

# Uplink Interference Alignment for OFDM Systems

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**Abstract**—This paper focuses on the uplink transmission in a cellular system in which the channels in two neighboring cells are modeled as two mutually interfering multiple access channels (2-IMAC). The base stations (BSs) are equipped with multiple antennas while each user has only one antenna and  $K$  users are active in each cell. By increasing the number of users, it is shown in [1] that for single-antenna BSs, using interference alignment, each cell can approach the interference-free degrees of freedom (DOF) provided that the bandwidth is sufficiently large. But as the number of users grows, the correlation between adjacent subcarriers increases and this degrades the performance of interference alignment. The proposed scheme incorporates the concept of cyclic delay diversity (CDD) to reduce the correlation among subcarriers with the help of multiple antennas at the BSs. Simulation results show the performance improvement of the proposed scheme.

**Keywords**—Interference alignment, degrees of freedom, cyclic delay diversity.

## I. INTRODUCTION

Interference management has become an important task in wireless networks because of the increasing number of users willing to exchange data simultaneously in a medium that has a broadcast nature. In most of the communication scenarios, exact capacity characterizations are difficult to find and asymptotic characterizations are preferred. The degrees of freedom (DOF) approach provides a means to approximate the scaling of network capacity with signal to noise ratio (SNR) in high SNR regime.

Authors in [2] presented a new scheme that achieves the total available degrees of freedom of the  $K$ -user Interference channel ( $K$ -user IC) assuming i.i.d. channel coefficients. Their scheme is based on interference alignment which aims to minimize the number of dimensions occupied by the total interference coming from other users. It has been shown in [2] that precoding at the transmitters and zero forcing at the receivers is sufficient to achieve the total DOF. The method of interference alignment has been also investigated in cellular networks [1], [3] in which downlink and uplink channels are modeled as interference broadcast channel (IBC) and interference multiple access channel (IMAC) respectively. The performance of these networks is mostly limited by the interference coming from the neighboring cells. Authors in [1]

presented a novel scheme called subspace interference alignment that asymptotically achieves the optimal DOF in G interfering multiple access channel (G-IMAC) when the number of users grows. In the simple two-cell scenario (2-IMAC) which is a good model for uplink channel, they show that when all users and BSs use  $N = K + 1$  subcarriers and all of them are equipped with one antenna, if the bandwidth is large enough so that the channel impulse responses (CIRs) have at least two significant taps, then each of the two interfering cells achieves the DOF of  $\frac{K}{K+1}$  almost surely. But their scheme is not optimal in terms of sum rate and bit error rate (BER) performance. For a fixed bandwidth when the number of users grows, spacing of the subcarriers will be reduced and adjacent subcarriers will be more correlated. This correlation degrades the performance of interference alignment.

In [6], it has been shown that there is a meaningful tradeoff between rate, diversity and interference alignment in a general network with multi-antenna transceivers. In a simple case, the antennas can be used to transmit similar messages thus providing more diversity or they can be used to produce appropriate directions for transmitting different messages in a way that the receiver is able to recover the messages. A transmitter diversity technique named cyclic delay diversity (CDD) was presented in [4], [5] that achieves the maximum diversity available in an orthogonal frequency division multiplexing (OFDM) system with multi-antenna transmitters.

In this paper we present a new scheme employing multiple antennas at the base stations to enhance the performance of the scheme presented in [1]. It is obvious that employing multiple antennas at BSs increases the DOF and allows to have a higher data rate but we need a more complicated interference alignment scheme to achieve the DOF and also a more complex receiver is needed. Even in the single-antenna case, we need complex receivers at each BS to perform joint detection of different messages. Due to the correlation mentioned earlier, the low-complexity receivers will have a poor performance. So we aim to exploit multiple antennas at BSs to reduce the correlations in order to use simple alignment schemes with low-complexity receivers. We show through simulations that the BER performance will be greatly improved when we use de-correlator receiver. When the number of users grows, our scheme allows to preserve the same performance by increasing the antennas at the BSs. We also show that careful

selection of the interference spaces at each BS will result in a better performance.

This paper is organized as follows. The system model is described in section II. In section III the concept of CDD is explained. The problem with single antenna BSs is discussed in section IV and analytical results are derived to optimize interference space. The proposed interference alignment scheme is introduced in section V. The performance of the proposed scheme is evaluated in section VI, followed by conclusion.

#### A. Notation

The following notations are used in this paper. Normal letters indicate scalar quantities, boldface letters represent vectors and boldface uppercase letters designate matrices. The trace, conjugate and Hermitian transpose of a matrix or vector are represented by  $tr(\cdot)$ ,  $(\cdot)^*$ ,  $(\cdot)^H$  respectively.  $diag(\cdot)$  stands for a diagonal matrix with its argument vector on its diagonal.  $\mathbf{I}_N$  denotes the identity matrix of size  $N$ . 2-norm of a vector and Frobenius norm of a matrix are represented by  $\|\cdot\|^2$ ,  $\|\cdot\|_F^2$  respectively.  $*$  denotes the convolution operation.

## II. SYSTEM MODEL

We consider the IMAC which is the case in cellular networks for uplink transmission. In this paper we consider two cells and in each cell  $K$  users want to communicate with their desired BS using the same frequency band. Fig. 1 illustrates this channel. Users in cells 1, 2 want to convey their messages to BSs 1, 2 respectively. Each user has one transmit antenna and each BS has  $N_r$  receive antennas. we use OFDM system to build the signal space and  $N$  is the number of subcarriers used for transmission. Users simultaneously transmit their data on these subcarriers. It is assumed that  $N = K + 1$ . The channels are slow varying during transmission of multiple OFDM symbols. It is assumed that the channel impulse responses have a small number of taps so that the channels are almost flat in frequency domain.

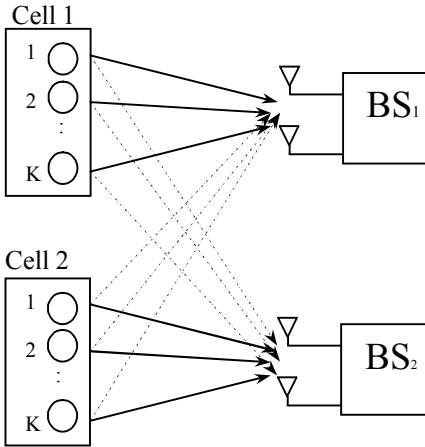


Fig. 1. Two interfering MAC

The channel impulse response from user  $k$  in cell  $i$  to antenna  $m$  of BS  $j$  is given by the  $1 \times N$  vector

$$\mathbf{h}_{km}^{ij} = [h_{km}^{ij}(0), h_{km}^{ij}(1), \dots, h_{km}^{ij}(D), 0, \dots, 0], \quad (1)$$

the channel coefficients  $h_{km}^{ij}(d)$  are complex Gaussian random variables with unit variance.  $D + 1$  is considered to be the number of significant taps for each channel impulse response. Complex Gaussian white noise with variance  $\sigma^2$  is added at each receive antenna on each subcarrier. The received signal at different antennas of BSs is given by

$$\begin{aligned} \mathbf{y}_m^{(1)} &= \sum_{k=1}^K \mathbf{H}_{km}^{11} \mathbf{v}_{k1} s_{k1} + \sum_{k=1}^K \mathbf{H}_{km}^{21} \mathbf{v}_{k2} s_{k2} + \mathbf{n}_m^{(1)}, \\ \mathbf{y}_m^{(2)} &= \sum_{k=1}^K \mathbf{H}_{km}^{22} \mathbf{v}_{k2} s_{k2} + \sum_{k=1}^K \mathbf{H}_{km}^{12} \mathbf{v}_{k1} s_{k1} + \mathbf{n}_m^{(2)}, \end{aligned} \quad (2)$$

for  $m = 1, \dots, N_r$ ,

where  $s_{ki}$  and  $\mathbf{v}_{ki}$ ,  $k = 1, \dots, K$ ,  $i = 1, 2$  represent the transmitted symbol and the precoding vector of user  $k$  in cell  $i$  respectively. Each user then transmits its vector over  $N$  available subcarriers.  $\mathbf{H}_{km}^{ij}$  shows the diagonal channel in frequency domain from user  $k$  in cell  $i$  to antenna  $m$  of BS  $j$ .  $\mathbf{y}_m^{(j)}$  and  $\mathbf{n}_m^{(j)}$  denote the vectors of received signal and additive noise at antenna  $m$  of BS  $j$ .

## III. OFDM RECIEVER WITH CDD

Fig. 2 shows an OFDM transceiver employing cyclic delays to exploit frequency diversity of the channel. Each antenna After down conversion (DC) and removing cyclic prefix (CP) introduces a cyclic delay in its resulting signal. Adding these delayed versions of the received signals together, the output signal,  $r(d)$ ,  $d = 0, \dots, N - 1$  will be derived. After removing CP, each antenna  $m$  has a signal denoted  $r_m(d) = x(d) * h_m(d)$ . Therefore  $r(d)$  will be the sum of delayed versions of the signals out of each antenna

$$\begin{aligned} r(d) &= \sum_{m=1}^{N_r} r_m((d - \Delta_m) \bmod N) \\ &= \sum_{m=1}^{N_r} x(d) * h_m((d - \Delta_m) \bmod N) \\ &= x(d) * \sum_{m=1}^{N_r} h_m((d - \Delta_m) \bmod N), \end{aligned} \quad (3)$$

where  $x(d)$  and  $h_m(d)$  for  $d = 0, \dots, N - 1$  denote the transmitted symbol and the channel impulse response to

antenna  $m$  respectively and  $\Delta_m$  is the delay introduced by antenna  $m$ . So the equivalent channel will be like this

$$h(d) = \sum_{m=1}^{N_r} h_m ((d - \Delta_m) \bmod N). \quad (4)$$

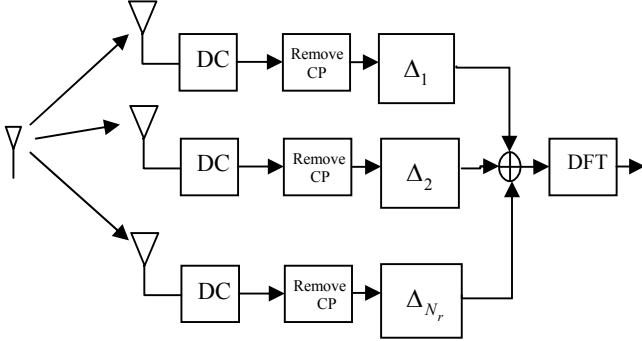


Fig. 2. OFDM transceiver with CDD

Therefore we can assume that the channel taps are delayed instead of the received signals.

#### IV. 2-IMAC

First we consider the 2-IMAC scheme presented in [1] in which there are  $K$  users in each cell and each of them want to transmit its message to the desired BS using  $N = K + 1$  subcarriers. All the users and BSs are equipped with a single antenna and all of them work on the same band. User  $k$  in cell  $i$  puts its symbol  $s_{ki}$  along a precoding vector  $\mathbf{v}_{ki}$  and transmits the resulting vector over  $K + 1$  subcarriers. So the received vector  $\mathbf{y}^{(i)}$  in BS  $i$  will be like this

$$\begin{aligned} \mathbf{y}^{(1)} &= \sum_{k=1}^K \mathbf{H}_k^{11} \mathbf{v}_{k1} s_{k1} + \sum_{k=1}^K \mathbf{H}_k^{21} \mathbf{v}_{k2} s_{k2} + \mathbf{n}^{(1)}, \\ \mathbf{y}^{(2)} &= \sum_{k=1}^K \mathbf{H}_k^{22} \mathbf{v}_{k2} s_{k2} + \sum_{k=1}^K \mathbf{H}_k^{12} \mathbf{v}_{k1} s_{k1} + \mathbf{n}^{(2)}, \end{aligned} \quad (5)$$

in which  $\mathbf{n}^{(i)}$  represents the noise vector at each BS and  $\mathbf{H}_k^{ij}$  is the diagonal frequency domain channel between user  $k$  in cell  $i$  and BS  $j$ . In [1], one direction ( $\mathbf{v}_r$ ) is allocated to interference coming from the other cell and so the precoding vectors are derived as follows

$$\mathbf{v}_{k1} = (\mathbf{H}_k^{12})^{-1} \mathbf{v}_r, \mathbf{v}_{k2} = (\mathbf{H}_k^{21})^{-1} \mathbf{v}_r. \quad (6)$$

Here we show that the directions allocated to interference in each cell need not be similar and if we choose different vectors, the performance can be improved.

Using different directions ( $\mathbf{v}_{r1}, \mathbf{v}_{r2}$ ) the precoding vectors are given by

$$\mathbf{v}_{k1} = (\mathbf{H}_k^{12})^{-1} \mathbf{v}_{r2}, \mathbf{v}_{k2} = (\mathbf{H}_k^{21})^{-1} \mathbf{v}_{r1}. \quad (7)$$

So the received vectors at BSs are given by

$$\begin{aligned} \mathbf{y}^{(1)} &= \sum_{k=1}^K \mathbf{H}_k^{11} (\mathbf{H}_k^{12})^{-1} \mathbf{v}_{r2} s_{k1} + \mathbf{v}_{r1} \sum_{k=1}^K s_{k2} + \mathbf{n}^{(1)}, \\ \mathbf{y}^{(2)} &= \sum_{k=1}^K \mathbf{H}_k^{22} (\mathbf{H}_k^{21})^{-1} \mathbf{v}_{r1} s_{k2} + \mathbf{v}_{r2} \sum_{k=1}^K s_{k1} + \mathbf{n}^{(2)}. \end{aligned} \quad (8)$$

(8) can be rewritten like this

$$\begin{aligned} \mathbf{y}^{(1)} &= \mathbf{H}_1 \mathbf{s}_1 + \mathbf{v}_{r1} \sum_{k=1}^K s_{k2} + \mathbf{n}^{(1)}, \\ \mathbf{y}^{(2)} &= \mathbf{H}_2 \mathbf{s}_2 + \mathbf{v}_{r2} \sum_{k=1}^K s_{k1} + \mathbf{n}^{(2)}, \end{aligned} \quad (9)$$

where  $\mathbf{s}_1, \mathbf{s}_2$  are vectors containing the transmitted symbols of each cell defined as

$$\mathbf{s}_1 = [s_{11}, s_{21}, \dots, s_{K1}]^T, \mathbf{s}_2 = [s_{12}, s_{22}, \dots, s_{K2}]^T$$

and  $\mathbf{H}_1, \mathbf{H}_2$  are defined as follows

$$\begin{aligned} \mathbf{H}_1 &= [\mathbf{H}_1^{11} (\mathbf{H}_1^{12})^{-1} \mathbf{v}_{r2}, \dots, \mathbf{H}_K^{11} (\mathbf{H}_K^{12})^{-1} \mathbf{v}_{r2}], \\ \mathbf{H}_2 &= [\mathbf{H}_1^{22} (\mathbf{H}_1^{21})^{-1} \mathbf{v}_{r1}, \dots, \mathbf{H}_K^{22} (\mathbf{H}_K^{21})^{-1} \mathbf{v}_{r1}] \end{aligned} \quad (10)$$

Using a linear filter  $\mathbf{F}_i$  at each BS, the interference term can be nullified

$$\mathbf{F}_i = \mathbf{I}_N - \frac{\mathbf{v}_{ri} \cdot \mathbf{v}_{ri}^H}{\mathbf{v}_{ri}^H \cdot \mathbf{v}_{ri}} \quad i = 1, 2. \quad (11)$$

So after suppressing the interference the output vector at each BS will be

$$\begin{aligned} \mathbf{F}_1 \mathbf{y}^{(1)} &= \mathbf{F}_1 \mathbf{H}_1 \mathbf{s}_1 + \tilde{\mathbf{n}}^{(1)}, \\ \mathbf{F}_2 \mathbf{y}^{(2)} &= \mathbf{F}_2 \mathbf{H}_2 \mathbf{s}_2 + \tilde{\mathbf{n}}^{(2)}, \end{aligned} \quad (12)$$

in which  $\tilde{\mathbf{n}}^{(i)} = \mathbf{F}_i \mathbf{n}^{(i)}$  for  $i = 1, 2$ . It should be noted that  $\mathbf{F}_i$  is a projection matrix and so it is unitary and it will not change the distribution properties of the noise and therefore the output noise  $\tilde{\mathbf{n}}^{(i)}$  for  $i = 1, 2$  will be a white noise with covariance matrix equal to  $\sigma^2 \mathbf{I}_N$ .

For each cell  $i$ , we aim to maximize Frobenius norm of  $\mathbf{F}_i \mathbf{H}_i$ ,

$$\begin{aligned} \|\mathbf{F}_i \mathbf{H}_i\|_F^2 &= \text{tr}(\mathbf{F}_i \mathbf{H}_i (\mathbf{F}_i \mathbf{H}_i)^H) \\ &= \text{tr}(\mathbf{F}_i \mathbf{H}_i \mathbf{H}_i^H \mathbf{F}_i^H) \\ &= \text{tr}(\mathbf{H}_i^H \mathbf{F}_i^H \mathbf{F}_i \mathbf{H}_i) \\ &= \text{tr}(\mathbf{H}_i^H \mathbf{F}_i \mathbf{H}_i). \end{aligned} \quad (13)$$

Last equality is resulted because  $\mathbf{F}_i$  is a projection matrix and so it is Hermitian and idempotent.

$\mathbf{H}_i$  can be decomposed into two matrices like this

$$\mathbf{H}_i = \mathbf{V}_j \bar{\mathbf{H}}_i \quad i = 1, 2, \quad j = 1, 2, \quad i \neq j, \quad (14)$$

in which  $\mathbf{V}_j = \text{diag}(\mathbf{v}_{rj})$  for  $j = 1, 2$ .  $\bar{\mathbf{H}}_1, \bar{\mathbf{H}}_2$  are matrices that their  $r$ th column is composed of the diagonal elements of  $\mathbf{H}_r^{11} (\mathbf{H}_r^{12})^{-1}$  and  $\mathbf{H}_r^{22} (\mathbf{H}_r^{21})^{-1}$  respectively for  $r = 1, \dots, K$ . It is obvious that the correlation among subcarriers directly translates into correlation among rows of  $\bar{\mathbf{H}}_i$  because  $r$ th row of  $\bar{\mathbf{H}}_i$  represents the channel between all users in cell  $i$  toward BS  $i$  in the  $r$ th subcarrier for  $i = 1, 2$ . If we use  $\mathbf{h}_{ir}^H$  to show the  $r$ th row of  $\bar{\mathbf{H}}_i$ , then (13) can be simplified to

$$\begin{aligned} \|\mathbf{F}_i \mathbf{H}_i\|_F^2 &= \text{tr}(\mathbf{H}_i^H \mathbf{F}_i \mathbf{H}_i) \\ &= \text{tr}(\bar{\mathbf{H}}_i^H \mathbf{V}_j^H \mathbf{F}_i \mathbf{V}_j \bar{\mathbf{H}}_i) \\ &= \text{tr}(\bar{\mathbf{H}}_i^H \mathbf{V}_j^H (\mathbf{I}_N - \frac{\mathbf{v}_{ri} \mathbf{v}_{ri}^H}{\mathbf{v}_{ri}^H \mathbf{v}_{ri}}) \mathbf{V}_j \bar{\mathbf{H}}_i). \end{aligned} \quad (15)$$

If we restrict the vectors  $\mathbf{v}_{r1}$  and  $\mathbf{v}_{r2}$  to have unit norm and defining  $\mathbf{G}_i$  as

$$\mathbf{G}_i = \mathbf{V}_j^H (\mathbf{I}_N - \mathbf{v}_{ri} \mathbf{v}_{ri}^H) \mathbf{V}_j \quad i = 1, 2, \quad (16)$$

then (15) results in

$$\begin{aligned} \|\mathbf{F}_i \mathbf{H}_i\|_F^2 &= \text{tr}(\bar{\mathbf{H}}_i^H \mathbf{V}_j^H (\mathbf{I}_N - \mathbf{v}_{ri} \mathbf{v}_{ri}^H) \mathbf{V}_j \bar{\mathbf{H}}_i) \\ &= \text{tr}([\mathbf{h}_{i1} \quad \dots \quad \mathbf{h}_{iN}] \mathbf{G}_i \begin{bmatrix} \mathbf{h}_{i1}^H \\ \vdots \\ \mathbf{h}_{iN}^H \end{bmatrix}) \\ &= \text{tr}(\sum_{m=1}^N \sum_{n=1}^N \mathbf{G}_i(m, n) \mathbf{h}_{im} \mathbf{h}_{in}^H), \end{aligned} \quad (17)$$

in which we have

$$\begin{aligned} \mathbf{G}_i(m, n) &= \begin{cases} [\mathbf{v}_{rj}^H(m) \cdot \mathbf{v}_{rj}(m)] [1 - \mathbf{v}_{ri}(m) \cdot \mathbf{v}_{ri}^H(m)] & m = n \\ -[\mathbf{v}_{rj}^H(m) \cdot \mathbf{v}_{rj}(n)] [\mathbf{v}_{ri}(m) \cdot \mathbf{v}_{ri}^H(n)] & m \neq n \end{cases} \end{aligned} \quad (18)$$

Rewriting (17) results in the following

$$\begin{aligned} \|\mathbf{F}_i \mathbf{H}_i\|_F^2 &= \sum_{m=1}^N \sum_{n=1}^N \mathbf{G}_i(m, n) \text{tr}(\mathbf{h}_{im} \mathbf{h}_{in}^H) \\ &= \sum_{m=1}^N \sum_{n=1}^N \mathbf{G}_i(m, n) \text{tr}(\mathbf{h}_{in}^H \mathbf{h}_{im}) \\ &= \sum_{m=1}^N |\mathbf{v}_{rj}(m)|^2 (1 - |\mathbf{v}_{ri}(m)|^2) \|\mathbf{h}_{im}\|^2 \\ &\quad - \sum_{m=1}^N \sum_{n=1, n \neq m}^N [\mathbf{v}_{rj}^H(m) \cdot \mathbf{v}_{rj}(n)] [\mathbf{v}_{ri}(m) \cdot \mathbf{v}_{ri}^H(n)] (\mathbf{h}_{in}^H \mathbf{h}_{im}). \end{aligned} \quad (19)$$

It is obvious from (19) that correlation of rows of  $\bar{\mathbf{H}}_i$  reduces the sum received power and using same vectors  $\mathbf{v}_{r1} = \mathbf{v}_{r2}$  is not a good choice because the coefficient of  $(\mathbf{h}_{in}^H \mathbf{h}_{im})$  in (19) always becomes positive and the correlation term reduces the sum power. With some modifications (19) can be further simplified to this

$$\begin{aligned} \|\mathbf{F}_i \mathbf{H}_i\|_F^2 &= \sum_{m=1}^N |\mathbf{v}_{rj}(m)|^2 \|\mathbf{h}_{im}\|^2 \\ &\quad - \left( \sum_{m=1}^N [\mathbf{v}_{rj}^H(m) \cdot \mathbf{v}_{ri}(m) \mathbf{h}_{im}] \right)^H \left( \sum_{m=1}^N [\mathbf{v}_{rj}^H(m) \cdot \mathbf{v}_{ri}(m) \mathbf{h}_{im}] \right) \end{aligned}$$

$$\begin{aligned}
&= \sum_{m=1}^N |\mathbf{v}_{rj}(m)|^2 \|\mathbf{h}_{im}\|^2 - (\bar{\mathbf{H}}_i^H \mathbf{z}_i)^H (\bar{\mathbf{H}}_i^H \mathbf{z}_i) \\
&= \sum_{m=1}^N |\mathbf{v}_{rj}(m)|^2 \|\mathbf{h}_{im}\|^2 - \mathbf{z}_i^H \bar{\mathbf{H}}_i \bar{\mathbf{H}}_i^H \mathbf{z}_i, \tag{20}
\end{aligned}$$

in which  $\mathbf{z}_i$  is the element-wise product of  $\mathbf{v}_{rj}^H$ ,  $\mathbf{v}_{ri}$  i.e.  $\mathbf{z}_i(m) = \mathbf{v}_{rj}^H(m) \cdot \mathbf{v}_{ri}(m)$  for  $m = 1, \dots, N$ .

The second term in (20) can be simply minimized over  $\mathbf{z}_i$  and then maximizing the first term yields the best choice of  $\mathbf{v}_{rj}$  and so  $\mathbf{v}_{ri}$ . But the point is that we should consider both cells ( $i = 1, 2$ ) and maybe an iterative procedure results in the optimum vectors.

In a suboptimal manner we propose to choose two orthogonal interference vectors one for each cell unlike [1] which employs the same interference vectors in both cells and we see that the performance slightly gets better. The reason is that when the channel is almost flat (e.g. two taps) then by using similar interference vectors, we will have desired vectors that are very correlated with the interference vector, so if we use orthogonal interference vectors then the interference vectors will be highly separated from the desired vectors.

## V. INTERFERENCE ALIGNMENT USING CDD

In previous section we explained that the correlation between subcarriers has a negative effect on the performance and we discussed to find the optimal interference vectors to reduce this effect. We observed that by appropriate choice of interference vectors the interference can be effectively eliminated but still there is a high correlation among desired vectors. If we want to use a decorrelator receiver which has a low complexity, this correlation among desired vectors would degrade the performance in terms of the received power and so the average BER. For a fixed bandwidth when the number of users grows, spacing of the subcarriers will be reduced and adjacent subcarriers will be more correlated. This correlation degrades the performance of interference alignment. Here we propose a new scheme that aims to reduce the correlations among subcarriers to enhance the performance. Our proposed scheme employs the interference alignment scheme used in [1] in a scenario consisting of multi-antenna BSs. Employing multiple antennas at BSs allows us to use the concept of CDD. If we use multiple antennas at each base station then the channels between each user in each cell toward base stations will become similar to the equivalent channel presented in section III. Therefore the equivalent channel from user  $k$  in cell  $i$  to BS  $j$  after adding the delayed versions out of different antennas will be as follows

$$h_k^{ij}(d) = \sum_{m=1}^{N_t} h_{km}^{ij}((d - \Delta_m) \bmod N), \tag{21}$$

where  $h_{km}^{ij}(d)$  is the channel from user  $k$  in cell  $i$  to antenna  $m$  of BS  $j$ . Therefore the equivalent channel in frequency domain can be evaluated as follows

$$\mathbf{H}_k^{ij}(l) = \sum_{m=1}^{N_t} H_{km}^{ij}(l) e^{-j \frac{2\pi}{N} l \Delta_m}, \quad l = 0, \dots, N-1. \tag{22}$$

We will have an effective channel that can be made more frequency selective than channels to individual antennas by choosing appropriate delays.

$\mathbf{h}_k^{ij}$  is a  $1 \times N$  vector representing the equivalent channel in time domain and  $\mathbf{H}_k^{ij}$  is the diagonal channel in frequency domain.

$$\begin{aligned}
\mathbf{h}_k^{ij} &= [h_k^{ij}(0), h_k^{ij}(1), \dots, h_k^{ij}(N-1)], \\
\mathbf{H}_k^{ij} &= \text{diag}([H_k^{ij}(0), H_k^{ij}(1), \dots, H_k^{ij}(N-1)]). \tag{23}
\end{aligned}$$

The spatial diversity is transformed into frequency diversity. Due to the increased frequency-selectivity, channel estimation becomes a more difficult task. Channel estimation for cyclic delay diversity (transmitter diversity) has been investigated in [7], where it has been shown that with  $N_t$  transmit antennas, the number of required pilot symbols is increased at least by a factor of  $N_t$  compared to a single transmit antenna system. It should be noted that only the effective channels need to be known at the transmitters and therefore the number of feedback bits to provide channel state information (CSI) at the transmitters is equal to that of single-antenna case.

Using a receiver with multiple antennas employing CDD we can make an effective frequency selective channel and reduce the correlation among subcarriers. Substituting the resulting  $\mathbf{H}_k^{ij}$  from (23) into (10) we will have the same equation as (12)

$$\begin{aligned}
\mathbf{F}_1 \mathbf{y}^{(1)} &= \mathbf{F}_1 \mathbf{H}_1 \mathbf{s}_1 + \tilde{\mathbf{n}}^{(1)}, \\
\mathbf{F}_2 \mathbf{y}^{(2)} &= \mathbf{F}_2 \mathbf{H}_2 \mathbf{s}_2 + \tilde{\mathbf{n}}^{(2)}, \tag{24}
\end{aligned}$$

$$\text{in which } \mathbf{F}_i = \mathbf{I}_N - \frac{\mathbf{v}_{ri} \cdot \mathbf{v}_{ri}^H}{\mathbf{v}_{ri}^H \cdot \mathbf{v}_{ri}} \quad i = 1, 2.$$

The sum rate for each cell can be evaluated as follows

$$R_i = \log \left| \mathbf{I}_N + \frac{1}{\sigma^2} \mathbf{F}_i \mathbf{H}_i \mathbf{Q}_s \mathbf{H}_i^H \mathbf{F}_i^H \right| \quad i = 1, 2. \quad (25)$$

As stated earlier, covariance matrix of  $\tilde{\mathbf{n}}^{(i)}$  is equal to  $\sigma^2 \mathbf{I}_N$  for  $i = 1, 2$ .  $\mathbf{Q}_s$  is the covariance matrix of the transmitted symbols and is assumed to be equal to  $P \mathbf{I}_K$  i.e. the users transmit independent streams with equal power  $P$ .

In the original scheme using only one antenna, if the channel is not frequency selective, then the precoding vectors are likely to have a high correlation with the reference interference vector (at one extreme when the channel is completely flat ( $D = 0$ ) then precoding vectors are scaled versions of the reference interference vector). By increasing frequency selectivity of the channel we can reduce this correlation and so it seems that we are pushing the interference power out of signal space.

## VI. NUMERICAL RESULTS

In this section we evaluate the performance of the proposed schemes in terms of sum rate and also bit error rate performance using decorrelator receiver. We suppose that all

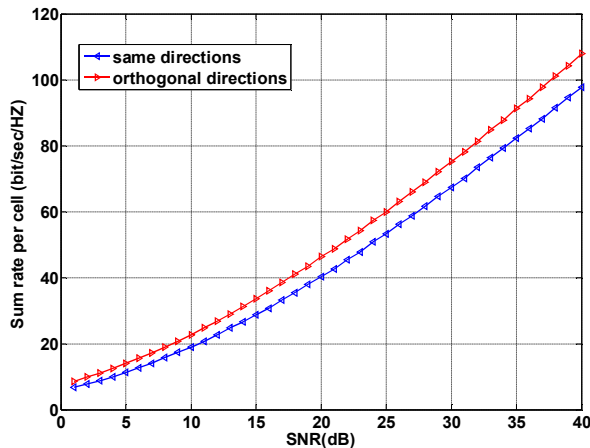


Fig. 3. Sum rate performance using different directions

channels have two taps ( $D = 1$ ).  $K = 11$  users are active in each cell so  $N = 12$ . Fig. 3 shows the enhancement in sum rate resulting from using two orthogonal interference vectors compared to the scheme that uses same vectors.

Fig. 4 shows the BER performance of the proposed scheme using decorrelator receiver, compared to the original scheme presented in [1]. 4-QAM modulation has been used in this part. 3 antennas are used at each base station and delays  $\{0, 3, 6\}$  are introduced.

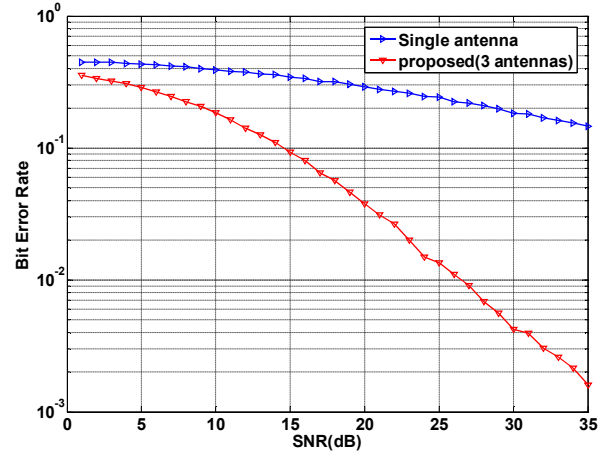


Fig. 4. BER performance of the proposed multi-antenna scheme

## VII. CONCLUSION

We proposed a new scheme that uses multiple antennas at base stations to enhance the performance of interference alignment. We also showed that effect of interference can be reduced by careful choice of the interference space. There are several issues that should be covered in future in order to make these schemes applicable. Channel estimation which will be a more challenging task when we use more antennas, and providing CSI for transmitters is of great importance.

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