JOINT NETWORK-CHANNEL CODING IN THE MULTIPLE-ACCESS RELAY CHANNEL: BEYOND TWO SOURCES

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ABSTRACT

The combination of *log-likelihood ratio* (LLR) quantization and network coding was previously shown to be a promising *compress-and-forward* strategy for the *multiple access relay channel* with two sources. In this paper, we generalize this approach to the case of more than two sources. Our proposed relay scheme consists of a scalar LLR quantizer for each source followed by a *network coding* (NC) step that suitably combines the quantizer outputs. We use the *information bottleneck method* to design the quantizers and the NC function. At the destination, an iterative message-passing decoder is used to jointly decode all source messages. Numerical simulations demonstrate the effectiveness of the proposed transmission strategy and its suitability for asymmetric source-relay channel conditions.

Index Terms— cooperative systems, relaying, network coding, quantization, iterative decoding

1. INTRODUCTION

We consider the *multiple-access relay channel* (MARC) with $N \geq 2$ sources, one relay and a destination node (cf. Fig. 1). We focus on the time-division MARC with half-duplex terminals where the sources and the relay transmit in separate time slots. These assumptions simplify practical implementation, in particular with regard to synchronization. *Network coding* (NC) [1] was recognized as an effective way to improve spectral efficiency in such a setting. Information-theoretic limits for the MARC with *decode-and-forward* (DF) and *compress-and-forward* (CF) protocols [2] have been established in [3,4]. Transmission schemes for the MARC with two sources have been developed e.g. in [5–8].

For $N = 2$, we have recently shown in [9] that scalar quantization of log-likelihood ratios (LLRs) followed by a NC step is a promising low-complexity CF strategy which is able to cope with unreliable and asymmetric source-relay channels. In this paper we generalize this approach to the case $N > 2$, which is relevant since the rate loss induced by the relay hop decreases as N increases. More specifically, our contributions are as follows:

- We propose a scalable CF-based transmission scheme for the MARC with $N \geq 2$ sources which is well suited for asymmetric source-relay channel conditions.
- We discuss network code design in the *information bottleneck* (IB) framework [10] and derive the network encoder and decoder for our scheme.
- We present an iterative joint network-channel decoder to jointly decode all source messages. We also discuss the scheduling of this message-passing decoder.

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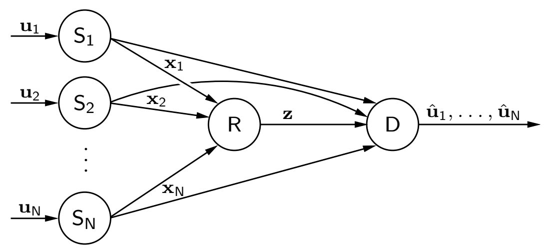

Fig. 1. The MARC with N sources.

- We compare our scheme to existing ones in terms of bit and frame error rates. Numerical results confirm that in our scheme *each* source can achieve a diversity order of two.

The remainder of this paper is organized as follows. Section 2 introduces the system model and describes the basic operation of all network nodes. In Section 3, we explain in detail the design of the LLR quantizers and the NC function at the relay. Section 4 addresses the network decoder as well as the iterative joint network-channel decoder and its scheduling. Numerical results are presented in Section 5 and conclusions are provided in Section 6.

2. SYSTEM MODEL

2.1. MARC Model

We consider the time-division MARC with N sources, $\mathsf{S}_1, \ldots, \mathsf{S}_\mathsf{N}$, a relay R, and a destination D as depicted in Fig. 1. The sources consecutively broadcast their independent messages in the first N time slots. In the $(N+1)$th time slot, the relay forwards to the destination a combination of the data it has received in the previous N time slots. Finally, the destination jointly decodes all signals received in the $N+1$ time slots. In this model, the sources do not overhear each others transmission. We assume that the total transmission time is shared equally among all sources and the relay (optimization of the time allocation is beyond the scope of this paper), i.e., each transmitting node uses the channel M times per time slot and, hence, there are $(N+1)M$ channel uses in total.

2.2. Channel Model

Source-Relay and Source-Destination Channels. Assuming Gaussian quasi-static fading channels, we have (all vectors in this expression are of length M)

$$\mathbf{y}_{ij} = d_{ij}^{-n/2} h_{ij} \mathbf{x}_i + \mathbf{w}_{ij}, \quad i \in \{1, 2, \ldots, \mathsf{N}\},\ j \in \{\mathsf{R}, \mathsf{D}\}, \quad (1)$$

where $\mathbf{x}_i$ is the signal transmitted by node i, $\mathbf{y}_{ij}$ is the corresponding receive signal at node j, d_{ij} is the distance between nodes i and j, n is the path-loss exponent, $\mathbf{w}_{ij} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ is white

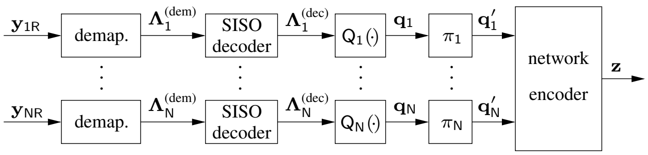

Fig. 2. Relay block diagram.

Gaussian noise (assumed zero-mean and circularly symmetric), and h_{ij} denotes the channel coefficient. We impose a transmit energy constraint at each node, i.e., we fix $E_s \triangleq \mathrm{E}\big[\|\mathbf{x}_i\|_2^2\big]$, where $\mathrm{E}[\cdot]$ and $\|\cdot\|_2$ denote expectation and ℓ_2-norm. The signal-to-noise ratio (SNR) for the link between node i and node j is given by $\gamma_{ij} = d_{ij}^{-n}|h_{ij}|^2 P_s/N_0$, where P_s denotes the transmit power. The average SNR is denoted by $\overline{\gamma}_{ij} = \mathrm{E}[\gamma_{ij}]$. The MARC is called *asymmetric* if $\gamma_{i_1\mathsf{R}} \neq \gamma_{i_2\mathsf{R}}$ for any i_1, i_2. We assume that each node only has receive *channel state information* (CSI), i.e., γ_{ij} is known at node j. In particular, the relay has no source-destination CSI.

Relay-Destination Channel. We assume a rate constraint on the relay-destination channel, such that the relay can reliably transmit data to the destination at a rate of at most R_0 bits per channel use (bpcu). This constraint imposes an upper bound on the entropy of the data produced by the NC function at the relay.

2.3. Sources

Source i ($i \in \{1, 2, \ldots, \mathsf{N}\}$) generates a length-$K_i$ sequence $\mathbf{u}_i \in \{0,1\}^{K_i}$ of independent and equally likely bits. The sequence $\mathbf{u}_i$ is channel encoded using a linear binary code $\mathcal{C}_i$ of rate $R_i = K_i/L_i$, yielding a length-L_i sequence $\mathbf{c}_i \in \{0,1\}^{L_i}$ of code bits. Next, the code bits are mapped to a signal constellation $\mathcal{A}_i$ of cardinality $|\mathcal{A}_i| = 2^{m_i}$, yielding length-$M$ sequences $\mathbf{x}_i \in \mathcal{A}^M$ of transmit symbols. The code rates are chosen as $R_i = K_i/(m_i M)$. For simplicity of exposition, we assume $K \triangleq K_i$, $L \triangleq L_i$ and, hence, $R \triangleq R_i$, $m \triangleq m_i$. The sum rate is then given by $R_s = mRN/(N+1)$ and the rate-loss due to the relay time slot is $R_\triangle = m/(N+1)$. This shows that the rate loss $R_\triangle$ decreases like $1/N$.

2.4. Relay

Fig. 2 shows a block diagram of the relay, which computes the integer-valued sequence $\mathbf{z}$ by jointly processing all signals $\mathbf{y}_{i\mathsf{R}}$ received in the first N time slots. The relay first performs soft demapping of each received signal, yielding an LLR vector $\mathbf{\Lambda}_i^{(\mathrm{dem})}$. These LLRs are passed to a soft-input soft-output (SISO) channel decoder which produces LLRs $\mathbf{\Lambda}_i^{(\mathrm{dec})}$ for the code bits $\mathbf{c}_i$. Next, $\mathbf{\Lambda}_i^{(\mathrm{dec})}$ is processed by a scalar quantizer $\mathsf{Q}_i(\cdot)$ with Q_i levels (cf. Section 3), i.e., $\mathbf{q}_i \in \mathcal{Q}_i^L$, where $\mathcal{Q}_i \triangleq \{1, 2, \ldots, Q_i\}$. The quantizer indices $\mathbf{q}_i$ are then interleaved, $\mathbf{q}'_i = \pi_i(\mathbf{q}_i)$, to avoid short cycles in the factor graph of the overall code (cf. Fig. 3). We note that one of the interleavers π_i can be omitted. Finally, the network encoder performs an element-wise mapping of all sequences $\mathbf{q}'_i$ to $\mathbf{z} \in \mathcal{Z}^L$, where $Z = |\mathcal{Z}|$ is chosen such that $H(z_k) \leq R_0$ ($k = 1, 2, \ldots, L$), where $H(\cdot)$ denotes entropy. Since $H(z_k) \leq \log_2(Z)$, the average compression rate per code bit $c_{i,k}$ is upper bounded by $\log_2(Z)/N$ bits, i.e., it also decreases like $1/N$. A detailed description of the quantization and NC stages is given in Section 3.

2.5. Destination

The destination jointly decodes the received signals $\mathbf{y}_{1\mathsf{D}}, \ldots, \mathbf{y}_{\mathsf{ND}}$, and $\mathbf{z}$ to obtain detected messages $\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_{\mathsf{N}}$. To this end, all source observations are first processed by soft demappers. Iterative joint network-channel decoding on the overall factor graph is then performed via the sum-product algorithm [11]. This corresponds to invoking the network and channel decoders according to a given schedule and exchanging extrinsic information between the individual decoders. Section 4 describes the decoder in more detail.

3. QUANTIZATION AND NETWORK CODING

3.1. LLR Quantization

The relay uses N scalar quantizers $\mathsf{Q}_i(\cdot)$ to map the LLRs $\mathbf{\Lambda}_i^{(\mathrm{dec})} \in \mathbb{R}^L$ to index vectors $\mathbf{q}_i \in \mathcal{Q}_i^L$, where $q_{i,k} = \mathsf{Q}_i(\Lambda_{i,k}^{(\mathrm{dec})})$. The design of the LLR quantizers is critical for the performance of the system. We aim at maximizing the mutual information $I(c_i; q_i)$ between the quantizer index q_i and the corresponding code bit c_i for a fixed number of quantization levels Q_i, i.e., we want to solve[1]

$$p^\star(q_i|\Lambda_i) = \underset{p(q_i|\Lambda_i)\in\{0,1\}}{\arg\max}\; I(c_i; q_i). \tag{2}$$

The deterministic quantizer $\mathsf{Q}_i(\cdot)$ is described by $p^\star(q_i|\Lambda_i) \in \{0,1\}$. Given the joint distribution $p(c_i, \Lambda_i) = p(\Lambda_i|c_i)p(c_i)$, the modified IB algorithm proposed in [9] allows us to find a locally optimal solution of (2). We note that the optimal choice of Q_i is SNR-dependent and $Q_i \leq 5$ is usually sufficient.

Since in most cases the joint distribution $p(c_i, \Lambda_i)$ has to be found using Monte Carlo simulations, quantizer design is performed offline. The relay stores a single set of quantizers for a sufficiently wide range of SNRs and then uses for each channel the quantizer that has been optimized for the corresponding average SNR $\overline{\gamma}_{i\mathsf{R}}$.

3.2. Network Coding

Let $\mathbf{q} = (q_1\ q_2\ \ldots\ q_{\mathsf{N}})^T$ denote a length-N vector consisting of the kth elements of the interleaved quantization index sequences $\mathbf{q}'_i$. Similarly, $\mathbf{c} = (c_1\ c_2\ \ldots\ c_{\mathsf{N}})^T$ denotes the vector of code bits corresponding to the quantization indices in $\mathbf{q}$. Furthermore, $\mathbf{a}_{\sim i}$ denotes the vector obtained by removing the ith element from $\mathbf{a}$.

Network encoding is performed at the relay using a deterministic function $g : \mathcal{Q}_1 \times \mathcal{Q}_2 \times \ldots \times \mathcal{Q}_{\mathsf{N}} \mapsto \mathcal{Z}$ which maps all $\mathbf{q}_i$, for each element $k = 1, 2, \ldots, L$ separately, to an integer $z = g(\mathbf{q}) \in \mathcal{Z}$, i.e., $\mathbf{z} = (z_1\ z_2\ \ldots\ z_L)^T$. The design of the NC function is motivated by the iterative decoding procedure at the destination (see Section 4 for details). To maximize the information exchange between the individual channel decoders we seek to maximize $I(c_i; z|\mathbf{c}_{\sim i})$ for each $i \in \{1, 2, \ldots, \mathsf{N}\}$. Loosely speaking, given perfect a priori information $\mathbf{c}_{\sim i}$, z should contain as much information about c_i as possible. However, since $I(c_i; z|\mathbf{c}_{\sim i})$ cannot be maximized for each i independently, we resort to maximizing a function of these mutual information expressions. Extending the case $N = 2$ (see [9]), we propose to use the average as relevant information, i.e.,

$$\begin{aligned} I_{\mathrm{rel}} &= \frac{1}{N}\sum_{i=1}^{N} I(c_i; z|\mathbf{c}_{\sim i}) = \frac{1}{N}\sum_{i=1}^{N} I(c_i; g(\mathbf{q})|\mathbf{c}_{\sim i}) \\ &= I\big(\mathbf{c}; g(\mathbf{q})\big) - \frac{1}{N}\sum_{i=1}^{N} I\big(\mathbf{c}_{\sim i}; g(\mathbf{q})\big) \\ &= I(\mathbf{c}; \mathbf{q}) - I\big(\mathbf{c}; \mathbf{q}|g(\mathbf{q})\big) \\ &\quad - \frac{1}{N}\sum_{i=1}^{N}\Big(I(\mathbf{c}_{\sim i}; \mathbf{q}_{\sim i}) - I\big(\mathbf{c}_{\sim i}; \mathbf{q}_{\sim i}|g(\mathbf{q})\big)\Big), \end{aligned} \tag{3}$$

where we used the chain rule of mutual information and the fact that $\mathbf{c} \leftrightarrow \mathbf{q} \leftrightarrow z = g(\mathbf{q})$ forms a Markov chain. For fixed Z, the NC

[1] For the sake of notational clarity, the bit position index k is suppressed since the operations on all bit position are identical.

Algorithm 1 *IB algorithm for finding the NC function*

Input: $p(\mathbf{c}, \mathbf{q})$, $\varepsilon > 0$, $n > 0$, N, $\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_{\mathsf{N}}$, $\mathcal{Z}$
Initialization: $\overline{C}_{-1} \leftarrow \infty$, $\delta \leftarrow \infty$, $\ell \leftarrow 0$, randomly initialize $C_0(z, \mathbf{q}) \in \mathbb{R}$ for all $z \in \mathcal{Z}$ and $\mathbf{q} \in \mathcal{Q}_1 \times \mathcal{Q}_2 \times \ldots \times \mathcal{Q}_{\mathsf{N}}$

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while δ ≥ ε and ℓ < n do
    for all q do
        z* ← arg min_{z∈Z} C_ℓ(z, q)
        p_ℓ(z|q) ← I{z = z*},  z ∈ Z
    end for
    p_ℓ(z) ← Σ_q p_ℓ(z|q) p(q)
    p_ℓ(c|z) ← Σ_q p(c, q) p_ℓ(z|q) / p_ℓ(z)
    C_ℓ(z, q) ← D(p(c|q) || p(c|z))
                 − Σ_i D(p(c_{∼i}|q_{∼i}) || p(c_{∼i}|z)) / N
    C̄_ℓ ← Σ_{z,q} p_ℓ(z|q) p(q) C_ℓ(z, q)
    δ ← |C̄_{ℓ−1} − C̄_ℓ| / |C̄_ℓ|
    ℓ ← ℓ + 1
end while
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optimization can be written as

$$g^{\star} = \arg\max_{g} I_{\text{rel}}. \tag{4}$$

Writing mutual information in terms of the Kullback-Leibler divergence $\mathrm{D}(\cdot\|\cdot)$, we can reformulate (4) as

$$p^{\star}(z|\mathbf{q}) = \underset{p(z|\mathbf{q})\in\{0,1\}}{\arg\min} \Big\{ \mathrm{E}\big[\mathrm{D}\big(p(\mathbf{c}|\mathbf{q})\|p(\mathbf{c}|z)\big)\big] - \frac{1}{N}\sum_{i=1}^{N} \mathrm{E}\big[\mathrm{D}\big(p(\mathbf{c}_{\sim i}|\mathbf{q}_{\sim i})\|p(\mathbf{c}_{\sim i}|z)\big)\big] \Big\}, \tag{5}$$

where g is specified by the deterministic mapping $p^{\star}(z|\mathbf{q})$. We note that randomized network coding cannot improve performance since it can be shown that optimizing with respect to all $p(z|\mathbf{q}) \in [0,1]$ in (5) yields the same minimizer $p^{\star}(z|\mathbf{q}) \in \{0,1\}$. Furthermore, the choice of the relevant information in (3) ensures that the data of the individual sources has large (little) impact on z if the respective source-relay SNR is high (low). Hence, user selection is of minor importance in our scheme, although it might still be useful to limit the decoding complexity at the destination.

The problem in (5) can be recognized as an instance of the IB problem, which can be solved using an appropriately modified version of the IB algorithm from [9] which is summarized in Algorithm 1 on the top of this page ($\mathrm{I}\{\cdot\}$ denotes the indicator function which equals 1 if the argument is true and 0 otherwise). The joint distribution $p(\mathbf{c}, \mathbf{q}) = \prod_{i=1}^{N} p(q_i|c_i)p(c_i)$ is required by Algorithm 1; $p(q_i|c_i)$ is obtained from the design of the scalar LLR quantizers and $p(c_i)$ is known a priori. Algorithm 1 delivers a locally optimal encoding $p(z|\mathbf{q}) \in \{0,1\}$, which can be implemented using an N-dimensional lookup table that is indexed by $\mathbf{q}$. Algorithm 1 can also be used to design the NC function on-the-fly during data transmission since $p(\mathbf{c}, \mathbf{q})$ is known once the relay has fixed the LLR quantizers. Hence, it is not necessary to store precomputed NC functions for all possible source-relay SNR combinations.

4. JOINT NETWORK-CHANNEL DECODER

The processing at the relay creates an equivalent discrete memoryless channel with transition pmf $p(z|\mathbf{c}') \triangleq p(z|c_1', c_2', \ldots, c_{\mathsf{N}}')$ that is known once the NC function is fixed; here $\mathbf{c}_i' = \pi_i(\mathbf{c}_i)$. The pmf

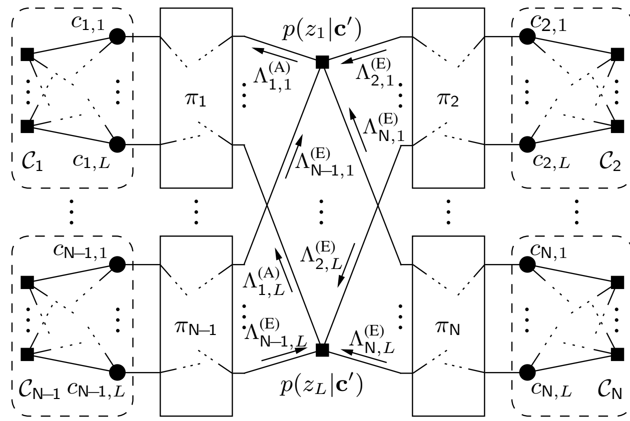


Fig. 3. Factor graph of the overall code.

$p(z|c_1', c_2', \ldots, c_{\mathsf{N}}')$ couples the code bits of all sources and hence enables joint decoding of the source codewords.

The operation of the joint network-channel decoder is such that the individual channel decoders iteratively exchange extrinsic LLRs via the network decoder (6)-(8). Fig. 3 shows the factor graph of the overall code, including extrinsic LLRs.

The network decoder computes a priori LLRs $\Lambda^{(\mathrm{A})}_{i,k}$ for the ith channel decoder using $N-1$ extrinsic LLRs $\Lambda^{(\mathrm{E})}_{j,k}$ (for all $j \neq i$) and the local function $p(z_k|c_1', c_2', \ldots, c_{\mathsf{N}}')$, where the respective value of z_k is known due to the transmission of the relay. The sum-product update rule for the a priori LLRs $\Lambda^{(\mathrm{A})}_{i,k}$ $(k = 1, 2, \ldots, L)$ is given by

$$\Lambda^{(\mathrm{A})}_{i,k} = \log \frac{\mu_{p\to c_{i,k}'}(c_{i,k}'=0)}{\mu_{p\to c_{i,k}'}(c_{i,k}'=1)}, \quad i = 1, 2, \ldots, N, \tag{6}$$

where

$$\mu_{p\to c_{i,k}'}(c_{i,k}') = \sum_{\mathbf{c}_{\sim i}'} p(z_k|\mathbf{c}') \prod_{j:j\neq i} \mu_{c_{j,k}'\to p}(c_{j,k}'), \tag{7}$$

and

$$\mu_{c_{j,k}'\to p}(c_{j,k}'=b) = \frac{\exp\big(-b\Lambda^{(\mathrm{E})}_{j,k}\big)}{1+\exp\big(-\Lambda^{(\mathrm{E})}_{j,k}\big)}, \quad b \in \{0,1\}. \tag{8}$$

Several message-passing schedules are possible, the most common being *flooding* and *serial* schedules. In the flooding schedule, *all* channel decoders update the extrinsic LLRs and in the next step the network decoder updates *all* a priori LLRs. In contrast, for the serial schedule only *one* channel decoder updates its extrinsic LLRs and then the network decoder updates the a priori LLRs only for the *next scheduled* channel decoder. The complexity per iteration of the joint network-channel decoder is the same for both schedules (N instances of both the channel decoder and the network decoder).

We mention two strategies for making the pmf $p(z|\mathbf{c}')$ used by the network decoder available to the destination. One possibility is to communicate $p(z|\mathbf{c}')$ directly from the relay to the destination. However, this approach leads to high communication overhead since $Z2^N$ probabilities have to be transmitted with sufficiently high accuracy. The second strategy is to store the set of scalar quantizers also at the destination and communicate only the N integer-valued indices which reflect the quantizer choice at the relay. Then, the relay runs Algorithm 1 to obtain $p(z|\mathbf{c}')$. This strategy is clearly preferable if computational overhead at the destination is cheaper

Table 1. Changes in the system parameters for $N = 2, 3, \ldots, 8$. Percentage changes relative to $N = 2$ are given in parentheses.

N	R	$\log_2(Z)/N$	R_Δ
2	0.75	1.16	0.67
3	0.67 (-11.1 %)	0.77 (-33.3 %)	0.50 (-25.0 %)
4	0.63 (-16.7 %)	0.58 (-50.0 %)	0.40 (-40.0 %)
5	0.60 (-20.0 %)	0.46 (-60.0 %)	0.33 (-50.0 %)
6	0.58 (-22.2 %)	0.39 (-66.7 %)	0.29 (-57.1 %)
7	0.57 (-23.8 %)	0.33 (-71.4 %)	0.25 (-62.5 %)
8	0.56 (-25.0 %)	0.29 (-75.0 %)	0.22 (-66.7 %)

than communication overhead on the relay-destination link. In any case, we assume that $p(z|\mathbf{c}')$ is available at the destination and neglect the signaling overhead.

5. NUMERICAL RESULTS

In this section we assess the performance of our proposed scheme by Monte Carlo simulations. We analyze the bit error rate (BER) and the frame error rate (FER) for the transmission over AWGN and quasi-static fading channels, respectively.

General Setup. Each source transmits $K = 512$ information bits and we fix the sum-rate to $R_s = 1$. The sources transmit using a QPSK signal constellation ($m = 2$) normalized to power P_s. The channel code is a recursive, systematic convolutional code with generator polynomial $[1\ 13/15]_8$ (in octal notation), punctured to a code rate of $R = (N+1)/(2N)$. We set $E_s = 2K/3$ and, hence, $P_s = 2(N+1)/(3N)$, which corresponds to $P_s = 1$ for $N = 2$ sources. Due to the processing at the relay, the overall code is a parallel concatenated code with block length NL, where $L = K/R$ is the block length of the individual codes. The path-loss exponent in (1) is chosen as $n = 3.52$, according to the Okumura-Hata model [12]. We assume that the source-destination channels are symmetric, i.e., $d \triangleq d_{i\mathsf{D}}$ and thus $\overline{\gamma}_{\mathsf{D}} \triangleq \overline{\gamma}_{i\mathsf{D}}$. The rate supported by the relay-destination channel is $R_0 = 2.45$ bpcu. The scalar quantizers at the relay use (at most) 5 quantization levels. Due to the rate constraint on the relay-destination channel we choose to design the NC function with $Z = 5$ which ensures that $H(z_k) \leq \log_2(Z) \leq R_0$. The joint network-channel decoder at the destination performs 5 iterations and uses a serial schedule unless noted otherwise. We have used random interleavers with depths equal to the block length L.

In our setting, the code rate R decreases as N increases and, at the same time, the rate per source at the relay decreases. The changes in the system parameters for varying N are summarized in Table 1. We observe that the decrease in compression rate (which is upper bounded by $\log_2(Z)/N$) is significantly stronger than the decrease of the code rate. The rate loss R_Δ is reduced by a factor of 3 when the number of sources is increased from 2 to 8.

Symmetric Case. We first study the performance of our proposed scheme in the symmetric MARC with AWGN channels ($h_{ij} \equiv 1$) and $d_{i\mathsf{R}} = 0.6754 \cdot d$, corresponding to $[\gamma_{i\mathsf{R}}]_{\mathsf{dB}} = [\gamma_{\mathsf{D}}]_{\mathsf{dB}} + 6\,\mathrm{dB}$. Fig. 4a shows the BER performance of the proposed scheme for 2 to 8 sources. For comparison the performance without relay (at the same sum rate) is also given.

We observe that the performance is significantly improved when we add more sources, except for the low-SNR regime where we observe an increased decoding threshold. The SNR gain saturates as the number of sources increases (law of diminishing returns). At a BER of 10^{-4} the system with 8 sources gains $1.6\,\mathrm{dB}$ over the system with 2 sources. We note that this behavior is observed for a wide range of sum rates and MARC geometries.

Decoder Scheduling. Next, we analyze the convergence behavior of the iterative joint network-channel decoder. We again consider the symmetric non-fading MARC with the same geometry as above. Fig. 4b depicts the BER performance for 5 sources and $0, 1, \ldots, 30$ decoder iterations for the serial schedule (solid lines) and the flooding schedule (dashed lines).

We observe that the serial schedule clearly outperforms the flooding schedule. With increasing SNR the decoder converges after fewer iterations and the difference between the schedules becomes less pronounced. For the serial schedule, 4 iterations are sufficient if $\gamma_{\mathsf{D}} \geq -2\,\mathrm{dB}$ and 2 iterations are sufficient if $\gamma_{\mathsf{D}} \geq -0.5\,\mathrm{dB}$. We note that the ordering of the individual channel decoders in the serial schedule has no measurable influence on the performance.

Asymmetric Case. We now turn our attention to the asymmetric MARC with AWGN channels and 4 sources. The geometry is such that $[\gamma_{1\mathsf{R}}]_{\mathsf{dB}} = [\gamma_{\mathsf{D}}]_{\mathsf{dB}} + 6\,\mathrm{dB}$, $[\gamma_{2\mathsf{R}}]_{\mathsf{dB}} = [\gamma_{\mathsf{D}}]_{\mathsf{dB}} + 13/3\,\mathrm{dB}$, $[\gamma_{3\mathsf{R}}]_{\mathsf{dB}} = [\gamma_{\mathsf{D}}]_{\mathsf{dB}} + 8/3\,\mathrm{dB}$, and $[\gamma_{4\mathsf{R}}]_{\mathsf{dB}} = [\gamma_{\mathsf{D}}]_{\mathsf{dB}} + 1\,\mathrm{dB}$. Fig. 5a shows the BER performance for each source for our proposed scheme (solid lines) and for a scheme which performs "boxplus encoding" (cf. [13]) at the relay (dashed lines). Additionally we show the performance of a scheme which performs plain "XOR encoding" at the relay when all source messages were decoded successfully. For comparison the performance without relay (at the same sum rate) is also given.

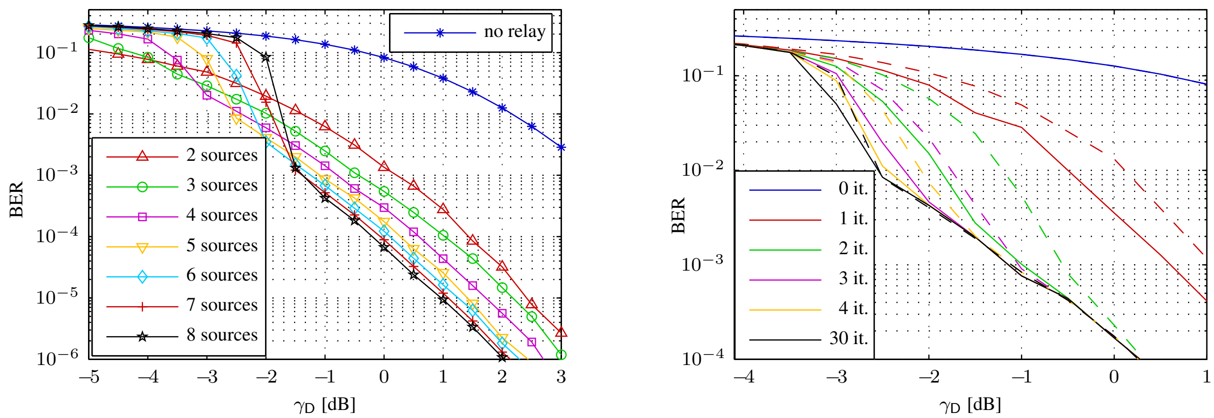
We observe that our proposed scheme is able to handle asymmetric channel conditions very well, especially in the low-SNR regime. In contrast, for the boxplus scheme the performance is the same for all sources if $\gamma_{\mathsf{D}} \leq 0\,\mathrm{dB}$. A major advantage of our scheme is that only a single set of scalar quantizers needs to be designed and stored, whereas for the boxplus scheme the quantizers have to be designed for all possible combinations of source-relay SNRs. At high SNR, our scheme is outperformed by the boxplus scheme for the sources with worse source-relay SNR. It might be possible to eliminate this drawback by adapting the relevant information (3) to the SNR. A system performing XOR encoding at the relay does even worse (for all sources) than a transmission without relay, since the probability of decoding *all* source messages is low for an asymmetric MARC and decreases with increasing N.

Quasi-Static Fading. We next consider the MARC with quasi-static fading channels, $h_{ij} \sim \mathcal{CN}(0, 1)$. The geometry of the MARC is chosen symmetric, i.e., $\overline{\gamma}_{\mathsf{R}} \triangleq \overline{\gamma}_{i\mathsf{R}}$ and the average source-relay and source-destination SNRs are related as $[\overline{\gamma}_{\mathsf{R}}]_{\mathsf{dB}} = [\overline{\gamma}_{\mathsf{D}}]_{\mathsf{dB}} + 4\,\mathrm{dB}$. Due to the fading, the MARC is in general asymmetric. Fig. 5b compares the average FER performance of our proposed scheme for 2 and 8 sources with a transmission without relay (at the same sum rate).

We observe that the performance is essentially unchanged when the number of sources is increased from 2 to 8. Moreover, our proposed scheme simultaneously provides a diversity order of two for *all* sources. A gain of approximately $9\,\mathrm{dB}$ compared to a transmission without relay is achieved for 8 sources at an FER of 10^{-2}. Finally, we note that the processing at the relay is simple since only a single set of scalar quantizers is needed and the NC function can be designed online and depends only on the average source-relay SNRs.

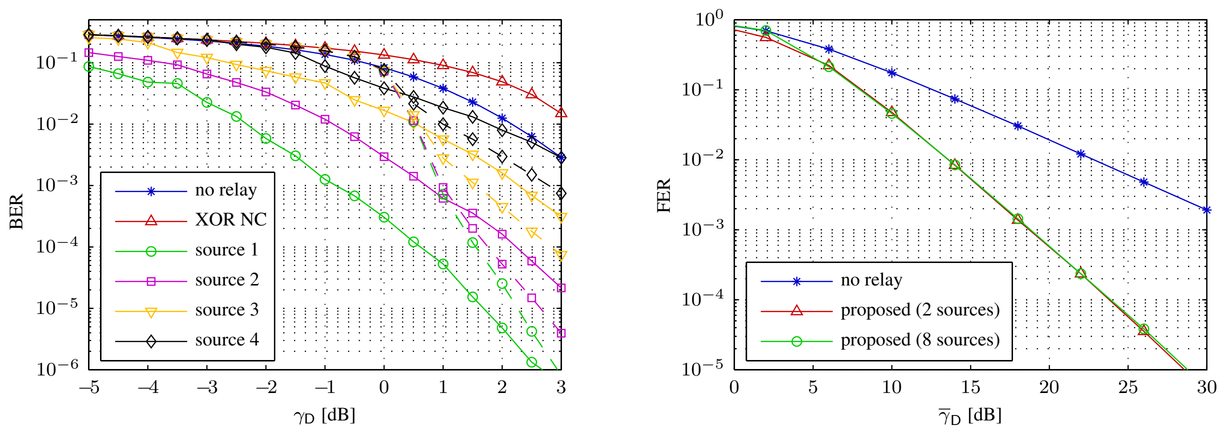
6. CONCLUSIONS

We have proposed a scalable CF-based transmission scheme for the MARC with more than two sources. This scheme performs optimal LLR quantization and NC based on the IBM at the relay. We have shown that the system performance can be improved substantially by adding more sources. The proposed scheme also achieves excellent performance for asymmetric source-relay channel conditions. We have furthermore analyzed the convergence behavior of the iterative joint network-channel decoder for which we proposed to use a serial



(a) (b)

Fig. 4. Symmetric Gaussian MARC: (a) BER performance and (b) decoder convergence with serial (solid) and flooding (dashed) schedules.



(a) (b)

Fig. 5. Asymmetric MARC: (a) AWGN channels and (b) quasi-static fading channels.

message-passing schedule. Finally, simulation results confirm that the proposed scheme simultaneously achieves a diversity order of two for all sources in quasi-static fading channels.

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