Numerical Integrator for the LLG Equation with Magnetostriiction

Marcus Page, Vienna University of Technology, Austria;  
Dirk Praetorius, Vienna University of Technology, Austria;  
Dieter Suess, Vienna University of Technology, Austria

Corresponding author: Marcus Page  
Institute for Analysis and Scientific Computing  
Vienna University of Technology – 1040 Wien, Wiedner Hauptstraße 8-10 – Austria  
Email: marcus.page@tuwien.ac.at

Ferromagnetic materials are utilized in a broad spectrum of devices and thus the understanding of the magnetic processes within such devices is essential for both industry and scientific community. In our work, we consider the Landau-Lifshitz-Gilbert (LLG) equation

\[ \mathbf{m}_t = -\alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{H}(\mathbf{m})) + \mathbf{m} \times \mathbf{H}(\mathbf{m}) \]

\[ \mathbf{m}(0) = \mathbf{m}_0 \quad \text{in} \ H^1(\Omega; S^2) \]

\[ \partial_n \mathbf{m} = 0 \quad \text{on} \ (0, \tau) \times \partial \Omega \]

\[ |\mathbf{m}| = 1 \quad \text{a.e. in} \ (0, \tau) \times \Omega, \]

which models the evolution of magnetization, coupled with the equation of elastodynamics

\[ \rho \mathbf{u}_{tt} - \nabla \cdot \mathbf{\sigma} = 0 \quad \text{in} \ (0, \tau) \times \Omega \]

to include the magnetostrictive effects. Here, \( \mathbf{H}(\mathbf{m}) \) denotes the total magnetic field and \( \mathbf{\sigma} \) denotes the so-called stress tensor.

We modify the approach of Alouges, cf. [1] to cover more general energy terms and combine it with the approach of Banas and Slodicka from [2] for the discretization of the second equation. As proposed by Goldenits et al in [3], the LLG equation is integrated by a linear-implicit time-splitting algorithm. In addition, the two equations can be decoupled which makes the implementation easier, since one only has to solve two linear systems per timestep. Under some stability assumptions, we prove unconditional convergence to a weak solution of the coupled system, where we do not need any assumptions on the regularity of the exact magnetization \( \mathbf{m} \) but only on the regularity of \( \mathbf{u} \) and \( \mathbf{\sigma} \).

