Interference Alignment over Partially Connected Interference Networks: Application to the Cellular Case

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Abstract—We consider the application of interference alignment (IA) to the cellular case with constant MIMO channel coefficients. Neglecting the weakest interferers in the network, we introduce an algorithm to obtain a partially connected (i.e., L-interfering) network, over which the feasibility of IA can be analyzed simply. We consider the application of IA over the obtained L-interfering network, while the rest of the interference is treated as noise. We benchmark this technique through extensive Monte-Carlo simulations, and introduce an approximate, compact expression for the ergodic mutual information achievable in the considered network setting.

I. INTRODUCTION

Interference alignment (IA) for the interference channel (IC) was introduced in [1], where it was shown to have the potential to maximize the achieved multiplexing gain in the system. The existence of an IA solution in the constant IC depends on the dimensions involved (i.e., number of aligned interferers, of independent data streams and of transmit/receive antennas), and was studied in [2], [3].

Attempts have been made to apply IA to cellular networks, e.g., [4] considered alignment inside spatial clusters of users, while inter-cluster interference was still present. In [5], alignment was proposed between intra-cell interference and inter-cell interference, however, the results do not generalize to the case of many interfering cells with constant MIMO channels. All of the above results have in common the fact that IA is considered over a fully connected IC. In this context, IA can be applied between an arbitrary number of users only if the signalling dimensions are allowed to grow to infinity. Partially connected (PC) networks, i.e., networks where some of the interference channels are assumed to have zero gain, have been considered in [6], where an achievable scheme is proposed for a limited subset of problem dimensions. In [7], IA over a partially connected model similar to the 1-dimensional Wyner model is considered. However, these results do not generalize to arbitrary topologies of the interference connections, and are therefore not applicable to cellular networks. [8] introduced the L-interfering condition on the topology of the interference connections, under which the question of IA feasibility is addressed by providing a sufficient condition for the system of IA equations to be proper.

The L-interfering IC from [8] is a desirable model for the cellular case since (i) it encompasses the case of the 2-dimensional Wyner model [9], and therefore is arguably fairly realistic, (ii) IA feasibility over such networks can be assessed simply (through the notion of properness) as a function of the problem dimensions, and (iii) it is applicable to arbitrarily large networks, since it enables IA among an infinite number of users, while using only a finite number of signalling dimensions.

The object of this article is to apply the results from [8] to a realistic cellular model. Our contributions are the following:

- We introduce a technique to map a (fully connected) cellular network with fading channels (including path-loss) into a PC network by neglecting the weak interference links, while guaranteeing the L-interfering property of the obtained PC network.
- We study the performance achieved by applying IA over the obtained PC network while treating the remaining interference as noise through Monte-Carlo simulations.
- We establish an approximate, compact expression for the ergodic mutual information achievable through IA in the network scenario at hand.

A key difference between the results herein and our previous work [8] is that the method proposed in this paper allows clusters of users doing IA to overlap, which was not allowed [4], resulting in possibly strong inter-cluster interference.

Notation: The conjugate transpose of a matrix \(A\) is denoted \(A^H\), \(\text{tr}(A)\) denotes its trace, and \(|A|_F\) is the Frobenius norm. \(\parallel \cdot \parallel\) is either the magnitude of a scalar, the determinant of a matrix or the cardinality of a set, depending on the argument used. The \(d\)-dimensional identity matrix is denoted \(I_d\).

II. CELLULAR MODEL

We consider a cellular network with a total of \(K\) cells, based on a homogeneous layout of hexagonal cells with radius \(R\). Access points (AP) lie on a regular grid and there is one user equipment (UE) per cell, which is uniformly distributed and associated with the AP at the center of its cell. The distance from AP\([k]\) to UE\([l]\) is denoted by \(r_{[kl]}\). A minimum distance of an associated AP-UE pair \(r_{[kl]} \geq r_{\text{min}}\) is assumed. All AP-UE pairs use the same resources. Only the downlink is investigated, where signals from the APs contribute interference to the non associated UEs. For the sake of simplicity, we restrict our analysis to a symmetric system: All APs and UEs have the same number \(M\) and \(N\) of antennas, respectively, and \(d\) beams are communicated over each link. The available transmit power per AP is denoted by \(P_{AP}\).
The $d$-dimensional signal of interest from $\text{AP}^k$, denoted $\mathbf{x}^k$, has i.i.d. circularly symmetric complex Gaussian distributed elements $\sim \mathcal{CN}(0, \mathbf{Q}_X^k)$ and the transmit power $P_{\text{AP}}$ is evenly split over the $d$ beams, i.e., $\mathbf{Q}_X^k = \frac{P_{\text{AP}}}{d} \mathbf{I}_d$.

The transmit signal from any $\text{AP}^k$ to $\text{UE}^l$ is impaired by flat fading and attenuation due to the environment, modelled as Rayleigh fading with path loss: Let $\mathbf{G}^l$ be a channel matrix with i.i.d. circularly symmetric complex Gaussian $\sim \mathcal{CN}(0,1)$ distributed elements. Taking also the path loss into account, the channel is $\mathbf{H}^l = \sqrt{\beta r^{-\alpha}} \mathbf{G}^l$, with $\alpha$ and $\beta$ depending on the channel model at hand.

Besides receiving signals from all APs, $\text{UE}^l$ is affected by an $N$-dimensional additive circularly symmetric complex Gaussian noise term $\mathbf{w}^l \sim \mathcal{CN}(0, \mathbf{Q}_N^l)$ with $\mathbf{Q}_N^l = \sigma_w^2 \mathbf{I}_N$.

At $\text{AP}^k$, an $(M \times d)$ precoding matrix $\mathbf{V}^k$ is used for beamforming. The resulting $N$-dimensional received signal $\mathbf{y}^l$ is given by

$$\mathbf{y}^l = \sum_{k=1}^{K} \mathbf{H}^l \mathbf{V}^k \mathbf{x}^k + \mathbf{w}^l \quad (1)$$

III. PARTIALLY CONNECTED NETWORKS AND INTERFERENCE ALIGNMENT

In a large cellular network, it is clear that the power of interference received from a distant transmitter vanishes due to the path loss term which decreases exponentially with the distance. This motivates us to approximate the considered cellular model by a PC network model. In particular, we consider the L-interfering $K$ user MIMO IC from [8], defined as follows: A maximum of $L$ APs out of $K-1$, which all interfere with $\text{UE}^l$, are indexed by the elements of the set $I(l) \subset \{1, \ldots, l-1, l+1, \ldots, K\}$. We also denote $I^{-1}(k) = \{i : k \in I(i)\}$, which contains the indices of the UEs which suffer from interference from $\text{AP}^k$. The L-interfering network model is used to implement interference alignment. Note that the PC model is merely an approximation of the fully connected case, and that non aligned (but hopefully weak) interference is still present in the network, conversely to the assumption in the two-dimensional Wyner model [9], where the interference from distant base stations is strictly zero.

A. Establishment of the L-interfering Network

Since the question of how to choose the index sets $I(\cdot)$ and $I^{-1}(\cdot)$ in a fair manner for the whole network is of particular importance, we introduce two signal to interference ratio (SIR) metrics for every $\text{AP}^k \cdot \text{UE}^l$ pair:

1) The instantaneous SIR, based on the channel norms,

$$\text{SIR}_{\text{in}}^{lk} = \frac{\mathbf{H}^l \mathbf{F}^{\dagger}_k}{||\mathbf{H}^l||^2_F} \forall k, l = 1 \ldots K, k \neq l.$$ (2)

2) The average SIR, based on the distances,

$$\text{SIR}_{\text{av}}^{lk} = \frac{\mathbf{E}_{\mathbf{R}}[\mathbf{H}^l \mathbf{F}^{\dagger}_k]}{\mathbf{E}_{\mathbf{R}}[||\mathbf{H}^l||^2_F]} = \left[\frac{\mathbf{R}^{lk}}{\mathbf{R}^{ll}}\right]^{-\alpha} \forall k, l = 1 \ldots K, k \neq l.$$ (3)

Each of the $K(K-1)$ elements in the set $\mathcal{S} = \{(l,k)\} | k = 1 \ldots K, k \neq l$ is used to index the SIR of a given interference link. The following greedy algorithm (Algorithm 1) builds an L-interfering network based on the partial SIRs defined above, by selecting first the links that have the lowest SIRs. Interference within this PC network will be mitigated through IA. We assume that the algorithm is executed by a central authority having full knowledge of the channel state. Note that

Algorithm 1 Defining index sets $I(\cdot)$ and $I^{-1}(\cdot)$

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while $\mathcal{S} \neq \{\}$ do
   (l, k) = arg min \text{SIR}_{\text{av}}^{lk}
   if $|I(l)| \leq L$ then
      $\mathcal{S} = \mathcal{S} \setminus \{(l, k)\}$
   else
      $I(l) = I(l) \cup \{k\}$
      $\mathcal{S} = \mathcal{S} \setminus \{(l, k)\}$
   end if
end while
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Algorithm 1 sometimes leads to a scenario where fewer than
L APs align their interference at a certain UE (i.e., $|I(l)| < L$ for some $l$). In particular this can happen when the signal from the desired link is very strong ($SIR^{[lk]}$ is high $\forall k$). In that case, it is intuitively not necessary to mitigate through IA the interference received by this user, and therefore the interference links arriving at user $l$ are not accounted for in the L-interfering network constructed by the proposed algorithm.

B. Spatial Interference Alignment

Interference alignment in the partially connected network is achieved if $(M \times d)$ channel precoding and $(N \times d)$ interference suppression matrices $V^{[k]}$ and $U^{[l]}$ can be found such that

$$U^{[l]H}H^{[lk]}V^{[k]} = 0 \quad \forall l \in 1 \ldots K, k \in I(l).$$

(4)

If a solution to the system of equations (4) exists, it is said to be feasible. In order to analyze the feasibility of IA, [2] introduced the concept of properness (meaning that the number of variables is at least as large as the number of equations for every subset of the equations). Feasibility is guaranteed almost surely in several important particular cases for proper systems (generic channel matrices, symmetric system, all users transmit $d$ streams and both $M$ and $N$ are divisible by $d$, see [3]). A sufficient condition for properness in the PC case was formulated in [8]. Adopted to the symmetric scenario at hand, the constraints that arise are given by

$$M + N - (L + 2)d \geq 0, \quad d \leq \min(M, N), \quad 2d \leq \max(M, N).$$

(5)

With IA, the received signal $y^{[l]}$ in (1) can be projected into a subspace free of interference from the $L$ aligned APs in the set $I(l)$. Thus, let us introduce the $d$-dimensional projected signal $y^{[l]}_p$ at UE $l^{[l]}$ as

$$y^{[l]}_p = U^{[l]H}y^{[l]} = \sum_{k \in I(l)} U^{[l]H}H^{[lk]}V^{[k]}x^{[k]} + U^{[l]H}w^{[l]}.$$  (6)

The $(N \times N)$ covariance matrix $Q^{[l]}_y$ of $y^{[l]}$ and the $(d \times d)$ covariance matrix $Q^{[l]}_y$ of $y^{[l]}_p$ conditioned on all involved channel matrices can be written respectively as

$$Q^{[l]}_y = Q^{[l]}_S + Q^{[l]}_Y + Q^{[l]}_I + Q^{[l]}_W$$

and

$$Q^{[l]}_y = U^{[l]H}Q^{[l]}_y U^{[l]}.$$  (7)

(8)

$$Q^{[l]}_y = U^{[l]H}\left(Q^{[l]}_S + Q^{[l]}_I + Q^{[l]}_W\right)U^{[l]},$$

where $Q^{[l]}_S$ accounts for the signal of interest, $Q^{[l]}_I$ for the $L$ aligned interferers and $Q^{[l]}_W$ for the residual interferers which are treated as noise, and are given by

$$Q^{[l]}_S = H^{[lk]}V^{[l]}Q^{[l]}_X V^{[l]H}H^{[lk]H}$$

and

$$Q^{[l]}_I = \sum_{k \in I(l)} H^{[lk]}V^{[k]}Q^{[k]}V^{[k]H}H^{[lk]H}$$

$$Q^{[l]}_W = \sum_{k \notin I(l), k \neq l} H^{[lk]}V^{[k]}Q^{[k]}V^{[k]H}H^{[lk]H}.$$  (10)

(11)

(12)

Note that (9) holds because $U^{[l]H}Q^{[l]}_I U^{[l]} = 0$ thanks to (4).

IV. ERGODIC MUTUAL INFORMATION AS PERFORMANCE METRIC

The mutual information in (bits/s/Hz) of a single AP$^{[l]}$ - UE$^{[l]}$ pair, for both received signals (1) and (6), conditioned on all involved channel matrices, is [10]

$$I(x^{[l]}, y^{[l]}; H^{[lk]}_{k,l=1\ldots K}) = \log_2 \left| Q^{[l]}_{y} \left( Q^{[l]}_{Y} - Q^{[l]}_{S} \right)^{-1} \right|$$

$$I(x^{[l]}, y^{[l]}_p; H^{[lk]}_{k,l=1\ldots K}) = \log_2 \left| Q^{[l]}_{y_p} \left( Q^{[l]}_{Y_p} - U^{[l]H}Q^{[l]}_{S} U^{[l]} \right)^{-1} \right|.$$  (13)

(14)

Since both terms are affected by the random channels, taking the expectation with respect to all channel matrices leads to the following respective ergodic mutual information values:

$$\ell^{[l]} = E_{H^{[lk]}_{k,l=1\ldots K}} \left\{ I(x^{[l]}, y^{[l]}; H^{[lk]}_{k,l=1\ldots K}) \right\},$$

$$\ell^{[l]} = E_{H^{[lk]}_{k,l=1\ldots K}} \left\{ I(x^{[l]}, y^{[l]}_p; H^{[lk]}_{k,l=1\ldots K}) \right\}.$$  (15)

(16)

A. Analytic Result

Let $C_{SU}(M, N, \Phi)$ denote the ergodic mutual information in (bits/s/Hz) in the single user MIMO case with $M$ transmit and $N$ receive antennas,

$$C_{SU}(M, N, \Phi) = E_{H^{[l]}} \left( \log_2 \left| I_N + H\Phi H^{[l]} \right| \right)$$

(17)

where $\Phi$ is a positive definite covariance matrix and the elements of $H$ are i.i.d. circularly symmetric complex Gaussian $\mathcal{CN}(0,1)$. Expression (17) can be evaluated in closed form also for arbitrary multiplicities of the eigenvalues of $\Phi$ [11], accounting for possibly different transmit powers at different antennas. Following [11], one can derive the ergodic mutual information of a single AP$^{[l]}$. - UE$^{[l]}$ pair in the symmetric $K$ user MIMO interference channel case in the absence of IA as

$$C_{MU,K} = C_{SU}(K M, N, \hat{\Theta}) - C_{SU}((K-1)M, N, \Theta).$$  (18)

The first term in (18) accounts for the relative entropy of all received signals including the desired one, while the second term accounts for the relative entropy of noise and interference only, $\hat{\Theta}$ and $\Theta$ are square block-diagonal matrices of dimension $K M$ and $(K-1)M$, respectively, where each $M \times M$ diagonal block is the transmit covariance of one user in the system.

We can apply (18) to evaluate $\ell^{[l]}$ for the IA case with projected receive signal by noticing that the equivalent channel matrices $U^{[l]H}H^{[lk]}V^{[k]}$ in (6) are Gaussian i.i.d. circularly symmetric with unit variance for $k \notin I(l)$, since $U^{[l]}$ and $V^{[k]}$ are unitary. Furthermore, the coefficients of the projected noise $U^{[l]H}w^{[l]}$ are Gaussian i.i.d. with variance $\sigma^2_w$. Taking the effective channel dimensions $(d \times d)$ in (6) into account, we have

$$\ell^{[l]} = C_{SU}((K - L)d, d, \tilde{\Psi}_l) - C_{SU}((K - L - 1)d, d, \Psi_l)$$

(19)

with

$$\tilde{\Psi}_l = \text{diag} \left( \frac{P_{\omega} \beta_{\omega} \tau_{\omega}}{d \sigma^2_w} I_d \right)_{k \in I(l)}$$

$$\Psi_l = \text{diag} \left( \frac{P_{\omega} \beta_{\omega} \tau_{\omega}}{d \sigma^2_w} I_d \right)_{k \in I(l), k \neq l}.$$  (20)

(21)
V. MONTE-CARLO SIMULATIONS

Let us consider a cell in the center of a network with $K = 56$ hexagonal cells (in order to mitigate border effects) while the channel and the user position inside a cell are random. We evaluate the ergodic mutual information by means of a Monte-Carlo simulation (averaging over 1000 realizations) as a function of the noise variance $\sigma_w^2$, while comparing the performance depending on:

- the type of receiver (with or without projection on the interference-free subspace, respectively associated with metrics $\iota$ and $\iota_p$),
- the maximum number $L$ of APs that align interference,
- the path loss exponent $\alpha$,
- the SIR metric used in Algorithm 1, $SIR_{av}$ and $SIR_{in}$.

We assume that the transmit power of each AP is $P_{AP} = 15$ mW per carrier. The number of antennas at each UE is $N = 2$ and the number of antennas at each AP increases with the number of interferers $L$ to be aligned in order to fulfill (5). The number of transmitted beams per AP-UE pair is $d = 1$. The cell radius is $r = 1000$ m, while a minimum AP-UE distance $r_{\text{min}} = 10$ m is enforced. The path loss constants are $\alpha = 3.76$ and $\beta = 10^{-1.53}$ with distance $r$ in (m).

The iterative algorithm from [12] was used in this work to solve (4) through alternatively minimizing leakage interference metrics of the type $\sum_{l=1}^{K} \lambda[l]$ with $\lambda[l] = \frac{\text{tr}(U[l]^H Q[l] U[l])}{H[l]}$ over $V[k]$ and $U[l]$ for $l \in \{2, 4, 6, 8\}$, as shown in Figure 2. Remarkable is the much better performance of $\iota$ compared to $\iota_p$. This interference alignment aims only for creating an interference-free subspace, without beamforming the desired signal into this space, the projection comes along with a loss of desired signal power. Better performance of $\iota$ compared to $\iota_p$ was also reported in [13] for the low $SNR$ regime, which is the case here since the not aligned interferers are treated as noise. As can be seen in this graph, the increase of the mutual information is quite limited for increasing number of aligned interferers.

A. Varying Number of Aligned Interferers

The dependence of the ergodic mutual information achieved in the center cell on the maximum number of APs which align their interference at each UE was studied for $L \in \{2, 4, 6, 8\}$, as shown in Figure 2. Since interference alignment aims only for creating an interference-free subspace, without beamforming the desired signal into this space, the projection comes along with a loss of desired signal power. Better performance of $\iota$ compared to $\iota_p$ was also reported in [13] for the low $SNR$ regime, which is the case here since the not aligned interferers are treated as noise. As can be seen in this graph, the increase of the mutual information is quite limited for increasing number of aligned interferers.

B. Varying Path Loss Exponent

The dependence of the ergodic mutual information on the path loss exponent $\alpha \in \{2.76, 3.76, 4.76, 5.76\}$ was studied. Following intuition, the mutual information is larger when the path loss is strong in the high $SNR$ regime. Since the desired signal is also affected by the path loss exponent, the curves get shifted horizontally to the right for increasing path loss.

C. Instantaneous Versus Average SIR Metric

The two metrics $SIR_{av}$ and $SIR_{in}$ that determine index sets $I(\cdot)$ and $I^{-1}(\cdot)$ through Algorithm 1 are compared for different values of aligned interferers and path loss exponents in Figure 4. The performance differences for $\iota_p$ only. Since the results are very similar for both metrics, the much higher
complexity for choosing the index sets of interferers based on the instantaneous $SIR_{in}$ seems not to be worthwhile.

VI. COMPARISON OF MONTE-CARLO SIMULATION AND ANALYTIC RESULT

We use the same system parameters as in Section V, now for both the iterative and the analytic result. Since accurate numeric evaluation of (19) is difficult to implement for large differences in the received powers, we restrict to the case with $K = 19$. With reference to Figure 1, only the second ring around a center UE is considered to be the interferers in the analytic result, namely the 12 APs, while APs are assumed to be perfectly aligned in the numeric simulation and therefore not taken into account in (19). Consequently, also the index set $I(1) = \{2 - 7\}$ is fixed in the numeric simulation. Furthermore, in this simulation, the distance of the desired link $r_{[11]}$ is assumed to be fixed.

In order to make use of the multiplicities of the eigenvalues of $\Psi$ and $\tilde{\Psi}$ as proposed in [11], we approximate the distances between APs to UE with the distance to AP. The resulting metric is denoted by $C_{MU}$.

Figure 5 shows the ergodic mutual information $\iota, \iota_p$ (evaluated through Monte-Carlo simulations) and $C_{MU}$ achieved in the center cell depending on the fixed distance of UE to AP. As can be seen, the curves corresponding to $\iota_p$ and $C_{MU}$ match very well.

VII. CONCLUSION

We considered the alignment of interference from a limited number of transmitters in a cellular network of arbitrary size. An algorithm was introduced to approximately map the cellular network into an L-interfering partially connected network. The performance of IA in this context was evaluated through both Monte-Carlo simulations and through approximation by a compact analytical formula.