Linear Finite-Difference Schemes for Energy Transport in District Heating Networks

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Abstract. This study evaluates five representative linear explicit finite difference schemes for solving the energy transport equation in district heating networks (DHN). Specifically for the QUICKEST scheme a robust modification to the original formulation is proposed which overcomes the problem originating from different flow situations at pipe junctions. All methods were put into the versatile state space representation for the purpose of simulating the propagation of two benchmark temperature profiles from the DHN plant to the distant consumers. It is shown that the modified QUICKEST scheme produced the most accurate results at the cost of small overshoots in regions of sharp gradients.

Keywords: District Heating Network, Energy Transport, Thermal Problems, Finite-Difference Schemes, QUICKEST

1 Introduction

The purpose of DHN is to provide adequate heat to a number of consumers from a central heat source through a network of pipes carrying hot water or steam. The advantage of such centralized heating production plants is that reducing emissions and limiting pollution is more effectively accomplished than in local installations at the consumers. From an operational planning and operational optimization point of view it is essential to have efficient and accurate simulation models of the thermal transients in the DHN available. Such transients occur during changes of weather conditions, like change of ambient temperature, during start-ups and shut-downs of district heating system operation, or due to the change of heat consumption of consumers [10]. Reducing operational costs may also be managed by keeping heat losses small. This can be achieved by lowering the supply temperature [3], which, on the other hand, implies higher pumping cost due to increased mass flow rates in the network because consumer demands still have to be satisfied. Hence, determining an optimal set of future operating points is highly desirable and real time simulation approaches in a

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model predictive setup play here an essential role.
Due to the complexity of DHN numerical schemes are necessary for predicting the transient thermal behavior. In [8] the node method and the element method are presented as two schemes which are commonly applied to thermal simulations in DHN. The element method was found to be inferior to the node method, both with respect to accuracy and computational cost. The main reason for the poor performance of the element method were found to be problems regarding artificial diffusion, which in turn could result in abnormal smoothing of sharp temperature profiles. The former is based on a first order upwind discretization scheme, whereas the node method keeps trace of how long time a water mass element which currently arrives at the outflow node has been on its way from the inflow node by addressing the mass flow time histories between inflow and outflow nodes. In [5] the node method approach and the commercial software TERMIS were used for predicting the temperature dynamics in the Naestved DHN. Results showed that discrepancies between the predicted and measured temperatures are pronounced for consumers at distant pipelines containing numerous bends and fittings. A new thermal transient approximation is presented in [10], which is based on a numerical scheme of third-order accuracy in space using Lagrange polynomials for interpolation.
In this work the third-order accurate QUICKEST scheme is presented and applied to solving the energy transport equation in case of DHN. Specifically for pipe junctions a robust modification to the original formulation is proposed. Together with four other finite difference schemes the modified QUICKEST is evaluated on two representative temperature profiles. Here, the linearity of the schemes is exploited through embedding into the versatile state space framework. This representation not only allows convenient and efficient real-time thermal simulation of an entire network but also easy implementation into a model predictive setup.

2 Numerical Schemes used for one-dimensional Transport Modelling

The general one-dimensional energy transport equation for a scalar $\phi(x,t)$ in case of pure advection with source term in non-conservation formulation can be written as [1, 7]

$$\frac{\partial \phi}{\partial t} + v\frac{\partial \phi}{\partial x} + S = 0 \tag{1}$$

$$\phi(x,0) = \phi_0(x),\ \phi(0,t) = \phi_i(t) \tag{2}$$

where S is a well defined source term, $v = v(x)$ is the velocity and $\phi_0(x), \phi_i(t)$ represent some boundary conditions. This partial differential equation (PDE) can be solved with finite differences schemes by defining a discrete computational grid in the $x-t$ plane with time increments Δt and spatial increments $\Delta x_{(l,m)}$ between grid points l and m. In the following the notation ϕ_j^n is used where the index j and n denote the discrete mesh points of space and time, respectively.

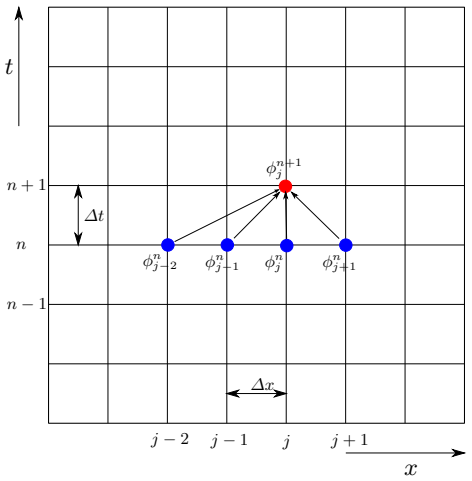

Fig. 1: Schematic representation of the uniform grid structure in space and time (QUICKEST stencil is shown).

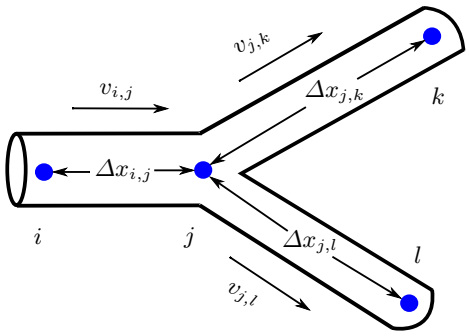

Fig. 2: Representation of a pipe junction in district heating networks assuming non-uniform grid.

3 Linear Finite Difference Schemes

3.1 First Order and Second Order Methods

Simple first order methods are the *first order upwind* (FOU) scheme and *Lax-Friedrichs* scheme (LF). The former can be written as

$$\phi_j^{n+1} = (1-c)\phi_j^n + c\phi_{j-1}^n - \Delta t \bar{S} \tag{3}$$

where $c = c_{j-1,j}$ is the so called Courant-Friedrichs-Lewy (CFL) [4] number, $c = v\Delta t/\Delta x$, describing the flow situation between grid points $(j-1, n)$ and (j, n). The time averaged source term $\bar{S}$ in (3) can be written as $\bar{S} = \frac{a}{2}(\phi_j^n + \phi_j^{n+1})$ with some parameter a. FOU is unconditionally bounded and highly stable, but introduces, due to being only first order accurate, significant amounts of artificial diffusion, which has the effect of smearing the profile [7]. Similarly, the LF scheme can be linearly formulated like the following

$$e\phi_j^{n+1} = d\phi_{j+1}^n + g\phi_{j-1}^n \tag{4}$$

with e, d and g being some coefficients that again depend on the Courant number and the source term parameter. The spatial accuracy of FOU and LF can be improved by choosing a more accurate spatial finite difference stencil. For instance, the *Lax - Wendroff* (LW) (5) results from using centered difference approximations and in the *Beam Warming* (BW) (6) method second order backward differences are applied for the first derivative and central differences for the

second derivative [7].

$$e\phi_j^{n+1} = d\phi_{j+1}^n + f\phi_j^n + g\phi_{j-1}^n \tag{5}$$

$$e\phi_j^{n+1} = f\phi_j^n + g\phi_{j-1}^n + h\phi_{j-2}^n \tag{6}$$

Second order methods are less diffusive compared to first order ones. However, they produce overshoots in regions of sharp gradients [7].

3.2 Third Order Method

QUICKEST: In [6] the so called *QUICKEST* method was proposed, which is third order accurate in the spatial grid size and regarded to be one of the best methods for convective transport [8]. After some rearrangements, the following linear relationship can be obtained

$$e\phi_j^{n+1} = d\phi_{j+1}^n + f\phi_j^n + g\phi_{j-1}^n + h\phi_{j-2}^n \; . \tag{7}$$

The idea of this method is to utilize left and right wall values ($\phi_{i,j}^*$, $\phi_{j,k}^*$) which are approximated by quadratic upstream interpolation.

Modified QUICKEST: As discussed in [8] the original algorithm presented in [6] gives incorrect results at pipe junctions or when diameters are changed. Therefore a slight modification was proposed, but still the obvious dilemma that more than one right wall value exist at pipe junctions was not resolved. If node j (Fig. 2) is used and velocity $v_{j,k}$ is close to zero unwanted unphysical overshootings occur at node j which are further propagated to node l. To overcome this problem a robust modified version of the original scheme is proposed, which is based on velocity weighted right wall values:

$$\phi_j^{n+1} = \phi_j^n + \frac{2\Delta t}{\Delta x_{i,j} + \Delta\tilde{x}_{j,k}} v_{i,j}(\phi_{i,j}^* - \tilde{\phi}_{j,k}^*) - \Delta t\bar{S} \tag{8}$$

$$\tilde{\phi}_{j,k}^* = \frac{\sum_{l=1}^N v_{j,l}\phi_{j,l}^*}{\sum_{l=1}^N v_{j,l}}; \qquad \Delta\tilde{x}_{j,k} = \frac{\sum_{l=1}^N v_{j,l}\Delta x_{j,l}}{\sum_{l=1}^N v_{j,l}} \; . \tag{9}$$

Validation: The modified QUICKEST was applied to a pipe junction with six grid variants (see Fig. 6). Boundary conditions were treated by creating mirror nodes (a, b, c) and using linear extrapolation (cp. [6]). In Table 1 the calculated L2 and weighted L2 error norm [11] for initial step signal and Gaussian signal with $\sigma = 200\,\mathrm{s}$ are shown. Clearly, finer spatial grid implies smaller error, but requires more computing time and computer memory. It was found that in general uniform grid gives good results and additional grid points close to the upstream boundary seems appropriate. Note that with fixed Δt a lower bound regarding the spatial discretization is given due to the *von Neumann* stability criterion. All presented numerical schemes are considered stable for $0 < c < 1$ [2, 6]. As discussed in [12], the computing time is approximately proportional to the square of the total number of grid points, because in order to keep the Courant number less than one, the time step size has to be reduced too.

Table 1: L2 and weighted L2 error for two benchmark signals.

		Grid					
		I	II	III	IV	V	VI
Step Signal	ϵ_{L2}	2.14	1.11	0.692	1.01	0.897	0.616
	ϵ_{L2_w}	2.10	1.07	0.677	0.996	0.917	0.616
Gaussian Signal	ϵ_{L2}	1.48	0.226	0.0337	0.204	0.0565	0.0423
	ϵ_{L2_w}	1.40	0.200	0.0282	0.190	0.0497	0.0423

4 Case Study: Energy Transport in DHN

In the special case of energy transport in DHN, (1) can be restated as [9]

$$\frac{\partial T}{\partial t}(x,t) + \frac{4\dot{m}(t)}{\rho D^2 \pi}\frac{\partial T}{\partial x}(x,t) + \frac{4h}{c_p \rho D}(T(x,t) - T_g) = 0 \tag{10}$$

with $\dot{m}$ [kg/s] denoting the mass flow through the specific pipe segment, D [m] the diameter and ρ [kg/m^3] the relative density of water. Here, the source term can be interpreted as heat losses to the surroundings which are composed of the heat transfer coefficient h [W/m^{2}K], the specific heat of water c_p [J/kgK] and the ground temperature T_g [K]. For simulation purposes a generic DHN

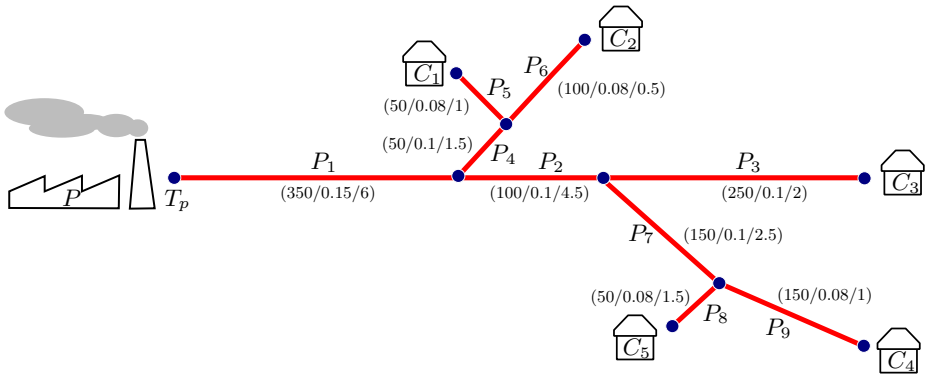

Fig. 3: Generic district heating network. The term in brackets are length in m / diameter in m / mass flow rate in kg/s.

as depicted in Fig. 3 was designed. For instance, each house could represent a substation or an entire appropriately aggregated subnetwork. For the mass flow rates only a constant steady flow situation was considered. This is reasonable since the dynamics of the flow in the network can be neglected in pure thermal simulations due to highly different response times [10]. It is also assumed that

heat losses were zero during the simulations. All numerical schemes were put into state space form (11) and implemented in MATLAB version 7.10. For the spatial discretization 50 m and for the temporal grid size 60 s was chosen.

$$\begin{aligned} \mathbf{T}^{n+1} &= \mathbf{A}(\dot{\mathbf{m}})\mathbf{T}^n + \mathbf{B}T^n_{Plant} \\ \mathbf{T}^n_{Con} &= \mathbf{C}\mathbf{T}^n \end{aligned} \tag{11}$$

The benchmark tests comprise two different plant inlet temperature profiles, namely a double Gaussian hill and a unit step (cp. [12]). The former should serve as an indicator on how a numerical scheme can resolve the interference of various peaks and valleys. The unit step, on the other hand, represents a discontinuity in the temperature profile. As expected, the first order methods give very smeared

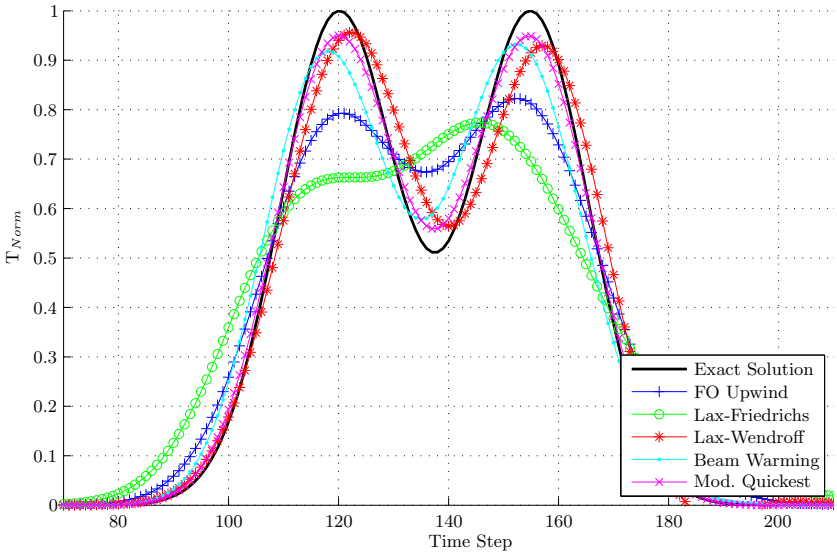

Fig. 4: Double Gaussian hill temperature profiles at consumer 4.

solutions compared to the higher order methods. Figure 4 also reveals that LW and BW produce phase errors at the double Gaussian signal, whereas the mod. QUICKEST performs best here. The step signal in Fig. 5 illustrates the typical behavior of higher order methods, that is producing unphysical oscillations near discontinuities. To overcome this phenomenon non-linear schemes are necessary [7]. Also first order schemes are free from under and over estimations, but show strong artificial diffusion effects.

5 Conclusion and Outlook

In this work a modification of the well-known QUICKEST scheme was proposed and together with four other finite difference methods successfully applied to

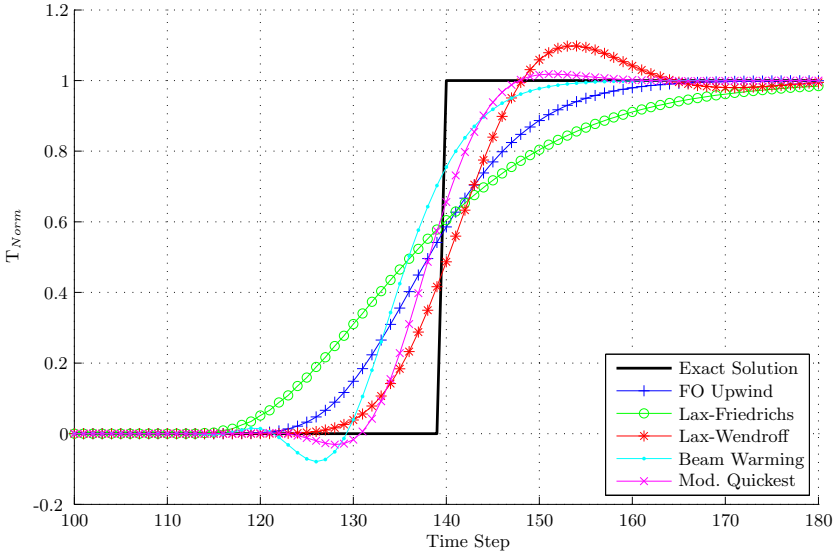

Fig. 5: Unit step temperature profiles at consumer 4.

thermal modelling of a district heating system. It was found that QUICKEST produces superior results at the cost of small unphysical overshoots in regions of sharp gradients. One main purpose of solving the energy transport equation with linear explicit schemes is the straightforward embedding into the model predictive control framework. State space formulations such as given in (11) allow to simulate an entire DH network as a first order difference equation. Forecasted temperature values can be recursively obtained through (const. $\dot{m}$)

$$\hat{\mathbf{T}}^{n+H_p} = \mathbf{A}^{H_p}\mathbf{T}^n + \mathbf{A}^{H_p-1}\mathbf{B}\hat{T}^n_{Plant} + \cdots + \mathbf{B}\hat{T}^{n+H_p-1}_{Plant} \ . \tag{12}$$

Thus, these predicted values could be utilized to determine optimal supply temperature trajectories in real time by minimizing some error criterion in a model predictive control setup. Detailed research in this direction will be attempted in future publications.

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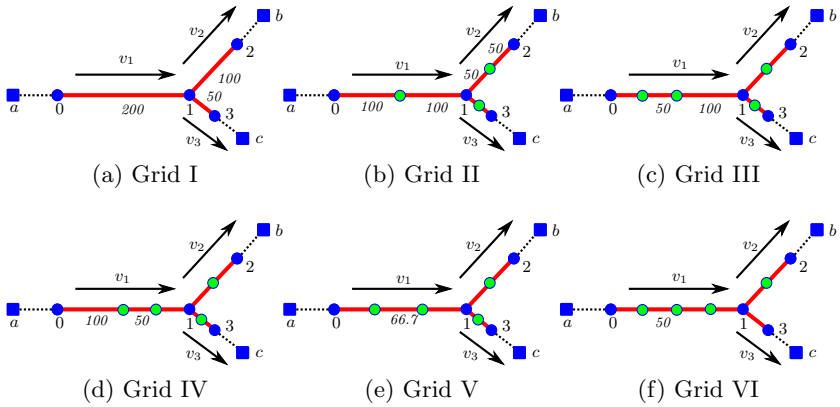

(a) Grid I (b) Grid II (c) Grid III

(d) Grid IV (e) Grid V (f) Grid VI

Fig. 6: Grid variants used for testing modified QUICKEST; $v = (1, 1.5, 0.5)$m/s.