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Soft Channel Encoding; A Comparison of Algorithms for Soft Information Relaying

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***Abstract*—Soft Decode and Forward using Soft-Input Soft-Output (SISO) BCJR encoder is a recent relaying technique. The main purpose of employing SISO BCJR encoder in the relay is to combine the benefits of traditional relaying techniques, i.e. "Decode and Forward" and "Amplify and Forward".**
We propose a novel approach to derive the estimated BER of the overall system and the equivalent source-relay-destination channel SNR. We consider the loss of mutual information due to applying SISO BCJR encoder in the relay and compare it with the mutual information loss due to applying an equivalent convolutional encoder. We also propose a novel SISO encoder which outperforms existing SISO BCJR encoder.

I. Introduction

Spatial transmit diversity by employing multiple antennas in the transmitter is one of the solutions to combat fading in wireless channel. In spite of the promising theoretical results, implementing multiple antennas in the user nodes can be practically infeasible, if not impossible, due to lack of space. A more recent approach to exploit spatial diversity is cooperation: several users work together to communicate with a common destination or even different destinations, so they can utilize transmit diversity by sharing resources and obtain better performance, i.e., higher throughput or lower error rates [1], [2], [3]. Two well-known relaying functions (e.g. [4]) are "Decode and Forward" (DF) and "Amplify and Forward" (AF). A more recent relaying function is "Soft Decode and Forward" (soft-DF), e.g. [5], [6], [7], where the intention is to combine the benefits of AF and DF. However, so far, the soft-DF technique has been evaluated in a scenario where distributed turbo coding [8] is applied.

The paper is organized as follows: in Section II we introduce our system model. In Section III-A the Soft-Input Soft-Output BCJR-algorithm, which is commonly used in literature [5], [6], [7] for soft channel *en*coding, is explained briefly; as an alternative algorithm, we introduce the "Averaging Soft Channel Encoder" in Section III-B. In Section IV we present methods to evaluate the performance of the system using hard and soft channel encoders at the relay. We finally present simulation results in Section V.

II. System Model

We consider a cooperative scenario in which a source node communicates with a destination via an intermediate relay node. We assume that there is *no* direct link between the source and the destination, which can, e.g., be due to the distance between the source and the destination. However, we deliberately make this assumption in order to evaluate only the effect of the soft information produced by the relay on the performance, thereby studying the concept of soft channel encoding as such, without mixing the concept with (sub-optimal) iterative decoding in a distributed Turbo coding scheme as it is commonly adopted in works on relaying. This is also the reason why we use a simple convolutional code, as in this case an optimum symbol-by-symbol decoder (BCJR algorithm) is available [9].

Fig. 1(a) shows the soft-relaying system under consideration (for the BCJR *en*coder; the other scheme we propose below has the same basic design but uses a different soft channel encoder). In the source node, a block of K data bits is encoded using a k/n convolutional encoder, modulated and transmitted towards the relay. The relay employs a Soft-Input Soft-Output (SISO) BCJR decoder for decoding the noisy codeword received via the source-relay link. The output of the BCJR decoder is fed into a soft channel encoder (further details below). Its output is scaled by the factor β to fulfil the power constraint of the relay and transmitted towards the destination. The destination employs the corresponding BCJR decoder for decoding the noisy codeword received via the relay-destination link. We assume AWGN channels in both the source-relay and relay-destination links. For simplicity we employ BPSK modulation in the source and the relay, although the magnitude of the transmitted BPSK modulation symbols is weighted according to the power scaling by the soft-encoded magnitudes of the code bits.

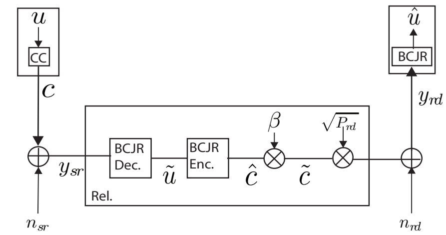
The signal received at the relay at each BPSK symbol time instant is

$$y_{\text{sr}} = \sqrt{P_{\text{sr}}} \cdot h_{\text{sr}} \cdot c + n_{\text{sr}}, \quad c \in \{\pm 1\} \tag{1}$$

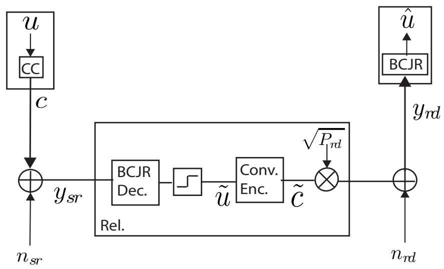
and the signal received at the destination equals

$$y_{\text{rd}} = \sqrt{P_{\text{rd}}} \cdot \beta \cdot h_{\text{rd}} \cdot \hat{c} + n_{\text{rd}}, \tag{2}$$

where $\beta = 1/\sqrt{\overline{|\hat{c}|^2}}$, with $\overline{|\hat{c}|^2}$ the average power of the transmitted channel symbols, averaged over each block of soft-encoded code bits resulting from each block of K data



(a) Soft DF, employing a BCJR soft channel encoder



(b) Hard DF, employing a conventional hard channel encoder

Fig. 1. System Model for both (a) Soft Decode and Forward and (b) Hard Decode and Forward. For Soft DF, β is chosen such that the average power of the signal $\tilde{c}$ is normalised to one. In (b) the convolutional encoder includes BPSK modulation of the bits to ± 1 signal values, so again the (average) power of $\tilde{c}$ is one.

bits. For simplicity we assume a non-fading scenario in this paper, and so h_{sr} and h_{rd} are unit power real values. The noise components, n_{sr} and n_{rd}, are zero mean real Gaussian random variables with variance N_0. The relay symbol $\hat{c}$ and its pdf will be explained in more detail in Section III.

In Fig. 1(b) we show the competing system design using hard decisions for the data bits after soft-input channel decoding at the relay, prior to hard re-encoding (by a classical convolutional encoder), modulation and transmission to the destination.

III. Soft Channel Encoding

In this section we discuss a commonly adopted scheme for soft channel encoding (Section III-A) and we propose a different, much simpler scheme (Section III-B).

A. BCJR Soft Channel-Encoder

The concept of the Soft BCJR *en*coder has been stated in the literature, e.g. [5], [6], [7], therefore due to space constraints, we will not further explain the SISO BCJR encoder.

B. Averaging Soft Channel-Encoder

The idea of the averaging soft channel encoder is to take the average of the magnitudes of the L-values of those data bits that are involved in a parity-check equation that would be used in a classical hard convolutional encoder. The sign is determined by the normal parity-check equations, i.e., by "xor"-operations on the data bits, with the "0/1"-output bits mapped to +1/-1 signs for the magnitude determined above by averaging.

Although such a soft encoder is rather simple it fulfills some properties that are desirable: if only the signs are considered, the result will be a valid channel code word. Moreover, it is sensible to allocate magnitudes to the coded bits that reflect the significances of the data bits to be encoded by the parity check equations. It wouldn't make sense to allocate large transmit power to data bits at the relay when they have been decoded (at the relay) with small reliabilities. The consequence would be that we communicate to the destination information that is actually very unreliable, but we would be making it strong by using large transmit power. On the other hand, the magnitude should not vanish, when only one of the bits involved in a parity check has a very small magnitude, as this would terminate the protection of all other bits as well. The latter point is interesting, as exactly this will happen when a soft decoding algorithm (such as the one described in Section III-A) is used as a soft encoder.

We would like to point out that we don't claim the proposed averaging soft channel encoder to be optimal or even "good" in any sense. But, as we will demonstrate below, the averaging soft channel encoder beats the soft BCJR encoder in bit error performance, which proves that this widely used soft encoder can not be the best choice.

IV. Performance Evaluation

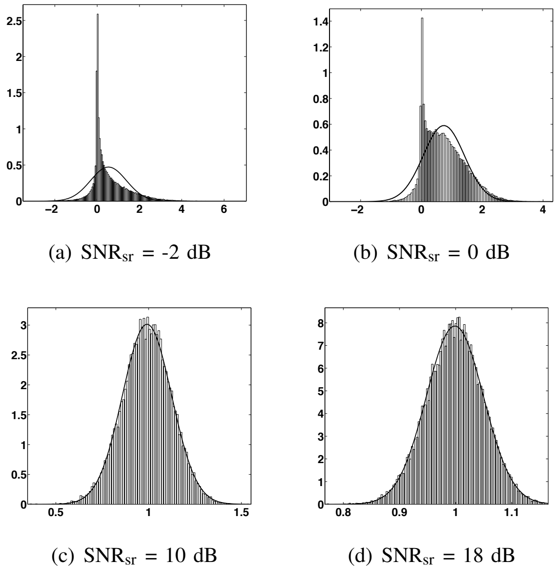
For proper decoding of the received noisy channel code word, the BCJR decoder in the destination needs to know the statistics of the received signal that will depend on the channel as well as on the (soft) relaying function used. As illustrated by Fig. 1, $y_{\text{rd}} = \tilde{c} + n_{\text{rd}}$, where n_{rd} is a zero mean Gaussian random variable with variance N_0 (receiver noise). To the best of our knowledge, there is no closed form solution for the probability density function (pdf) of $\tilde{c}$ for non-trivial relaying functions. Hence, we have measured histograms that describe the conditional pdfs $p(\tilde{c}|c=1)$; the pdfs $p(\tilde{c}|c=-1)$ would be symmetric.

As an example, Fig. 2 shows the pdfs $p(\tilde{c}|c=1)$ for different source-relay channel SNRs when a feed-forward BCJR soft channel encoder is applied in the relay. Due to the lack of an analytical description, in literature (e.g. [6], [7]) the pdfs are usually modeled by additive Gaussian random variables with non-zero mean (that depends on the input bit $c \in \{-1, +1\}$), i.e.,

$$\tilde{c} = \mu_{\tilde{c}} c + n_{\tilde{c}}, \quad n_{\tilde{c}} \sim \mathcal{N}(0, \sigma_{\tilde{c}}^2) \tag{3}$$

As illustrated by Fig. 2, the Gaussian assumption is not very accurate, especially at low SNR_{sr}, and, therefore, a performance degradation for low SNR is expected.

To evaluate the performance of the soft channel encoding algorithms, we start by analyzing the mutual information loss that occurs, due to the use of a soft channel encoder in the relay. We continue with the evaluation of the estimated BER of the overall system, and, based on that, we calculate the equivalent "receive SNR" at the destination. We compare the results of "soft" and "hard" relaying (algorithms as shown in Fig. 1).



(a) SNR_{sr} = -2 dB

(b) SNR_{sr} = 0 dB

(c) SNR_{sr} = 10 dB

(d) SNR_{sr} = 18 dB

Fig. 2. Conditional probability density functions $p(\tilde{c}|c = +1)$ for a (7,5) soft BCJR encoder. The best fit of a Gaussian pdf is laid behind the plots to demonstrate the difference of reality and common model assumption.

A. Mutual Information (Loss)

Mutual information, $I(U; \tilde{U})$, can be used to measure the amount of the information that soft (or hard) data bits, $\tilde{u}$, at the relay carry about the data symbols, u, transmitted by the source. As illustrated by Fig. 1, the two system models, *soft/hard* DF, use two different (*soft/hard*) channel encoders. The intention of calculating mutual information is to measure the mutual information loss, [10], due to different channel encoders.

The mutual information $I(U; \tilde{U})$ [10], [11] between the (binary) transmitted data bits, $u \in \{+1, -1\}$, and the L-values $\tilde{u}$ is given by

$$I(U;\tilde{U}) = \frac{1}{2} \cdot \sum_{u'=\pm 1} \int_{-\infty}^{+\infty} p(\tilde{u} \mid u = u') \times \text{ld} \frac{2 \cdot p(\tilde{u} \mid u = u')}{p(\tilde{u} \mid u = +1) + p(\tilde{u} \mid u = -1)} d\tilde{u}\,, \tag{4}$$

with $p(\tilde{u} \mid u)$ the conditional pdf of the L-values at the relay (see Fig. 1) given the input bits u. We have measured this pdf, similarly as the ones for the code bits ($p(\tilde{c}|c = 1)$) but we have omitted the plots due to lack of space.

To characterize $I(U; \tilde{U})$ associated with hard-DF, we model the source-channel-relay link as a Binary Symmetric Channel (BSC) in which the channel input is a data bit. The output bit of the channel is flipped with probability q. Hence, mutual information for such a BSC is given (e.g. [10]) by

$$I(U; \tilde{U}) = 1 - H_2(q), \tag{5}$$

with $H_2(q) \doteq -q \cdot \text{ld}(q) - (1 - q) \cdot \text{ld}(1 - q)$ the standard binary entropy function.

Using the same approach, one can calculate the mutual information $I(C; \tilde{C})$, too. A comparison of the mutual informations $I(U; \tilde{U})$ and $I(C; \tilde{C})$ for both the *hard/soft* DF is useful in characterizing the performance of the soft channel encoders. Numerical results will follow in Section V.

B. Equivalent Receive SNR at the Destination

The bit error rates at the destination and the equivalent receive SNR (equivalent for a substitute AWGN channel) at the destination are closely related, with the equivalent receive SNR providing extra insight into the communication process.

1) Hard DF: Calculating the equivalent receive SNR at the destination for hard DF is somewhat cumbersome, due to the error-prone relay. Since the relay decodes and forwards both the correct and erroneous frames, the distribution of the received signal at destination is no longer Gaussian. The common approach to estimate the SNR at the destination is to model the source-relay-destination link as an equivalent AWGN channel with channel SNR_{eq} that depends on both the source-relay and the relay-destination channel qualities.

The total bit error probability is given by

$$P_{\text{tot}}(e \mid \gamma_{\text{sr}}, \gamma_{\text{rd}}) = P_b(e \mid \gamma_{\text{sr}})[1 - P_b(e \mid \gamma_{\text{rd}})] + [1 - P_b(e \mid \gamma_{\text{sr}})]P_b(e \mid \gamma_{\text{rd}})\,, \tag{6}$$

where γ and $P_b(e)$ are the corresponding channel SNR and the bit error probabilities for the two links (source-relay and relay-destination) involved.

Calculating P_{tot} using simulations is straightforward but one can also calculate it using the complementary error function. The bit error probability of convolutional codes under symbol-by-symbol MAP decoding can be approximated by

$$P_b(e) \approx \frac{1}{2}\text{erfc}\Big(\sqrt{\frac{\mu_{\text{out}}^2}{2\sigma_{\text{out}}^2}}\Big) = \frac{1}{2}\text{erfc}\Big(\sqrt{\frac{1}{2}\gamma_{\text{out}}}\Big), \tag{7}$$

(e.g. [11]) where μ_{out}^2 and σ_{out}^2 are, respectively, the mean and variance of the data bit L-values at the output of the BCJR decoder, and $\gamma_{\text{out}} = \mu_{\text{out}}^2/\sigma_{\text{out}}^2$. Similarly, μ_{in}^2 and σ_{in}^2 would define $\gamma_{\text{in}} = \mu_{\text{in}}^2/\sigma_{\text{in}}^2$ for the input L-values of the symbol-by-symbol MAP decoder.

Fig. 3 illustrates $\gamma_{\text{out}} = f(\gamma_{\text{in}})$ for a (7,5) convolutional code, decoded by a BCJR decoder. Using regression analysis, γ_{out} can be modeled by a polynomial according to

$$\gamma_{\text{out}} = f(\gamma_{\text{in}}) \approx 3.38 \cdot 10^{-3}\gamma_{\text{in}}^3 - 0.12\gamma_{\text{in}}^2 + 2.38\gamma_{\text{in}} + 2.56. \tag{8}$$

$P_b(e \mid \gamma_{\text{sr}})$ and $P(e \mid \gamma_{\text{rd}})$ in (6) can be computed using (7) and (8) (with $\gamma_{\text{in}} \in \{\gamma_{\text{sr}}, \gamma_{\text{rd}}\}$). Given P_{tot}, the equivalent SNR_{out}, $\gamma_{\text{eq-out}}$, follows from

$$\gamma_{\text{eq-out}} = 2\Big(\text{erfc}^{-1}(2P_{\text{tot}})\Big)^2. \tag{9}$$

By substituting (9) into

$$\gamma_{\text{eq}} = f^{-1}(\gamma_{\text{eq-out}}), \tag{10}$$

the equivalent source-relay-destination SNR, γ_{eq}, is computed.

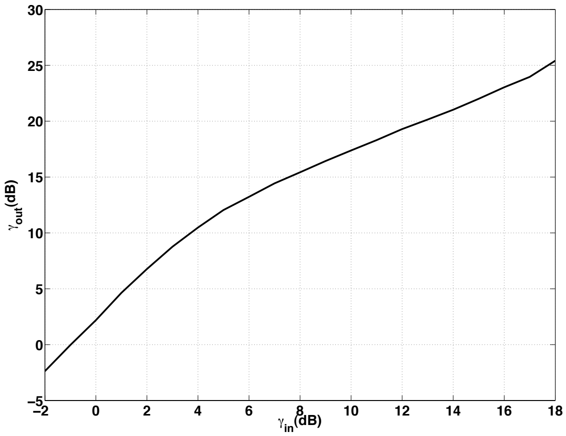


Fig. 3. $\gamma_{\text{out}} = f(\gamma_{\text{in}})$ for a (7,5) convolutional code under symbol-by-symbol MAP (BCJR) decoding

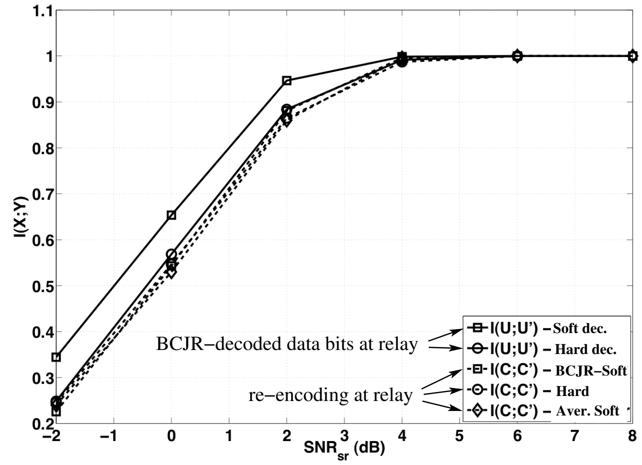


Fig. 4. Mutual information for hard and soft channel encoding (BCJR and "averaging" encoders) compared with mutual information between the original data bits U and the BCJR soft-decoded data bits U' (with soft and with hard decisions after decoding) at the relay prior to re-encoding.

2) Soft DF: One might exploit the Gaussian assumption of (3) for calculating SNR_{eq} of the soft DF schemes, but, as illustrated by Fig. 2, the Gaussian assumption is not accurate, especially at low SNR. Therefore, in order to calculate SNR_{eq}, we use Monte-Carlo simulations in the destination to determine $\gamma_{\text{eq-out}}$. With $\gamma_{\text{eq-out}}$, calculating γ_{eq} using (10) is straightforward. The estimated BER for soft DF can then be determined by substituting $\gamma_{\text{eq-out}}$ in (7).

We can, however, apply this approach with good justification only to the BCJR soft channel encoder, as it is known from experiments ([11]) that the distribution of the BCJR-decoded data-bit L-values at the destination is essentially Gaussian: this assumption is required implicitly for (7) to hold. When the averaging soft channel encoder is used, the distribution of the code bits is such that after transmission and decoding at the destination, the distribution of the BCJR-decoded data bits L-values is *not* Gaussian. Therefore, we will investigate the equivalent receive SNR only for the BCJR soft channel encoder.

V. Simulation Results

The simulated cooperative scenario is illustrated by Fig. 1. It has the following characteristics: the source applies a [7,5] convolutional encoder for encoding information frames of length 2000 bits; for simplicity we also use BPSK modulation. The modulated code symbols are scaled by the source's transmit power constraint and are sent to the relay. We assume that receive signal in the relay is corrupted by zero mean real Gaussian receiver noise with a variance of $N_0 = 1$. In order to decode the received noisy codeword in the relay, the relay employs a standard soft-in/soft-out symbol-by-symbol MAP (BCJR) decoder [9]. The L-values of the BCJR-decoded information bits (or equivalently their a-posteriori probabilities) are then used for calculating the L-values of the code bits by the BCJR soft channel encoder. For simplicity the BCJR encoder has the trellis structure of the [7,5]-convolutional code, although other codes, and hence other trellis structures, could be used. The output L-values for the code bits of each code word are then normalized by β in order to fulfill a unit-power constraint, so the same average power as in conventional "hard" BPSK modulation is used. The unit-power code-bit L-values are then scaled by the (square-root) of the relay's transmit power constraint (P_{rd}) and transmitted towards the destination. The destination applies a BCJR decoder that takes hard decisions to obtain the output data bits $\hat{u}$.

We also simulate another scenario, in which the relay performs conventional DF, i.e., hard encoding at the relay (Fig. 1(b)).

Fig. 4 compares the mutual informations $I(U;\tilde{U})$ of the data bits U and their decoded counterparts $\tilde{U}$ (with soft and hard decisions after soft-input BCJR decoding, see Figs. 1(a) and 1(b)) at the relay as well as the mutual informations $I(C;\tilde{C})$ of the code bits C at the source and the (soft and hard) re-encoded code bits $\tilde{C}$ at the relay. As expected the mutual information $I(U;\tilde{U})$ for the soft-DF scheme is larger than the one for hard-DF. This confirms the received wisdom that "soft is better than hard". The reason is the quantization (for hard DF) applied on the L-values of the data bits at the output of the BCJR decoder in the relay: this quantization obviously destroys information.

In hard-DF the relay applies a conventional convolutional encoder to compute the transmitted code bits but in soft-DF a (BCJR or Averaging) soft channel encoder is used. Because of applying different encoders, the mutual information loss[1] explained in Section IV-A will be different. As illustrated by Fig. 4, the mutual information loss is more severe for soft-DF (for both soft encoders) than for hard DF. In fact, although the input data to the channel encoder of soft-DF

[1]Any processing of data can only cause a loss of information: data processing theorem, [10].

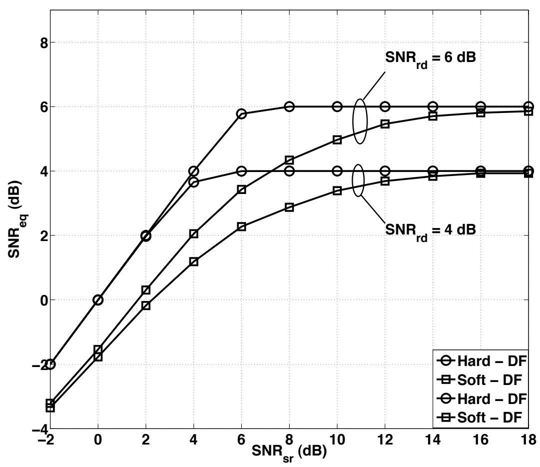


Fig. 5. Equivalent source-relay-destination SNR; for soft DF, a BCJR soft channel encoder is used.

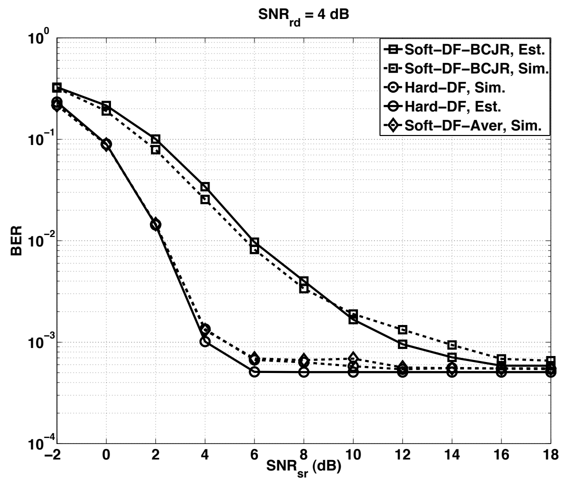


Fig. 6. Bit-error-rate performance of hard DF and soft DF (the latter employing BCJR and averaging soft channel encoders). Because of the limited quality of SNR_{rd} = 4 dB of the relay-destination link, the curves saturate at large source-relay channel quality SNR_{sr}.

contains more information than the input data of hard-DF, there is still a slight mutual information loss at the output of the hard convolutional encoder. This indicates that both soft channel encoding schemes actually destroy more information by data processing than the hard encoding algorithm has less information at its input. Therefore, we conclude that neither the BCJR soft channel encoder nor the averaging soft channel encoder do a good job

Fig. 5 shows the equivalent channel SNR for both the hard DF algorithm and the soft DF algorithm using a BCJR soft channel encoder. The SNR_{eq} curve of soft DF merges with the SNR_{eq} curve of hard-DF at high SNR_{sr}, although a small difference remains. The explanation is that the relay usually performs error free decoding at high SNR; therefore the hard-DF algorithm is very close to optimum at high SNR. But for soft-DF, the transmitted symbol from the relay, $\tilde{c}$, is Gaussian distributed. Because of the unit power constraint of BPSK modulation that we have to enforce for soft encoding in an average sense as well, we have an average power of $\text{P}(\tilde{c}) = \mu_{\tilde{c}}^2 + \sigma_{\tilde{c}}^2 = 1$. Since $\sigma_{\tilde{c}}^2 > 0$, we find that $|\mu_{\tilde{c}}| < 1$ must hold, so some of the transmitted code bits will have smaller instantaneous power than the hard-encoded symbols (which all have "one"): this will cause the slight SNR_{eq} degradation in comparison with hard-DF for large values of SNR_{sr}. The SNR_{eq} of the system is bounded by $\min\{\gamma_{\text{sr}}, \gamma_{\text{rd}}\}$, i.e. $\gamma_{\text{eq}} < \min\{\gamma_{\text{sr}}, \gamma_{\text{rd}}\}$.

Fig. 6 illustrates the bit error rates of the system for SNR_{rd}=4dB. The estimated curves for both the hard and soft DF (for BCJR soft channel encoding) confirm the simulation results. The largest BER gap between hard DF and soft DF (with BCJR soft channel encoding) appears in an SNR_{sr} range between 3–7 dB, which corresponds to a similar gap shown in Fig. 5. It is rather striking that hard DF works much better than soft DF with BCJR soft channel encoding at all SNRs.

The situation is, however, different for soft DF with the averaging soft channel encoder: although the performance of hard DF is also better, it is only slightly so. Of course, this means that hard DF is still the method of choice, but it also proves that BCJR soft channel encoding is a much worse algorithm in the given context. It is an open question (and part of future work by the authors), if there is at all a soft DF scheme that can perform better than hard DF within the framework of our system model.

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