ISBN 978-3-200-02328-4

Performance Evaluation of LTE Advanced Downlink Channel Estimators

Michael Meidlinger and Qi Wang

Institute of Telecommunications, Vienna University of Technology
Gusshausstrasse 25/389, A-1040 Vienna, Austria
Email: {mmeidlin, qwang}@nt.tuwien.ac.at
Web: http://www.nt.tuwien.ac.at/research/mobile-communications

***Abstract*—In this paper we apply and assess the performance of different channel estimation algorithms to the new reference signal structure standardized for LTE Advanced in the 3GPP Release 10 standards. The different reference signals are presented and they are employed to obtain channel estimates for demodulation and feedback calculation. Furthermore, we point out the advantages and drawbacks of the LTE Advanced reference signals in comparison to the LTE reference symbol structure. Finally we provide simulation results to evaluate the capabilities of the designed channel estimators. All data is available online for reproducibility.**

I. Introduction and Overview

The 3GPP Long Term Evolution (LTE) Standard Release 10 [1], commonly referred to as Long Term Evolution Advanced (LTE-A), supports a variety of new features compared to Release 8 [2] in order to reach the targets for 4G communications. These new properties, such as Coordinated Multipoint (CoMP) and enhanced Multiple-Input Multiple-Output (MIMO) transmission call for new Reference Signals (RS) that guarantee efficient implementation of those.

From a channel estimation point of view, Multi User MIMO (MU-MIMO) transmission makes out the major difference between LTE and LTE-A. In order to fully utilize the benefits of this transmission scheme, the additional degrees of freedom in the spatial domain have to be taken into account. This is done by introducing a transmit mode that is capable of dynamically adjusting the number of downlink transmission layers [3]. Consequently, the channel as we knew it from LTE no longer exists in that regular form and we have to find means to obtain it in a way that guarantees both efficient equalization and keeps the computational effort for the estimation small.

In this paper, we derive channel estimators fulfilling the above requirements by using the LTE-A RS structure. First of all, we will briefly discuss how channel estimation was done in LTE and why this approach is suboptimal for the case of LTE-A. The main Section III then introduces the channel model and the RS structure of LTE-A. Furthermore, estimators based on Least Squares (LS) and Linear Minimum Mean Square Error (LMMSE) strategies are derived. Afterwards, we point out the consequences that come along with the usage of the new RS structure. Section V then evaluates the performance of the estimators using the standard compliant LTE-A Link Level Simulator [4, 5]. Finally, Section VI concludes the work.

II. Reference Signals in LTE

In Release 8, downlink channel estimation is based on Cell-specific Reference Signals (CRS). The term CRS stems from the fact that all User Equipments (UEs) in a given cell can employ the CRS to estimate the channel from the evolved base station (eNodeB) to their location. In that sense, CRS are used both for demodulation and feedback calculation. Although this approach seems self-evident at a first glance, it imposes several restrictions on the system: Since the feedback for the eNodeB is obtained via CRS, they are inserted after the precoding. Thus, precoding information has to be conveyed to the User Equipment (UE). As a result, such a design does not allow for non-codebook based precoding with reasonable overhead. This makes up a radical restriction, when considering that according to [3], the codebooks are the limiting factors regarding the system performance of LTE-A MU-MIMO transmission. In addition, this constraint complicates efficient MU-MIMO equalization, as we will explain in Section IV-A.

Moreover, CRS were not designed to support eight antenna transmissions. Therefore, extending CRS to eight antennas would lead to an unacceptable increase in overhead.

III. LTE Advanced Channel estimation

In order to overcome the drawbacks mentioned in Section II, LTE-A proposes the usage of a RS structure that distinguishes between feedback calculation and demodulation. The former is done by using CSI Reference Signals (CSI-RS), a type of RS that is very sparse in time and frequency. CSI-RS are intended to estimate the *physical* channel over the entire transmission band and hence are inserted in the *antenna domain*, i.e. after the precoding. In contrast, UE-specific RS or Demodulation-RS (DM-RS) aim to estimate the *effective* channel, i.e., the channel together with the precoder and are consequently inserted in the *layer domain* before the precoder.

A. Demodulation Reference Signals

Figure 1 shows the distribution of the DM-RS over two resource blocks for the case of M transmission layers and normal Cyclic Prefix [1]. We denote the time index as k,

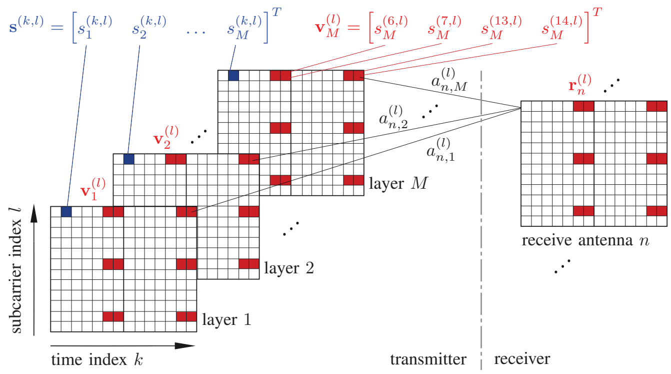

Fig. 1. Overview of the time-frequency domain

subcarrier index as l, layer index as m and receive antenna index as n. Here the vector $\mathbf{s}^{(k,l)} \in \mathbb{C}^{M\times 1}$ represents the symbols transmitted on time-frequency position (k,l). For the moment, we assume the channel to be quasi-static and the precoder to be fixed within one subframe. At the receiver, we then observe the vector

$$\mathbf{r}^{(k,l)} = \underbrace{\mathbf{H}^{(l)}\mathbf{P}^{(l)}}_{=:\mathbf{A}^{(l)}}\mathbf{s}^{(k,l)} + \mathbf{n}^{(k,l)}\,, \tag{1}$$

where $\mathbf{H}^{(l)} \in \mathbb{C}^{N_r\times N_t}$, $\mathbf{P}^{(l)} \in \mathbb{C}^{N_t\times M}$, $\mathbf{A}^{(l)} \in \mathbb{C}^{N_r\times M}$ and $\mathbf{n}^{(k,l)} \in \mathbb{C}^{N_r\times 1}$ are the channel, the precoding matrix, the effective channel and the noise at subcarrier index l, respectively. The integers N_t and N_r denote the number of transmit and receive antennas.

On top of that, Figure 1 also depicts the vectors $\mathbf{v}_m^{(l)}$, $m \in \{1,2,\dots,M\}$, of dimension 4 that subsume the RS at subcarrier l and layer m.

With these assumptions we can write the received RS vector at subcarrier l and receive antenna n as follows:

$$\mathbf{r}_n^{(l)} = \sum_{m=1}^{M} a_{n,m}^{(l)}\mathbf{v}_m^{(l)} + \mathbf{n}_n^{(l)}\,. \tag{2}$$

In Equation (2), $a_{n,m}^{(l)}$ denotes the entry at position (n,m) of $\mathbf{A}^{(l)}$ and $\mathbf{n}_n^{(l)}$ represents the additive noise.

To ensure appropriate demodulation, the LTE-A Standard requires the vectors $\mathbf{v}_m^{(l)}$ to have a certain structure, depending on the number of layers M.

If, for instance, $M = 8$, the Standard defines 4 out of 8 vectors to be zero and the remaining 4 vectors to be mutually orthogonal. The exact layer indices of the zero vectors depend on the subcarrier index l. However, this dependence is the same for all pilot subcarrier indices $l \in \mathcal{L} = \{l_1, l_2, \dots, l_I\}$, where I is the ammount of DM-RS in the frequency domain. This means if, e.g., the layer indices of the zero vectors are (3,4,7,8) we can state that

$$\mathbf{0} = \mathbf{v}_3^{(i)} = \mathbf{v}_4^{(i)} = \mathbf{v}_7^{(i)} = \mathbf{v}_8^{(i)}\,, \tag{3}$$

$$\mathbf{v}_1^{(i)} \perp \mathbf{v}_2^{(i)} \perp \mathbf{v}_5^{(i)} \perp \mathbf{v}_6^{(i)}\,, \qquad \forall i \in \mathcal{L}. \tag{4}$$

Combining Equation (2) and Equation (3) we obtain

$$\mathbf{r}_n^{(l)} = a_{n,1}^{(l)}\mathbf{v}_1^{(l)} + a_{n,2}^{(l)}\mathbf{v}_2^{(l)} + a_{n,5}^{(l)}\mathbf{v}_5^{(l)} + a_{n,6}^{(l)}\mathbf{w}_6^{(l)} + \mathbf{n}_n^{(l)} \tag{5}$$

and after taking the inner product with $\mathbf{v}_1^{(l)}$, using Equation (4) and dividing by $\|\mathbf{v}_1^{(l)}\|^2$, Equation (5) becomes

$$\underbrace{\frac{\langle \mathbf{r}_n^{(l)}, \mathbf{v}_1^{(l)}\rangle}{\|\mathbf{v}_1^{(l)}\|^2}}_{=:\tilde{r}_{n,1}^{(l)}} = a_{n,1}^{(l)} + \underbrace{\frac{\langle \mathbf{n}_n^{(l)}, \mathbf{v}_1^{(l)}\rangle}{\|\mathbf{v}_1^{(l)}\|^2}}_{=:\tilde{n}_{n,1}^{(l)}}\,. \tag{6}$$

Now we can take up two different positions: We can either assume that all quantities in Equation (6) are deterministic and consequently regard $\mathbf{n}_n^{(l)}$ as a deterministic error that we ignore, which will lead to the LS solution, or we assume them to be random with known second order statistics, which leads to the LMMSE solution.

1) Least Squares Channel Estimator: The LS Channel Estimator can be formulated as

$$\hat{a}_{n,m}^{(l)\,\mathrm{LS}} = \arg\min_{a_{n,m}^{(l)}} \left\| \mathbf{r}_n^{(l)} - \sum_{m=1}^{M} a_{n,m}^{(l)}\mathbf{v}_m^{(l)} \right\|_2^2 \tag{7}$$

and the solution can be obtained by generalizing Equation (6) to arbitrary layers m:

$$\hat{a}_{n,m}^{(l)\,\mathrm{LS}} = \frac{\langle \mathbf{r}_n^{(l)}, \mathbf{v}_m^{(l)}\rangle}{\|\mathbf{v}_m^{(l)}\|^2}. \tag{8}$$

Appropriate interpolation has to be performed between the subcarrier indices l where there are no reference symbols.

2) Linear Minimum Mean Squared Error Estimator: The LMMSE Estimator takes into account the autocorrelation matrix in the frequency domain and the second order noise statistics: If we assume that the elements of $\mathbf{n}_n^{(l)}$ in Equation (6) are i.i.d. complex-valued Gaussian random variables with variance σ^2, $\tilde{n}_{n,m}^{(l)}$ is again distributed according to $\mathcal{CN}(0,\sigma^2)$. Generalizing (6) to arbitrary elements $a_{n,m}^{(l)}$ and assembling them in a vector over frequency yields

$$\underbrace{\begin{bmatrix} \tilde{r}_{n,m}^{(l_1)} \\ \tilde{r}_{n,m}^{(l_2)} \\ \vdots \\ \tilde{r}_{n,m}^{(l_I)} \end{bmatrix}}_{\tilde{\mathbf{r}}_{n,m}} = \underbrace{\begin{bmatrix} a_{n,m}^{(l_1)} \\ a_{n,m}^{(l_2)} \\ \vdots \\ a_{n,m}^{(l_I)} \end{bmatrix}}_{\mathbf{a}_{n,m}^{\mathrm{ref}}} + \underbrace{\begin{bmatrix} \tilde{n}_{n,m}^{(l_1)} \\ \tilde{n}_{n,m}^{(l_2)} \\ \vdots \\ \tilde{n}_{n,m}^{(l_I)} \end{bmatrix}}_{\tilde{\mathbf{n}}_{n,m}}\,. \tag{9}$$

Since we want to estimate over the whole frequency range $l = 1,2,\dots,L$ we can reformulate (9) in terms of the vector $\mathbf{a}_{n,m} = \left[a_{n,m}^{(1)}, a_{n,m}^{(2)}, \dots, a_{n,m}^{(L)}\right]^T$:

$$\tilde{\mathbf{r}}_{n,m} = \mathbf{D}\mathbf{a}_{n,m} + \tilde{\mathbf{n}}_{n,m}\,. \tag{10}$$

In equation Equation (10) the matrix $\mathbf{D} \in \{0,1\}^{I\times L}$ extracts the elements on the reference signal positions of $\mathbf{a}_{n,m}$. Given that we know (have estimated) the autocorrelation matrix $\mathbf{R}_{\mathbf{a}_{n,m}}$ of our effective channel elements as well as $\mathbf{R}_{\tilde{\mathbf{n}}_{n,m}} = \mathrm{E}\{\tilde{\mathbf{n}}_{n,m}\tilde{\mathbf{n}}_{n,m}^H\} = \mathbf{I}\sigma^2$, one recognizes that Equation (10) is essentially the problem of estimating the random vector $\mathbf{a}_{n,m}$ out of the received random vector $\tilde{\mathbf{r}}_{n,m}$. The well known solution that minimizes the Mean Squared Error by using a linear estimator is given as follows:

$$\hat{\mathbf{a}}_{n,m}^{\text{LMMSE}} = \mathbf{R}_{\mathbf{a}_{n,m}} \mathbf{D}^T (\mathbf{D}\mathbf{R}_{\mathbf{a}_{n,m}} \mathbf{D}^T + \mathbf{I}\sigma^2)^{-1} \tilde{\mathbf{r}}_{n,m} \,. \quad (11)$$

In principle, using a LMMSE Estimator in practice requires the effective channel not to change too quickly over time because we need to assume ergodicity and perform time averaging in order to obtain an estimate of $\mathbf{R}_{\mathbf{a}_{n,m}}$. As we will see in Section IV-A, a practical channel will not fulfill this requirement. However, a suitable strategy to circumvent this problem was proposed in [6].

B. Channel-State Information Reference Signals

In order to provide the eNodeB with Channel State Information (CSI), in addition to the effective channel $\mathbf{A}^{(l)}$, the UEs also have to estimate the physical channel $\mathbf{H}^{(l)}$. For this reason LTE-A defines CSI-RS that have properties both of the DM-RS and the CRS used in LTE:

- Similar to the CRS, the CSI-RS are inserted in the antenna domain and span over the entire transmit band, so they can be employed by all UEs simultaneously for feedback calculation.
- In order to keep the overhead small, the CSI-RS use the same strategy as the UE-RS to accomplish orthogonality. They can be subsumed in orthogonal vectors over time similar to the DM-RS and thus all the equation derived in Section III-A remain valid with minor, straightforward differences.

Moreover, CSI-RS are just inserted every K_0-th subframe, where $K_0 \in \{5, 10, 20, 40, 80\}$ can be chosen according to the coherence time of the channel.

IV. Consequences

A. MU-MIMO Channel Estimation

Figure 2 shows a possible resource block allocation for a eNodeB with four transmit antennas operating in LTE-A transmit mode 9 [7]. In this mode, the number of layers M can change dynamically from one resource block to the next. Rectangles in the same color represent resource blocks allocated for one and the same user, white rectangles symbolize the absence of transmission. Using DM-RS, the UE can now directly estimate the effective channel (i.e. the mapping from those layers to its receive antennas) for each resource block where it is scheduled individually, invert it and cancel the interference from other users without the computational effort of multiplying by the precoder. Furthermore, since $M \leq N_\text{t}$, the dimension of the estimate will be lower on average compared to estimates based on CRS.

B. UE Mobility

In contrast to Section III, where we have assumed $M = 8$, we also want to address the case where $M \leq 4$. For this scenario we can weaken our assumption that $\mathbf{H}^{(l)}$ and $\mathbf{P}^{(l)}$ are flat over a whole subframe and assume them to be constant just over *half* a subframe. The reason for that is that now we receive just a superposition $M \leq 4$ vectors in Equation (2),

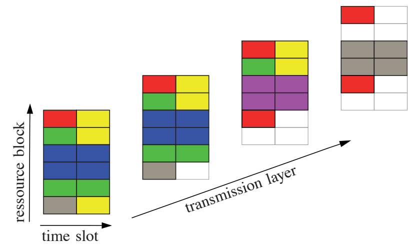

Fig. 2. Example of an ressource block allocation

so we can split up the reference signals in two subvectors of dimension 2

$$\mathbf{v}_m^{(l)} = \begin{bmatrix} \mathbf{v}_{m,1}^{(l)} \\ \mathbf{v}_{m,2}^{(l)} \end{bmatrix} \quad (12)$$

and still preserve orthogonality, if we reformulate Equation (2) in terms of $\mathbf{r}_{n,i}^{(l)}$ and claim constraints similar to (3) and (4) to hold for the subvectors $\mathbf{v}_{m,i}^{(l)}$ with $i = 1, 2$. Using this approach, one can improve the system performance in high mobility scenarios: First of all, rapidly changing channels can now be estimated more precisely and secondly the precoder is now allowed to change on a slot basis. On the downside the performance of estimators based on $\mathbf{v}_{m,i}^{(l)}$ will decrease in static scenarios as the Mean-Squared-Error increases by the factor two.

Regarding CSI-RS, they are suitable in channels with long coherence times only: Because they are inserted just every K_0-th subframe, closed loop transmission imposes rigid constrains on the stationarity of the mobile channel.

C. Overhead

Table I shows the relative overhead of the different types of RS in percents for a given number of layers/antennas. The values for the CSI-RS need to be devided by K_0.

TABLE I
Relative overhead in percents

M/N_t	1	2	4	8
CRS	4.76	9.52	14.28	–
UE-RS	7.14	7.14	14.28	14.28
CSI-RS	1.19	1.19	2.38	4.76

V. Simulation Results

In this section, we provide numerical results obtained from a standard compliant LTE-Advanced link level simulator [4, 5]. Corresponding parameters are listed in Table II. All simulations consider just the single user case with identity precoding in order not to blur the effect of the estimators. Furthermore, we used the bootstrap algorithm [8] to calculate the 95% confidence intervals.

Figure 3 shows the normalized channel Mean Squared Error in pilot subframes (This means that for the CSI-RS we considered just every K_0-th subframe). What one can see here is that the LS estimators tend to show an error floor for high

SNR. The reason for this effect is that the LS estimators rely on (in our case linear) interpolation between the RS positions. CSI-RS, DM-RS and CRS are inserted in average every 6th, 4th and 3rd subcarrier, that's why the effect increases in this order. Noteworthy at this plot is the performance of the LMMSE estimator based on CSI-RS that comes very close to the LS estimator based on DM-RS with less than 17% of the overhead. Considering that the LMMSE estimator for CSI-RS has to be calculated just every K_0-th subframe, we can neglect its computational complexity. What can also be observed is the 3 dB gain of the estimators using 4 DM-RS over those using 2 DM-RS.

Figure 4 then shows how the estimator performance and the overhead from Table I map into coded throughput. According to Table I, for 4×4 transmission, the overhead for DM-RS and CSI-RS together should be bigger than for CRS and hence the throughput of LTE-A with perfect channel knowledge should be below that of LTE. This seeming contradiction can be understood by the fact that there are no synchronization signals transmitted in the band where there is LTE-A transmission, as they would overlap with the DM-RS. Quite remarkable is the gap between the estimators using 4 DM-RS and 2 DM-RS. From this we conclude that for static scenarios a UE should use 4 RS instead of 2 RS even if $M \leq 4$.

TABLE II
SIMULATION PARAMETERS

Parameter	Value
Bandwidth	1.4 MHz
Number of Subcarriers L	72
Cyclic Prefix	normal [1]
Subcarrier spacing	15 kHz
Transmission setting	4×4
Modulation & coding	adaptive
Channel model	ITU PedB [9]
Equalizer	ZF
UE speed	0 km/h

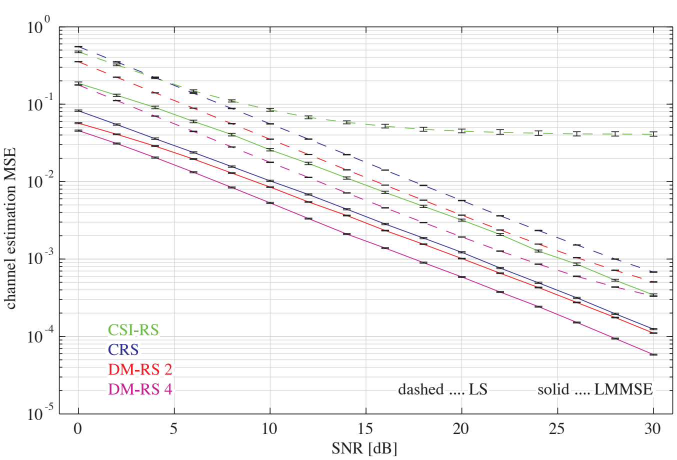

Fig. 3. Channel estimation MSE

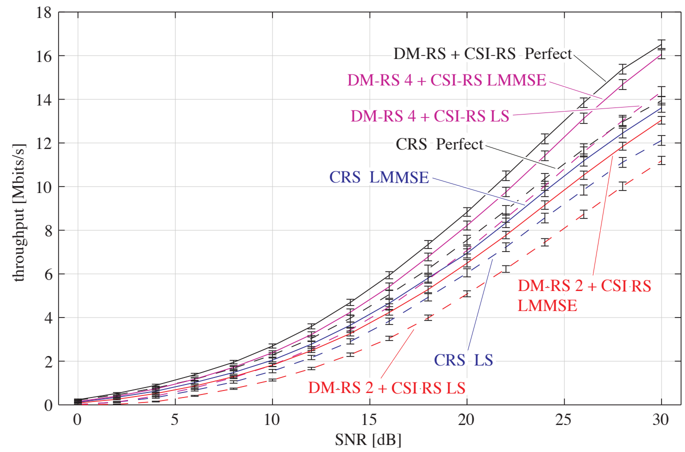

Fig. 4. Coded throughput

VI. Conclusion

In this work, we derived channel estimators for LTE-A downlink transmission. We showed that efficient receiver side processing can be achieved by using DM-RS inserted before the precoding. In static scenarios, UEs should, if possible, be scheduled in two consecutive resource blocks over time in order to use the full 4 DM-RS. In doing so, the throughput can be increased by several Mbit/s. For slowly varying channels, CSI-RS with insignificant overhead can be employed to obtain channel estimates at a high precision with negligible complexity.

Acknowledgments

The authors would like to thank the LTE research group for continuous support and lively discussions. This work has been funded by A1 Telekom Austria AG.

References

[1] 3GPP, "Technical specification group radio access network; (E-UTRA) and (E-UTRAN); physical channels and modulation; (release 10)," Tech. Rep., Dec. 2010. [Online]. Available: http://www.3gpp.org/ftp/Specs/html-info/36211.htm

[2] ——, "Technical specification group radio access network; (E-UTRA) and (E-UTRAN); physical channels and modulation; (release 8)," Tech. Rep., Sep. 2008. [Online]. Available: http://www.3gpp.org/ftp/Specs/html-info/36211.htm

[3] J. Duplicy, B. Badic, R. Balray, R. Ghaffar, P. Horáth, F. Kaltenberger, R. Knopp, I. Z. Kovács, H. T. Nguyen, D. Tandur, and G. Vivier, "MU-MIMO in LTE Systems," *EURASIP Journal on Wireless Communications and Networking*, 2011.

[4] "LTE simulator homepage." [Online]. Available: http://www.nt.tuwien.ac.at/ltesimulator/

[5] C. Mehlführer, J. C. Ikuno, M. Šimko, S. Schwarz, M. Wrulich, and M. Rupp, "The Vienna LTE Simulators - Enabling Reproducibility in Wireless Communications Research," *EURASIP Journal on Advances in Signal Processing*, 2011.

[6] C. Mehlführer, S. Caban, and M. Rupp, "An accurate and low complex channel estimator for OFDM WiMAX," in *Proc. Third International Symposium on Communications, Control, and Signal Processing (ISCCSP 2008)*, St. Julians, Malta, Mar. 2008, pp. 922–926. [Online]. Available: http://publik.tuwien.ac.at/files/pub-et_13650.pdf

[7] 3GPP, "Technical specification group radio access network; (E-UTRA) and (E-UTRAN); physical layer procedures; (release 10)," Tech. Rep., Dec. 2010. [Online]. Available: http://www.3gpp.org/ftp/Specs/html-info/36213.htm

[8] B. Efron and D. V. Hinkley, *An Introduction to the Bootstrap (CRC Monographs on Statistics & Applied Probability 57)*, 1st ed. Chapman & Hall/CRC, 1994.

[9] "Recommendation ITU-R M.1225: Guidelines for evaluation of radio transmission technologies for IMT-2000," Tech. Rep., 1997.