Joint Network-Channel Coding for the Asymmetric Multiple-Access Relay Channel

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***Abstract*—In this paper we consider the time-division *multiple-access relay channel* (MARC). We propose a low-complexity compress-and-forward-based transmission scheme that consists of a scalar quantization of log-likelihood ratios (LLRs), followed by a suitably defined network code. This scheme is well suited also for asymmetric source-relay channel conditions. We use the *information bottleneck method* (IBM) for designing the LLR quantizers and we propose a modified IBM algorithm with more favorable numerical properties. Finally, we assess the performance of the proposed scheme in AWGN and fading channels using numerical simulations, including a comparison with existing transmission schemes.**

I. Introduction

In the *multiple-access relay channel* (MARC) two sources transmit data to one destination node with the help of a single relay. We focus on the time-division MARC with half-duplex terminals and a three-phase transmission protocol which simplifies a practical implementation. Furthermore, we use *network coding* (NC) [1] to improve performance.

Information-theoretic limits for the MARC with *decode-and-forward* (DF) and *compress-and-forward* (CF) protocols [2], have been established in [3], [4]. For practical NC-based transmission schemes for the MARC see e.g. [5]–[8].

In [9], soft information about network-coded bits is forwarded by the the relay to the destination in order to deal with unreliable source-relay channels. This idea has been extended to a CF-based NC scheme in which the relay performs scalar quantization (SQ) of the network-coded soft information [10]. Since the processing in [10] is suboptimal for asymmetric source-relay channels, [11] proposed a vector quantization (VQ) that combined the NC and the quantization. However, there exists no efficient encoder for the VQ proposed in [11]. In this paper, we propose a different scheme which has about the same complexity as SQ while retaining the benefits of VQ without noticeable performance loss. More specifically, our contributions are as follows:

- We propose to circumvent VQ by using two independent scalar quantizers followed by a suitably designed NC at the relay.
- We present a unifying framework of our proposed scheme and the schemes in [10], [11] based on message passing on the overall code's factor graph.
- We review the problem of quantizer design in the information bottleneck framework [12] and propose a numerically stable and efficient modification of the iterative information bottleneck algorithm.
- We compare our scheme with existing ones in terms of bit and frame error probability. Numerical results confirm that our scheme achieves a diversity order of two in quasi-static fading channels.

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The remainder of this paper is organized as follows. Section II introduces the system model and describes the basic operation of all network nodes. In Section III, we explain in detail the proposed relay processing and compare it to existing schemes. In Section IV we discuss the design of LLR quantizers and we propose a modified information bottleneck algorithm. Numerical results are presented in Section V and conclusions are provided in Section VI.

II. System Model

MARC Model. We consider the three-phase time-division MARC with two sources S_1 and S_2, a relay R, and a destination D. The sources consecutively broadcast independent messages in the first two phases. In the third phase, the relay forwards a combination of the data it has received in the previous two time slots to the destination. Finally, the destination jointly decodes the signals received in all three time slots. In this model, the sources do not overhear each others transmission.

The total number of channel uses per transmission is $M = M_1 + M_2 + M_{\mathsf{R}}$, where M_i denotes the channel uses of the respective node (throughout the paper, subscripts 1, 2, R, and D refer to source 1, source 2, relay, and destination). The fraction of channel uses of node i is denoted by $\Delta_i = M_i/M$.

Channel Model. Assuming quasi-static fading, we have (all vectors in this expression are of length M_i)

$$\mathbf{y}_{ij} = d_{ij}^{-n/2} h_{ij} \mathbf{x}_i + \mathbf{w}_{ij}, \tag{1}$$

where $\mathbf{x}_i$ is the signal transmitted by node i, $\mathbf{y}_{ij}$ is the corresponding receive signal at node j, d_{ij} is the distance between nodes i and j, n is the path-loss exponent, $\mathbf{w}_{ij} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ is white Gaussian noise (assumed zero-mean and circularly symmetric), and h_{ij} denotes the fading coefficient. We impose an average transmit power constraint at each node, i.e.,

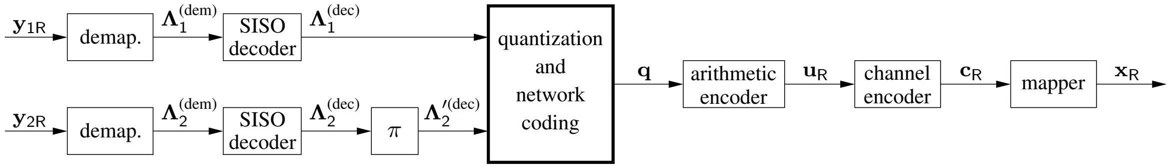


Figure 1: Relay block diagram.

$\mathrm{E}\{\|\mathbf{x}_i\|_2^2\}/M_i = 1$, where $\mathrm{E}\{\cdot\}$ and $\|\cdot\|_2$ denote expectation and ℓ_2-norm. The signal-to-noise ratio (SNR) for the link between node i and node j is given by $\gamma_{ij} = d_{ij}^{-n}|h_{ij}|^2/N_0$. The average SNR is denoted by $\overline{\gamma}_{ij} = \mathrm{E}\{\gamma_{ij}\}$. The MARC is called *asymmetric* if $\gamma_{1\mathrm{R}} \neq \gamma_{2\mathrm{R}}$. We assume that each node only has receive channel state information (CSI), i.e., γ_{ij} is known at node j. In particular, the relay has no source-destination CSI.

Sources. Each source generates a length-K_i sequence $\mathbf{u}_i \in \{0,1\}^{K_i}$ of independent and equally likely bits. The sequence $\mathbf{u}_i$ is channel encoded using a linear binary code $\mathcal{C}_i$ of rate $R_i = K_i/N_i$, yielding a length-N_i sequence $\mathbf{c}_i \in \{0,1\}^{N_i}$ of code bits. Next, the code bits are mapped to a signal constellation $\mathcal{A}_i$ of cardinality $|\mathcal{A}_i| = 2^{m_i}$, yielding length-$M_i$ sequences $\mathbf{x}_i \in \mathcal{A}^{M_i}$ of transmit symbols. The code rates are chosen as $R_i = K_i/(m_i M_i)$. We assume that both sources use identical channel code block lengths, i.e., $N_1 = N_2 \triangleq N$.

Relay. Fig. 1 shows a block diagram of the relay. The signal $\mathbf{x}_\mathrm{R}$ transmitted by the relay to the destination in the third time-slot is obtained by jointly processing the signals $\mathbf{y}_{1\mathrm{R}}$ and $\mathbf{y}_{2\mathrm{R}}$ received in the first two time slots. First, the relay performs soft-demapping of each received signal, yielding a vector of log-likelihood ratios (LLRs) $\mathbf{\Lambda}_i^{(\mathrm{dem})}$. These LLRs are passed to a soft-input soft-output (SISO) channel decoder which produces LLRs $\mathbf{\Lambda}_i^{(\mathrm{dec})}$ for the code bits $\mathbf{c}_i$. The code bit LLRs for source S_2 are interleaved to avoid short cycles in the factor graph of the overall code (cf. Fig. 2). Then, joint NC and Q-level quantization is performed on $\mathbf{\Lambda}_1^{(\mathrm{dec})}$ and $\mathbf{\Lambda}_2'^{(\mathrm{dec})}$, resulting in a sequence of integers $\mathbf{q} \in \mathcal{Q}^N$, with $\mathcal{Q} \triangleq \{1,2,\ldots,Q\}$. A more detailed description of the joint NC and quantization stage is given in Section III. Since Q is not necessarily a power of 2 and $\mathbf{q}$ may contain residual redundancy, we use arithmetic coding of $\mathbf{q}$ to obtain a binary sequence $\mathbf{u}_\mathrm{R}$. After a cyclic redundancy check (CRC) is added, the sequence $\mathbf{u}_\mathrm{R}$ is channel-encoded and mapped to the transmit signal $\mathbf{x}_\mathrm{R}$.

Destination. The destination jointly decodes the received signals $\mathbf{y}_{1\mathrm{D}}$, $\mathbf{y}_{2\mathrm{D}}$, and $\mathbf{y}_{\mathrm{RD}}$ to obtain detected messages $\hat{\mathbf{u}}_1$, $\hat{\mathbf{u}}_2$. To this end, each observation is first processed by a soft demodulator. Next, the message transmitted by the relay is decoded; if residual errors are detected via the CRC, both source messages are decoded separately and the relay transmission is discarded to avoid error propagation. Otherwise, we assume that the arithmetic decoding yields the correct sequence $\mathbf{q}$ at the destination.

The joint quantization and NC stage at the relay creates an equivalent time-invariant discrete memoryless channel with

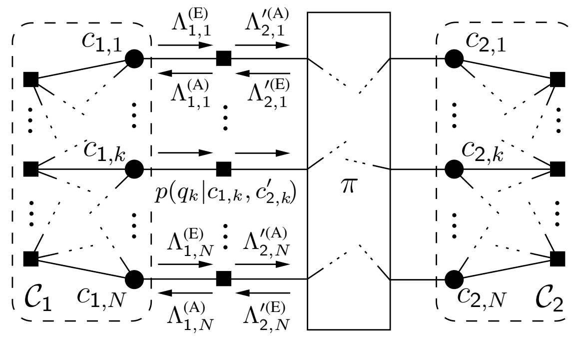


Figure 2: Factor graph of the overall code.

probability mass function (pmf)

$$p(\mathbf{q}|\mathbf{c}_1,\mathbf{c}_2') = \prod_{k=1}^{N} p(q_k|c_{1,k},c_{2,k}').$$

where $\mathbf{c}_2' = \pi(\mathbf{c}_2)$ denotes the interleaved version of $\mathbf{c}_2$. In the factor graph of the overall code that is shown in Fig. 2, the pmfs $p(q_k|c_{1,k},c_{2,k}')$ correspond to factor nodes that couple the code bits of the two sources and hence enable joint iterative decoding of the source codewords.

With $\Lambda_{i,k}^{(\mathrm{E})}$ denoting the extrinsic LLRs obtained from decoding $\mathcal{C}_i$, the a priori LLRs exchanged by the two constituent codes can be shown to be given by

$$\Lambda_{1,k}^{(\mathrm{A})} = \log \frac{\exp\big(\Lambda_{q,k}\big) + \exp\big(\Lambda_{2,k}'^{(\mathrm{E})} + \Lambda_{q,k}^{(1)}(0)\big)}{\exp\big(\Lambda_{2,k}'^{(\mathrm{E})}\big) + \exp\big(-\Lambda_{q,k}^{(2)}(1)\big)}, \qquad (2)$$

$$\Lambda_{2,k}'^{(\mathrm{A})} = \log \frac{\exp\big(-\Lambda_{q,k}\big) + \exp\big(\Lambda_{1,k}^{(\mathrm{E})} + \Lambda_{q,k}^{(2)}(0)\big)}{\exp\big(\Lambda_{1,k}^{(\mathrm{E})}\big) + \exp\big(-\Lambda_{q,k}^{(1)}(1)\big)}. \qquad (3)$$

Here, we used the auxiliary LLRs (cf. [11])

$$\begin{aligned} \Lambda_{q,k} &\triangleq \log \frac{p(q_k|c_{1,k}=0, c_{2,k}'=1)}{p(q_k|c_{1,k}=1, c_{2,k}'=0)}, \\ \Lambda_{q,k}^{(1)}(b) &\triangleq \log \frac{p(q_k|c_{1,k}=0, c_{2,k}'=b)}{p(q_k|c_{1,k}=1, c_{2,k}'=b)}, \\ \Lambda_{q,k}^{(2)}(b) &\triangleq \log \frac{p(q_k|c_{1,k}=b, c_{2,k}'=0)}{p(q_k|c_{1,k}=b, c_{2,k}'=1)}, \end{aligned} \qquad (4)$$

where the respective value of q_k is known due to the transmission of the relay.

Using (2) and (3) we can perform iterative joint network-channel decoding by running the sum-product algorithm [13] on the factor graph of the overall code. We note that (2), (3) are applicable to the decoding of any joint network-channel

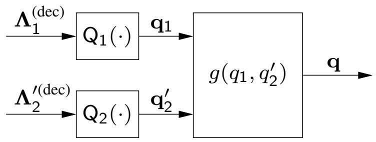


Figure 3: Coding strategy at the relay.

code that can be represented in terms of a discrete memoryless channel $p(q_k|c_{1,k}, c_{2,k})$. Indeed, the specific joint network-channel code affects only the LLRs in (4) but not the structure of the message updates.

III. Quantization and Network Coding

In this section we describe the quantization and NC stage in Fig. 1 in more detail. In [10], this stage consisted of soft XOR NC, implemented via a boxplus "$\boxplus$" (cf. [14]) of the code bit LLRs $\Lambda_{1,k}^{(\mathrm{dec})}$ and $\Lambda_{2,k}'^{(\mathrm{dec})}$, followed by SQ. With this scheme, abbreviated as sXOR-SQ, the function node $p(q_k|c_{1,k}, c'_{2,k})$ in Fig. 2 becomes a conventional parity check node and the message updates (2) and (3) simplify to boxplus operations. This scheme has low complexity but is not suited for the asymmetric MARC. For that reason, [11] proposed to combine NC and quantization via a two-dimensional VQ (2D-VQ) that can be adapted to asymmetric channel conditions at the price of high complexity and memory requirements.

Proposed Scheme. Our scheme aims at combining the low complexity of sXOR-SQ with the improved performance of 2D-VQ over asymmetric channels. The key idea is to replace the VQ with two independent scalar quantizers followed by a NC step. We use two (possibly different) scalar quantizers $\mathsf{Q}_1(\cdot)$ and $\mathsf{Q}_2(\cdot)$ that map the LLRs $\Lambda_{1,k}^{(\mathrm{dec})}$ and $\Lambda_{2,k}'^{(\mathrm{dec})}$ to index vectors $\mathbf{q}_1 \in \mathcal{Q}_1^N$ and $\mathbf{q}_2 \in \mathcal{Q}_2^N$ with $\mathcal{Q}_i \triangleq \{1, \ldots, Q_i\}$; these quantizers are individually optimized for the respective source-relay channel conditions (see Section IV for details regarding the quantizer design). The quantization is followed by element-wise NC using a deterministic function $g : \mathcal{Q}_1 \times \mathcal{Q}_2 \mapsto \mathcal{Q}$, i.e., $q_k = g(q_{1,k}, q_{2,k})$. This function is designed according to

$$g^\star = \arg\max_g \big\{ I(c_1; q|c_2') + I(c_2'; q|c_1) \big\}. \tag{5}$$

This problem can be recognized as an instance of the information bottleneck problem which can be solved using the algorithm proposed in Section IV. Even though (5) is a two-dimensional information bottleneck problem, it is computationally much less demanding than the VQ design since the cardinality of the set of values to be encoded is only $Q_1 \times Q_2$. Indeed, the resulting function g can equivalently be written as an integer-valued matrix $\mathbf{G} \in \mathcal{Q}^{Q_1 \times Q_2}$ with $[\mathbf{G}]_{ij} = g(i, j)$, $i \in \mathcal{Q}_1$, $j \in \mathcal{Q}_2$. Therefore, the encoding amounts to a simple lookup in $\mathbf{G}$ with q_1 as row index and q_2' as column index. We note that $\mathbf{G}$ can be found on-the-fly during data transmission since the distributions $p(c_i, q_i)$, $i \in \{1, 2\}$, required for solving (5) are known once the relay has fixed the scalar quantizers.

The complexity and memory requirements of our proposed method are very low since it only involves SQ and a lookup in a (small) matrix. Furthermore, the schemes from [10], [11] need to store (two-dimensional) quantizers for all asymmetric channel conditions, i.e., for all SNR combinations $(\gamma_{1\mathsf{R}}, \gamma_{2\mathsf{R}})$. In contrast, for our scheme it is sufficient to store a single set of scalar quantizers for a sufficiently wide range of source-relay SNRs. Moreover, the individual quantization allows us to perform the interleaving at the relay after the quantizer, thereby yielding a significant reduction of the interleaver memory. Finally, the numerical results in Section V show that there is no noticeable performance penalty resulting from splitting the 2D-VQ into two separate SQs with a subsequent NC step.

Interpretation. Fig. 4a shows the mapping $(\Lambda_1\ \Lambda_2') \mapsto q$ (different colors represent different values of q) for sXOR-SQ and symmetric channels. In this case, the quantization regions are characterized by the set of points for which $\Lambda_1 \boxplus \Lambda_2'$ equals a SQ boundary. For the same scenario, our proposed method leads to quantization regions whose boundaries are straight lines, cf. Fig. 4b. These quantizer regions can equivalently be obtained by quantization of the approximate boxplus

$$\Lambda_1 \widetilde{\boxplus} \Lambda_2' \triangleq \operatorname{sign}(\Lambda_1)\operatorname{sign}(\Lambda_2') \min\{|\Lambda_1|, |\Lambda_2'|\} \geq \Lambda_1 \boxplus \Lambda_2'. \tag{6}$$

Fig. 4c shows the mapping that is obtained for an asymmetric MARC with 2D-VQ. This mapping can be viewed as a generalization of the boxplus operation to asymmetric channel conditions. This interpretation is justified by the corresponding decoding rules (2) and (3), which simplify to the boxplus operation for symmetric channels. The mapping obtained for this scenario with our method is depicted in Fig. 4d. Again, the boundaries are straight lines, i.e., we obtain an approximation of the mapping in Fig. 4c that can be viewed as an asymmetric generalization of the boxplus approximation (6).

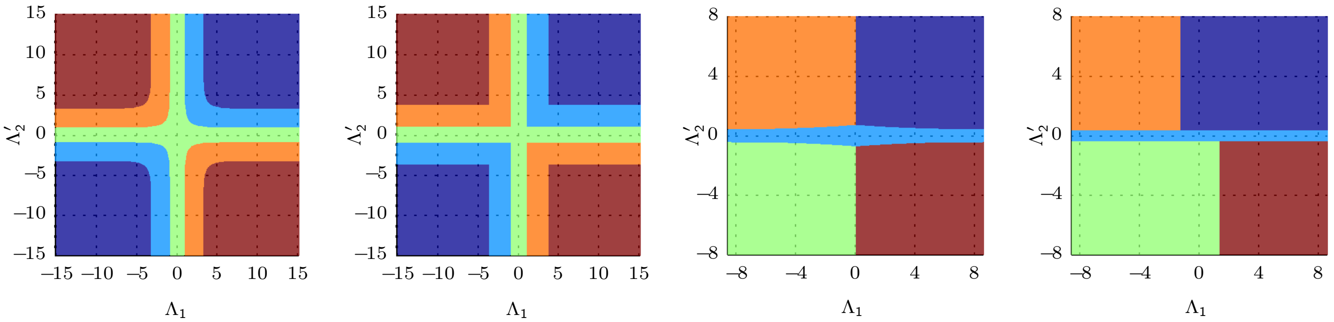
IV. Quantizer Design

Information Bottleneck Method. The information bottleneck method (IBM) amounts to performing data compression with mutual information as fidelity criterion [12]. It has first been applied to LLR quantizer design in [10]. We are interested in SQ of an LLR $\Lambda \in \mathbb{R}$ with Q levels, yielding the quantization index $q \in \mathcal{Q} = \{1, \ldots, Q\}$, with the aim of preserving as much information possible about the corresponding code bit c. Given the joint distribution $p(c, \Lambda)$, the iterative IBM algorithm (see [12], [10]) finds a probabilistic mapping $p(q|\Lambda)$ that corresponds to a local optimum of the problem

$$p^\star(q|\Lambda) = \arg\max_{p(q|\Lambda)} I(c; q). \tag{7}$$

Proposed Algorithm. It can be shown that the maximizer in (7) is in fact a deterministic mapping, i.e., $p^\star(q|\Lambda) \in \{0, 1\}$. Hence, our goal is to design a *deterministic* quantizer. This requires the parameter β in the conventional IBM algorithm (cf. [12]) to be chosen large which in turn often causes numerical instability. Therefore we next propose a modified algorithm in which we directly incorporate the constraint that the resulting quantizer must be deterministic. A similar approach has independently been proposed in [15].

The cost function in the IBM algorithm is given by $\overline{C} = \mathrm{E}\{C(q, \Lambda)\}$ with $C(q, \Lambda) = \mathrm{D}(p(c|\Lambda) \| p(c|q))$, where $\mathrm{D}(\cdot\|\cdot)$



(a) sXOR-SQ (b) proposed (c) 2D-VQ (d) proposed

Figure 4: Comparison of quantizer mappings with $Q = 5$ for (a), (b) the symmetric case ($\gamma_{1\mathrm{R}} = \gamma_{2\mathrm{R}} = 1.5\,\mathrm{dB}$) and (c), (d) the asymmetric case ($\gamma_{1\mathrm{R}} = 1.5\,\mathrm{dB}$, $\gamma_{2\mathrm{R}} = -3.5\,\mathrm{dB}$).

denotes the Kullback-Leibler divergence. The key idea is to find $q^\star = \arg\min_q C(q,\Lambda)$ and set $p(q|\Lambda) = \mathrm{I}\{q = q^\star\}$ in each iteration ($\mathrm{I}\{\cdot\}$ denotes the indicator function which equals 1 when the argument is true and 0 otherwise). This update ensures that the average cost $\overline{C}$ decreases in each iteration and therefore the algorithm converges to a locally optimal deterministic mapping $p(q|\Lambda)$. The proposed algorithm is summarized in Algorithm 1 (ℓ denotes the iteration index).

Algorithm 1 *scalar, deterministic IBM quantizer design*

Input: $p(c,\Lambda)$, $\varepsilon > 0$, $L > 0$, $\mathcal{Q}$
Initialization: $\overline{C}_{-1} \leftarrow \infty$, $\delta \leftarrow \infty$, $\ell \leftarrow 0$, randomly initialize $C_0(q,\Lambda) \in \mathbb{R}_+$ for all q and Λ

while $\delta \geq \varepsilon$ **and** $\ell < L$ **do**
  for all Λ **do**
    $q^\star \leftarrow \arg\min_{q\in\mathcal{Q}} C_\ell(q,\Lambda)$
    $p_\ell(q|\Lambda) \leftarrow \mathrm{I}\{q = q^\star\}$, $q \in \mathcal{Q}$
  end for
  $p_\ell(q) \leftarrow \sum_\Lambda p_\ell(q|\Lambda)p(\Lambda)$
  $p_\ell(c|q) \leftarrow \sum_\Lambda p(c,\Lambda)p_\ell(q|\Lambda)/p_\ell(q)$
  $C_\ell(q,\Lambda) \leftarrow \mathrm{D}(p(c|\Lambda)\|p_\ell(c|q))$
  $\overline{C}_\ell \leftarrow \sum_{q,\Lambda} p_\ell(q|\Lambda)p(\Lambda)C_\ell(q,\Lambda)$
  $\delta \leftarrow (\overline{C}_{\ell-1} - \overline{C}_\ell)/\overline{C}_\ell$
  $\ell \leftarrow \ell + 1$
end while

Our algorithm is numerically more stable than the original IBM algorithm. Moreover, with the substitutions $c \rightarrow \mathbf{c} = (c_1\ c_2)$, $\Lambda \rightarrow \mathbf{\Lambda} = (\Lambda_1\ \Lambda_2)$, and a suitable choice of the relevant information (cf. (5)), it is straightforward to extend Algorithm 1 to the design of 2D-VQ.

Discussion. An important difference between SQ and VQ in the IBM context is the encoding (quantization) operation and the storage of the quantizer. In SQ, it is sufficient to store the boundaries of the quantization intervals and compare the LLR to those boundaries. In VQ, however, the quantization regions are difficult to characterize and the encoding requires a very large lookup table for the mapping $(\Lambda_1\ \Lambda_2) \mapsto q$, thereby rendering the scheme practically infeasible.

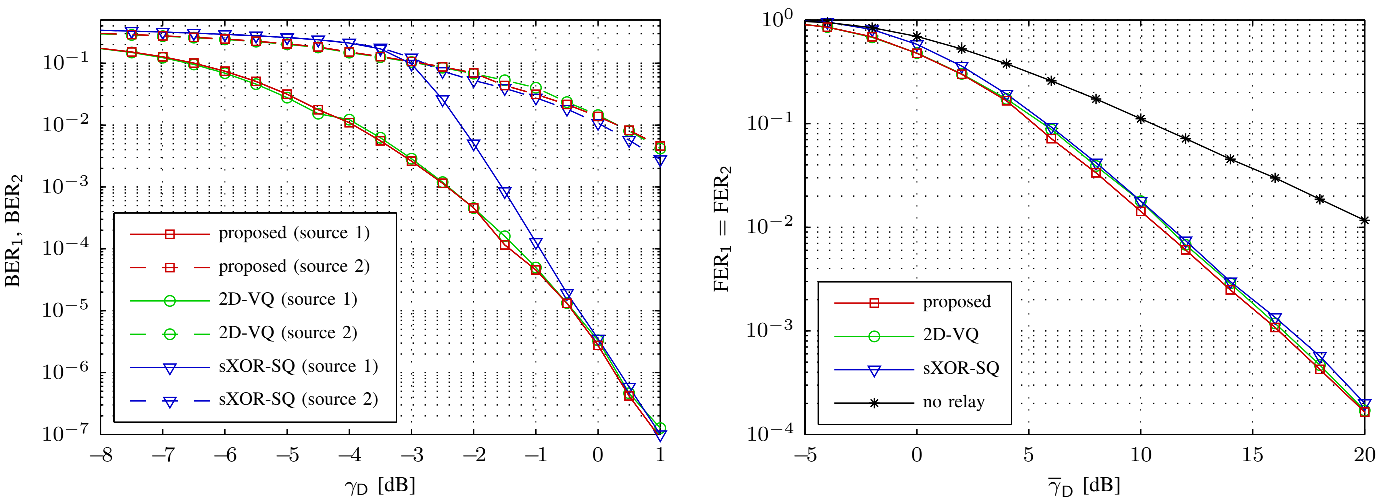
V. Numerical Results

In this section we assess the performance of our proposed scheme by Monte Carlo simulations. We analyze the bit error rate (BER) and the frame error rate (FER) for the transmission over AWGN and quasi-static fading channels, respectively.

General Setup. The sources transmit $K_1 = K_2 = 1024$ information bits, encoded by a recursive, systematic convolutional code with generator polynomial $[1\ 13/15]_8$ (in octal notation). Due to the processing at the relay, the overall code is a parallel concatenated code. The sources use QPSK constellations. We assume that the transmit frame is shared equally among the nodes, i.e., $\Delta_i = 1/3$. The path-loss exponent in (1) is chosen as $n = 3.52$, in accordance with the Okumura-Hata model [16]. We assume that the source-destination channels are symmetric, i.e., $d_{1\mathrm{D}} = d_{2\mathrm{D}} \triangleq d$ and thus $\overline{\gamma}_{1\mathrm{D}} = \overline{\gamma}_{2\mathrm{D}} \triangleq \overline{\gamma}_{\mathrm{D}}$. All quantizers were designed with $Q = 5$ quantization levels. We further assume that the destination can perfectly recover $\mathbf{q}$ from $\mathbf{y}_{\mathrm{RD}}$, i.e., the SNR on the relay-destination channel is sufficiently high. This assumption is not too restrictive, e.g., when the relay is deployed by an operator. The joint network-channel decoder at the destination performs four iterations to exchange information between the constituent codes. The results shown in Fig. 5 have been obtained by simulating 10^5 transmissions per SNR operating point for each scheme. Each transmission used a different random interleaver whose depth equals the block length.

AWGN Channels. We first compare our proposed scheme with 2D-VQ and sXOR-SQ for the case of AWGN channels. Here, we consider an asymmetric MARC with $d_{1\mathrm{R}} = 0.6754\cdot d$ and $d_{2\mathrm{R}} = 0.9367\cdot d$, corresponding to $[\gamma_{1\mathrm{R}}]_{\mathrm{dB}} = [\gamma_{\mathrm{D}}]_{\mathrm{dB}} + 6\,\mathrm{dB}$, $[\gamma_{2\mathrm{R}}]_{\mathrm{dB}} = [\gamma_{\mathrm{D}}]_{\mathrm{dB}} + 1\,\mathrm{dB}$. The solid (dashed) curves in Fig. 5a show the BER performance for source S_1 (S_2).

Here, the main observation is that our proposed scheme performs as well as the 2D-VQ scheme, while having a considerably lower computational complexity. In contrast to [11], we moreover observe that the sXOR-SQ scheme is able to handle asymmetric source-relay channels to some extent. Finally, we note that the performance gap between our scheme and the sXOR-SQ scheme decreases as γ_{D} increases.



(a) asymmetric AWGN channels, $Q = 5$

(b) block fading channels, $Q = 5$

Figure 5: Error rate performance in AWGN and quasi-static fading channels.

Quasi-Static Fading. Fig. 5b compares the FER performance of our proposed scheme with the sXOR-SQ scheme and a transmission without relay (at the same sum rate) for the case of quasi-static fading channels, $h_{ij} \sim \mathcal{CN}(0, 1)$. The geometry of the MARC in this case is chosen symmetric, i.e., $\overline{\gamma}_{1\mathsf{R}} = \overline{\gamma}_{2\mathsf{R}} \triangleq \overline{\gamma}_{\mathsf{R}}$. The source-relay and source-destination SNRs are related as $[\overline{\gamma}_{\mathsf{R}}]_{\mathsf{dB}} = [\overline{\gamma}_{\mathsf{D}}]_{\mathsf{dB}} + 4\,\mathrm{dB}$, corresponding to $d_{1\mathsf{R}} = d_{2\mathsf{R}} = 0.7698 \cdot d$. Due to the fading, the MARC is in general asymmetric.

We observe that our proposed scheme achieves a diversity order of two and outperforms the sXOR-SQ scheme by $0.5\,\mathrm{dB}$. Again, the proposed scheme performs not worse than the 2D-VQ scheme; in fact, it slightly outperforms the 2D-VQ scheme at medium SNR. A gain of approximately $10\,\mathrm{dB}$ is achieved compared to a transmission without relay at a FER of 10^{-2}. It is important to note that the main advantage of our scheme in this setting is that only a single set of scalar quantizers needs to be designed and stored. In contrast, for the 2D-VQ and sXOR-SQ schemes, the quantizers have to be designed for sufficiently many combinations of source-relay SNRs. (Note that the statistics of the LLRs to be quantized in the sXOR-SQ scheme depends on all source-relay SNRs.)

VI. Conclusions

We have proposed a transmission scheme for the MARC based on CF with optimal LLR quantization at the relay. By performing individual SQ for each source-relay channel we are able to achieve excellent performance also for asymmetric channel conditions while keeping the computational complexity of our scheme low. We have reviewed quantization in the information bottleneck context and we proposed an improved algorithm for the design of deterministic LLR quantizers. Finally, numerical simulation results confirm that the proposed scheme performs well in AWGN channels and achieves a diversity order of two in quasi-static fading channels.

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