$ than

$$R(f_h) = X\sum_{l=0}^{L}\sum_{k=0}^{K}\alpha_{k,l}X(f_h)e^{-j2\pi(q_1\frac{h}{H}+q_2)\frac{(T_l+\tau_{k,l})_{\text{norm}}}{q_1}} \quad (16)$$

where $q_1 = \frac{BW}{f_g}$ and $q_2 = \frac{f_c}{f_g}$.

In this paper we have considered two cases:

- The channels between the transmitter and each of the sensor are mutually uncorrelated. All the channel parameters are generated according to IEEE 802.15.3a channel model.
- The channels between the transmitter and each of the sensors are mutually correlated and this correlation is reflected in the delay of clusters which are the same for all channels, as a consequence of scattering on the scatterers with the known positions. Other parameters, including delay of components within the cluster are generated according to the IEEE 802.15.3a channel model.

In the latter case the cluster delay T_l is the sum of the propagation delay from the transmitter to the l-th scatterer and propagation delay from the l-th scatterer to the appropriate sensors. In the simulation signal is first delayed by propagation delay, then passed through one of these two channels and at the end, the noise is added to it.

V. Localization algorithm

Various algorithms for localization in UWB systems are analyzed in [7]. In the case of the localization based on the TDOA method, every TDOA measurement defines hyperbola which contains location of the transmitter. If we use three reference sensors it is feasible to calculate two distance differences (from the TDOA measurements)

$$d_{m1} = \sqrt{(x-x_m)^2+(y-y_m)^2} - \sqrt{(x-x_1)^2+(y-y_1)^2} \quad (17)$$

for $m = 2,3$ and $d_{m1} \equiv d_m - d_1 = c(T_m - T_1) = c\text{TDOA}_{m1}$. On that way two different hyperbolas are defined. Position of the transmitter is at the intersection of hyperbolas, and can be found from Eq.(17) and relation :

$$d_1 = \sqrt{(x-x_1)^2+(y-y_1)^2} \quad (18)$$

Combining this two we get system of one quadratic and two linear equations.

It is feasible to linearize this system by using one additional sensor:

$$\begin{aligned} d_1^2 &= x^2+y^2 \\ d_m^2 &= (x_m-x)^2+(y_m-y)^2, m = 2,3,4 \end{aligned} \quad (19)$$

Subtracting the first equation from the other three we get linear system

$$Ax + By + Cd_1 = N \quad (20)$$

where $A = -2x_m$,$B = -2y_m$, $C = -2d_{m1}$,
and $N = d_{m1}^2 - x_m^2 - y_m^2$.
Solution of this system is $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$ where $D = [A\ B\ C]$,$D_x = [N\ B\ C]$,and $D_y = [A\ B\ N]$
We will use this calculation in this paper to find location of the transmitter.

To estimate time difference of arrival we first cross-correlate a signal on the reference sensor with signals on every sensor in the array. After that we find delay that corresponds to a correlation peak on every sensor. Transmitter and sensors are not synchronized, thus estimated delays consists of propagation delay and timing offset which is the same for all sensors. By subtracting these delays we get TDOA which we use in the previously described algorithm.

We will use CEP(Circular error probability) to evaluate positioning accuracy. For location estimation on hyperbola CEP is approximated with the accuracy up to 10 % as CEP $\approx 0.75\sqrt{\sigma_{\text{x}}^2+\sigma_{\text{y}}^2}$ where σ_x^2 and σ_y^2 are variances of estimated position.

VI. Simulation results

Simulation was performed in Matlab. In uncorrelated case, the simulation setup consists of five sensors deployed in a circle and of the transmitter placed within the circle. In correlated case, additionally two scatterers are presented as it is described in IV. SNR is 30 dB on one OFDM symbol duration. For both cases the simulation is repeated 100 times (red x is position estimate), and CEP is drawn.

Fig. 2 and Fig. 3 are showing CEP in the correlated and in the uncorrelated case, respectively.

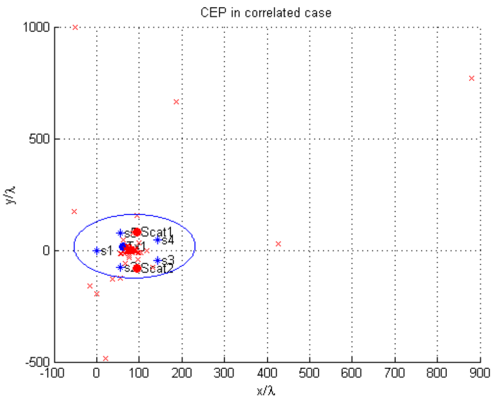

Fig. 2. CEP in the case when the channels between transmitter and each of sensors in array are correlated.

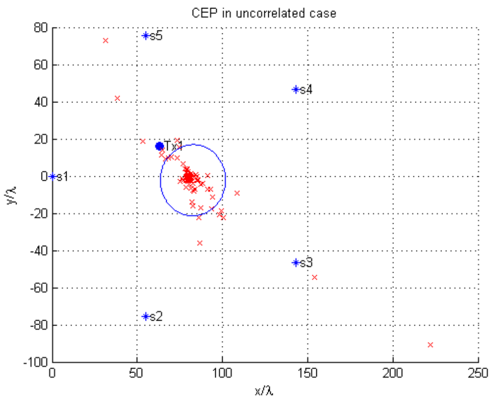

Fig. 3. CEP in the case when the channels between transmitter and each of sensors in array are uncorrelated.

In the first case, when the channel from the transmitter to one of the sensors is in a bad condition, than channels to the other sensors are also in bad conditions. Consequently, large error in location estimation can occur. Although the number of these estimates are relatively small compared to the total number of estimations, it significantly increases the error.

The error can be reduced if we discard the position estimates with significant error (outside of sensor array circle). Fig. 4 shows CEP in the case when the channels between transmitter and each of sensors in array are correlated, when we discard the position estimates which are outside of sensor array circle. Fig. 5 shows CEP in the case when these channels are uncorrelated. When discarding the position estimates with significant error the positioning accuracy is similar for both cases.

VII. Summary and Conclusions

In this paper, we analyze localization in MB OFDM UWB systems based on TDOA method. We proposed new space-temporal signal model in IEEE 802.15.3a channel in which cluster delays in the channels between the transmitter and each sensor in sensor array are correlated due to the scattering on the scatterers with the known position. We showed that this correlation decreases localization accuracy significantly.

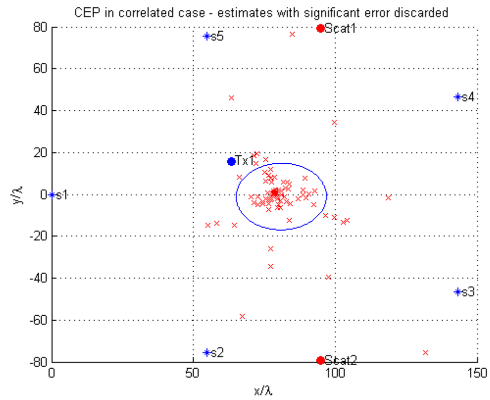

Fig. 4. CEP in the case when the channels between transmitter and each of sensors in array are correlated, when we neglect the position estimates with significant error (outside of sensor array circle).

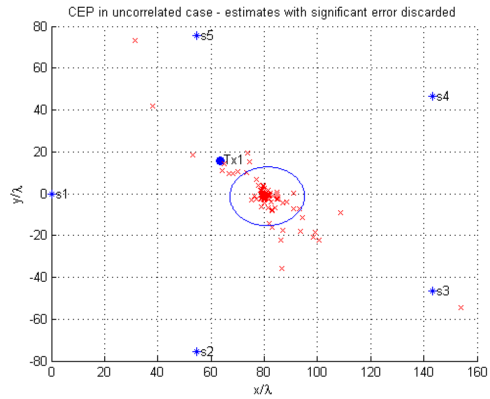

Fig. 5. CEP in the case when the channels between transmitter and each of sensors in array are uncorrelated, when we neglect the position estimates with significant error (outside of sensor array circle).

If we discard the position estimates with significant error, MB OFDM UWB systems are suitable for indoor localization based on TDOA principle.

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