Instabilities in the Quark-Gluon Plasma *

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*“Dans la vie, rien n’est à craindre, tout est à comprendre.” Marie Curie
Overview

Quark-gluon Plasma
- Early Universe
- Heavy Ion Collision
- QCD Phase diagram
- QGP signatures

Hard Expanding Loops (HEL)
- Momentum Anisotropy
- Weibel instabilities
- Scales QGP
- Hard (Thermal) Loops - Boltzmann - Vlasov
- Notations for Bjorken expansion
- Hard-Expanding-Loop formalism

Plasma Instabilities
- Expanding 1D+3V Abelian plasma
- Expanding 3V plasma
- Conclusions
History of the Universe

Key:
- W, Z bosons
- photon
- quark
- meson
- gluon
- electron
- baryon
- muon
- tau
- neutrino
- galaxy
- ion
- star
- atom
- black hole

Particle Data Group, LBNL, © 2008. Supported by DOE and NSF
Relativistic Heavy Ion Collider (RHIC)

Quark-gluon Plasma
Early Universe
Heavy Ion Collision
QCD Phase diagram
QGP signatures
Hard Expanding Loops (HEL)
Plasma Instabilities

Au+Au ions $\sqrt{s_{NN}} = 200\text{GeV/nucleon pair}$, p+p, d+A
QCD Phase diagram

Schematic QCD phase diagram
QGP signatures

- Small viscosity (elliptic flow $v_2$)
- Jet quenching
- Experimental observation of $T > T_c$
- High $p_T$ suppression of hadrons (for central collisions)
- Rapid thermalization:
  estimates from hydrodynamical computation $\sim 1 \text{fm/c}$
Elliptic flow

Pressure gradients generate positive elliptic flow $v_2$

$$\frac{d^2 N}{d\phi dp_T} = N_0 (1 + 2v_2(p_T)\cos(2\phi) + ..) \quad (1)$$

The elliptic flow is quantified by the anisotropy of particle production with respect to the reaction plane $v_2 = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle$

Illustration of the reaction plane definition.
PHOBOS data on Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, compared to hydrodynamic model for various $\eta/s$ ratios.

P. Romatschke, U. Romatschke hep-th/0706.1522
Jet Quenching

Single CMS event Pb-Pb collision at $\sqrt{s_{NN}} = 2.76\, \text{TeV/nucleon pair}$
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Early conditions at RHIC - T

\[ T = 300 - 600 \text{MeV} > 2 \times T_c \]
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Momentum Anisotropy

Expansion rate and isotropization via interactions balance

Expansion rate is much faster than the interaction time scale
\( 1/\tau \gg 1/\tau_{\text{int}} \)

System is momentarily isotropic

Momentum space anisotropy time dependence at the early stages of a heavy ion collision
Weibel instabilities

Illustration of the mechanism of filamentation instabilities.
Filaments and active solar region from NASA’s Solar Dynamics Observatory
Scales of weakly coupled QGP

- $T$: energy of hard particles

- $gT$: thermal masses, Debey screening mass, Landau damping, plasma instabilities [Mrowczynski 1988, 1993, ..]

- $g^2T$: magnetic confinement, color relaxation, rate for small angle scattering

- $g^4T$: rate for large angle scattering, $\eta^{-1}T^4$
Hard (Thermal) Loops - Boltzmann - Vlasov

With color-neutral background distribution \( v \cdot \partial f_0(p, x, t) = 0 \), \( v^\mu = p^\mu/p^0 \) gauge covariant Boltzmann-Vlasov:

\[
v \cdot D \partial f_a(p, x, t) = g v_\mu F_\mu^\nu \partial_\nu (p) f_0(p, x, t) = -g(E_a + v \times B_a) \cdot \nabla_p f_0, \quad (2)
\]

\[
D_\mu F_\mu^\nu = j_\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(p, x, t). \quad (3)
\]

So far mostly stationary \( f_0(p) \) with \( \partial_\mu f_0 \equiv 0 \)

- isotropic: \( f_0(p) = f_0(|p|) \), \( \nabla_p f_0 \propto v \)

\[
v \cdot D \delta f_a(p, x, t) = -gE_a \cdot \nabla_p f_0 \quad (stable) \quad (4)
\]

- anisotropic: \( f_0(p) \), \( \nabla_p f_0 \not\propto v \)

\[
v \cdot D \delta f_a(p, x, t) = -g(E_a + v \times B_a) \cdot \nabla_p f_0 \quad (unstable!) \quad (5)
\]
Discretized Hard Loop Effective Theory

**Auxiliary field formulation:** [Mrowczynski, Rebhan & Strickland 2004]

\[ \delta f^a(x; p) = -g W^a_\mu(t, x; v) \partial^\mu(p) f_0(p) \]  

(6)

\[ [v \cdot D(A)] W_\mu(x; v) = F_{\mu\gamma}(A) v^\gamma \]  

(7)

where \( v^\mu \equiv p^\mu/|p| = (1, v) \)

\[ j^\mu(x) = -g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{2|p|} p^\mu \frac{\partial f(p)}{\partial p^\nu} W^\nu(x; v), \]  

(8)

**Hard Loop effective theory:** (hard) scale \(|p|\) integrated out for real-time lattice simulation: discretize also velocity space in "disco balls"

\[ D_\sigma(A) F^{\sigma\mu} = j^\mu(x) = \frac{1}{N} \sum_v v^\mu \mathcal{W}_v(x) \]  

(9)
It is convenient to switch to comoving coordinates

\[ t = \tau \cosh \eta, \quad \beta = \tanh \eta, \]
\[ z = \tau \sinh \eta, \quad \gamma = \cosh \eta, \quad (10) \]

i.e., a coordinate system with metric \( ds^2 = d\tau^2 - dx^2_{\perp} - \tau^2 d\eta^2 \).

We introduce the notation

\[ \tilde{x}^\alpha = (x^\tau, x^i, x^\eta) = (\tau, x^1, x^2, \eta) \quad (11) \]

with indices from the beginning of the Greek alphabet for these new coordinates. In addition to space-time rapidity \( \eta \), we also introduce momentum space rapidity \( y \) for the massless particles according to

\[ p^\mu = p_{\perp} (\cosh y, \cos \phi, \sin \phi, \sinh y). \quad (12) \]
Hard-Expanding-Loop formalism

\[
\text{With } \quad p^\beta \partial_\beta \left[ \partial^\alpha_{(p)} f_0(p_{\perp}, p_\eta) \right] \bigg|_{p^\mu = \text{const.}} = 0 \quad (13)
\]

we can commute \( p \cdot D \) and thus solve gauge-covariant Vlasov equation in comoving coordinates

\[
p . D \delta f_a(p, x, t) \bigg|_{p^\mu = \text{const.}} = g p^\beta F^a_{\beta \alpha} \partial^\alpha_{(p)} f_0(p, x, t). \quad (14)
\]

Introducing auxiliary fields \( W^a_\alpha(\tau, x^i, \eta; \phi, y) \) similar to the auxiliary field \( W^\nu(x, v) \) of the hard-loop formalism

\[
\delta f^a(x; p) = -g W^a_\alpha(\tau, x^i, \eta; \phi, y) \partial^\alpha_{(p)} f_0(p_{\perp}, p_\eta) \quad (15)
\]

that obey

\[
v \cdot D W^a_\alpha(\tau, x^i, \eta; \phi, y) \bigg|_{\phi, y} = v^\beta F^a_{\alpha \beta} \quad (16)
\]

where \( v^\alpha \equiv \frac{p^\alpha}{|p_{\perp}|} = \left( \cosh(y - \eta), \cos \phi, \sin \phi, \frac{\sinh(y - \eta)}{\tau} \right) \).
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Expanding 1D+3V Abelian plasma

The proper-time evolution of the canonical field momentum of a single Abelian mode.
The proper-time dependence of the chromo-field energy densities and the energy gain rate times an extra factor of $\tau_0$ resulting from non-Abelian run initialized with Fukushima, Gelis, and McLerran (FGM) initial conditions.
Expanding 1D+3V non-Abelian plasma

The comparison of the longitudinal and transverse pressures for the fields and particles resulting from a typical non-Abelian run initialized with FGM (CGC inspired) initial conditions.
Expanding 1D+3V Abelian plasma

Fourier spectrum of the color-traced conjugate field momentum obtained from Abelian run with FGM initial conditions.
Expanding 1D+3V non-Abelian plasma

Fourier spectrum of the color-traced conjugate field momentum obtained from non-Abelian run with FGM initial conditions.
Expanding 3D+5V plasma

Preliminary runs from the HEL 3d codes in Abelian and non Abelian setup with different lattice sizing's in the longitudinal $\eta$ direction, but identical transverse size and $\mathcal{W}$ auxiliary field numbers.
Unstable transverse modes

Influence of different initial conditions for a specific mode with $\nu = 30$
Visualization of the space-time development of color correlations in a non-Abelian plasma instabilities in Bjorken expansion.
Conclusions

Non-abelian plasma instabilities accelerate isotropization and thermalization of the Quark Gluon Plasma.

Large amplitude turbulent field configurations can have an important effect on Quark Gluon Plasma transport such as momentum broadening, energy loss, plasma viscosity, ...

In the 1D+3V Hard Expanding Loop (HEL) 1D we found that the exponential (in $\sqrt{\tau}$) growth in the Abelian (weak-field) phase is only mildly weakened when nonlinearities through non-Abelian self-interactions of the collective fields set in.

The previous 1D HEL code has been extended to full 3D+5V. Final results including different initial conditions are being computed.
Thank you.
Conjugate Momenta

\[
\partial_\tau E_i = +\tau j^i + \frac{1}{\tau} D^2_\eta A^i + \tau g^2 i[A^{j\neq i}, i[A^{j\neq i}, A^i]] 
\] (17)

\[
\partial_\tau E^\eta = -\tau j^\eta + \frac{ig}{\tau} [A^i, D_\eta A^i] 
\] (18)

Gauss law

\[
j^\tau = +\frac{1}{\tau} D_\eta E^\eta - \frac{ig}{\tau} [A_i, E^i] 
\] (19)

with

\[
E^i \equiv \tau \partial_\tau A_i, \quad E^\eta \equiv \frac{1}{\tau} \partial_\tau A_\eta
\] (20)
The proper-time dependence of the chromo-field energy densities from a run with a single non-Abelian mode seeded with random noise.
The proper-time dependence of the chromo-field energy densities with a single non-Abelian mode with decoupled hard particle currents ($j = 0$).
The proper-time dependence of the longitudinal and transverse pressure with a single non-Abelian mode with decoupled hard particle currents ($j = 0$).