NUMERICAL INTEGRATOR FOR THE LLG EQUATION WITH MAGNETOSTRICTION

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Ferromagnetic materials are utilized in a broad spectrum of devices and thus the understanding of the magnetic processes within such devices is essential for both industry and scientific community. In our work, we consider the Landau-Lifshitz-Gilbert (LLG) equation

$$\begin{aligned} \mathbf{m}_t &= -\alpha \mathbf{m} \times \big(\mathbf{m} \times \mathbf{H}(\mathbf{m})\big) + \mathbf{m} \times \mathbf{H}(\mathbf{m}) \\ \mathbf{m}(0) &= \mathbf{m}_0 \quad \text{in } H^1(\Omega;\mathbb{S}^2) \\ \partial_n \mathbf{m} &= 0 \quad \text{on } (0,\tau) \times \partial\Omega \\ |\mathbf{m}| &= 1 \quad \text{a.e. in } (0,\tau) \times \Omega, \end{aligned}$$

which models the evolution of magnetization, coupled with the equation of elastodynamics

$$\rho \mathbf{u}_{tt} - \nabla \cdot \boldsymbol{\sigma} = 0 \quad \text{in } (0,\tau) \times \Omega$$

to include the magnetostrictive effects. Here, $\mathbf{H}(\mathbf{m})$ denotes the total magnetic field and $\boldsymbol{\sigma}$ denotes the so-called stress tensor.

We modify the approach of ALOUGES, cf. [1] to cover more general energy terms and combine it with the approach of BANAS and SLODICKA from [2] for the discretization of the second equation. As proposed by GOLDENITS ET. AL in [3], the LLG equation is integrated by a linear-implicit time-splitting algorithm. In addition, the two equations can be decoupled which makes the implementation easier, since one only has to solve two linear systems per timestep. Under some stability assumptions, we prove unconditional convergence to a weak solution of the coupled system, where we do not need any assumptions on the regularity of the exact magnetization $\mathbf{m}$ but only on the regularity of $\mathbf{u}$ and $\boldsymbol{\sigma}$.

[1] F. Alouges, *A new finite element scheme for Landau-Lifchitz equations*. Discrete and Continuous Dynamical Systems Series S, Vol. 1, 2008, p.187-196

[2] L. Banas, M. Slodicka *Error estimates for Landau-Lifshitz-Gilbert equation with magnetostriction*. Applied Numerical Mathematics, Vol. 56, 2006, p.1019-1039.

[3] P. Goldenits, G. Hrkac, M. Page, D. Praetorius, D. Süss, *Effective Simulation of the Dynamics of Ferromagnetism*. work in progress, 2012.