Compress-and-Forward in the Multiple-Access Relay Channel: with or without Network Coding?

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***Abstract*—We consider compress-and-forward-based transmission strategies for the multiple-access relay channel. In particular, we study the impact of the capacity on the relay-destination link on the usefulness of network coding in this context. To this end, we compare a transmission scheme with network coding to a simple forwarding scheme without network coding. For both schemes we design optimal log-likelihood ratio (LLR) vector quantizers using the information bottleneck method. Moreover, we provide closed-form expressions for the LLR statistics at the relay which are required for the vector quantizer design. Numerical simulation results show that the usefulness of network coding depends strongly on the capacity of the relay-destination channel as well as on the number of sources.**

I. INTRODUCTION

In this paper, we consider the *multiple-access relay channel* (MARC) with $N \geq 2$ sources, one relay and one destination (cf. Fig. 1). Our focus is on *compress-and-forward* (CF) [1] schemes for the time-division MARC with half-duplex terminals. For this scenario, a CF-based "noisy" *network coding* [2] scheme with scalar *log-likelihood ratio* (LLR) quantization at the relay and iterative joint network-channel decoding at the destination was proposed in [3]. However, the impact of the capacity on the relay-destination link on the usefulness of network coding in this context has not been sufficiently studied to date. Clearly, in case of an error-free relay-destination link there is no incentive for network-coded cooperation since all source signals can be individually forwarded with arbitrary precision. Intuitively, network coding is expected to be beneficial when the capacity of the relay-destination channel is low or the number of sources is large.

In this paper, we compare an extended version of the scheme in [3] to a scheme without network coding where the relay forwards a compressed representation of each source signal separately. More specifically, our contributions are as follows:

- We extend the scheme of [3] to more than two sources and we propose to use the information bottleneck method [4] to design an optimal *vector* quantizer for the network-coded LLRs at the relay.
- As baseline we use a forwarding scheme which performs separate vector quantization for each source.
- We provide a closed-form expression for the LLR statistics in the network-coded case. These statistics are required for vector quantizer design and are shown to be excellent approximate characterizations of the boxplus operation (cf. [5]).
- We numerically compare the two transmission schemes mentioned above for different rate constraints on the relay-destination link.

Funding by FWF Grant S10606 and WWTF Grant ICT08-44.

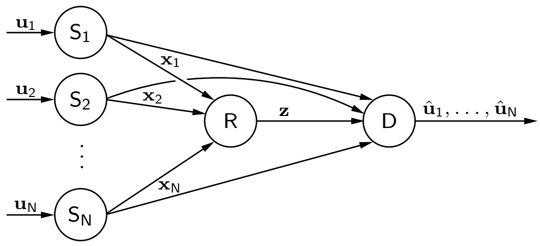

Figure 1: The MARC with N sources.

This paper is organized as follows. Section II describes the system model and the operation of all network nodes. In Section III, we explain the relay processing and we analytically characterize the LLR statistics in the network-coded case. In Section IV we discuss the design of the vector quantizers employed at the relay. Numerical results and conclusions are provided in Sections V and VI, respectively.

II. SYSTEM MODEL

A. MARC Model

We consider the time-division MARC with N sources, $\mathsf{S}_1, \ldots, \mathsf{S}_\mathsf{N}$, a relay R, and a destination D as depicted in Fig. 1. The sources consecutively broadcast their messages in the first N time slots. In the $(N+1)$th time slot, the relay forwards to the destination a compressed representation of the received source data. Finally, the destination decodes the source and relay messages. We assume that the sources do not overhear each others transmission, that the relay has no side information, and that the number of channel uses is the same for all transmitting nodes.

B. Channels

Assuming Gaussian channels, we have

$$\mathbf{y}_{ij} = d_{ij}^{-n/2}\mathbf{x}_i + \mathbf{w}_{ij}, \quad i \in \{1, 2, \ldots, N\},\ j \in \{\mathsf{R}, \mathsf{D}\}, \quad (1)$$

where $\mathbf{x}_i$ is the signal transmitted by node i, $\mathbf{y}_{ij}$ is the corresponding receive signal at node j, d_{ij} is the normalized distance between nodes i and j, n is the path-loss exponent,

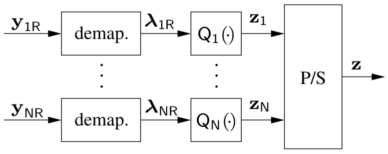

Figure 2: Relay block diagram without network code.

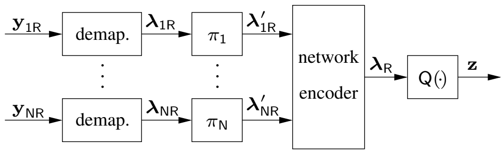

Figure 3: Relay block diagram with network code.

and $\mathbf{w}_{ij} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ is zero-mean and circularly symmetric white Gaussian noise. The average power of each source is normalized to 1. The signal-to-noise ratio (SNR) for the link between nodes i and j thus equals $\gamma_{ij} = d_{ij}^{-n}/\sigma^2$. Each node has only receive *channel state information* (CSI), i.e., γ_{ij} is known at node j; the relay has no source-destination CSI.

We assume a rate constraint on the relay-destination channel, such that the relay can reliably transmit data to the destination at a rate of at most R_0 bits per channel use (bpcu).

C. Sources

Source i ($i \in \{1, 2, \ldots, N\}$) generates a length-$K_i$ sequence $\mathbf{u}_i \in \{0,1\}^{K_i}$ of independent and uniformly distributed bits. The sequence $\mathbf{u}_i$ is encoded using a linear binary code $\mathcal{C}_i$ of rate $R_i = K_i/L_i$, yielding a length-L_i sequence $\mathbf{c}_i \in \{0,1\}^{L_i}$ of code bits. Next, the code bits are mapped to a signal constellation $\mathcal{A}_i$ of cardinality $|\mathcal{A}_i| = 2^{m_i}$, yielding a sequence $\mathbf{x}_i \in \mathcal{A}^{L_i/m_i}$ of transmit symbols. For simplicity of exposition, we assume $K \triangleq K_i$, $L \triangleq L_i$, $m \triangleq m_i$, and hence, $R \triangleq R_i$. The sum rate is then given by $R_s = mRN/(N+1)$.

D. Relay

Without Network Code. Fig. 2 shows a block diagram of the relay without network code. The relay first performs soft demapping of the receive signals $\mathbf{y}_{i\mathsf{R}}$, resulting in LLR vectors $\boldsymbol{\lambda}_{i\mathsf{R}} \in \mathbb{R}^L$. Next, each $\boldsymbol{\lambda}_{i\mathsf{R}}$ is passed through an M_i-dimensional vector quantizer $\mathsf{Q}_i(\cdot)$ with Q_i levels (cf. Section IV), i.e., $\mathbf{z}_i = \mathsf{Q}_i(\boldsymbol{\lambda}_{i\mathsf{R}})$. Then, the relay concatenates the N integer-valued sequences $\mathbf{z}_i$ in a single sequence $\mathbf{z}$ that is transmitted to the destination in time slot $N+1$.

With Network Code. A block diagram of the relay with network code is depicted in Fig. 3. First, soft demappers compute the LLR sequences $\boldsymbol{\lambda}_{i\mathsf{R}} \in \mathbb{R}^L$ as before. The LLRs are then interleaved, $\boldsymbol{\lambda}'_{i\mathsf{R}} = \pi_i(\boldsymbol{\lambda}_{i\mathsf{R}})$, in order to avoid short cycles in the resulting overall code. Next, the network encoder maps all N sequences $\boldsymbol{\lambda}'_{i\mathsf{R}}$ element-wise to $\boldsymbol{\lambda}_\mathsf{R}$ (cf. Section III-B). The transmit vector $\mathbf{z}$ is obtained by quantizing $\boldsymbol{\lambda}_\mathsf{R}$ using an M-dimensional vector quantizer $\mathsf{Q}(\cdot)$ with Q levels.

We note that in both cases the relay operation has low complexity. In contrast to [3] the relay does not perform soft channel decoding which enables low-delay operation.

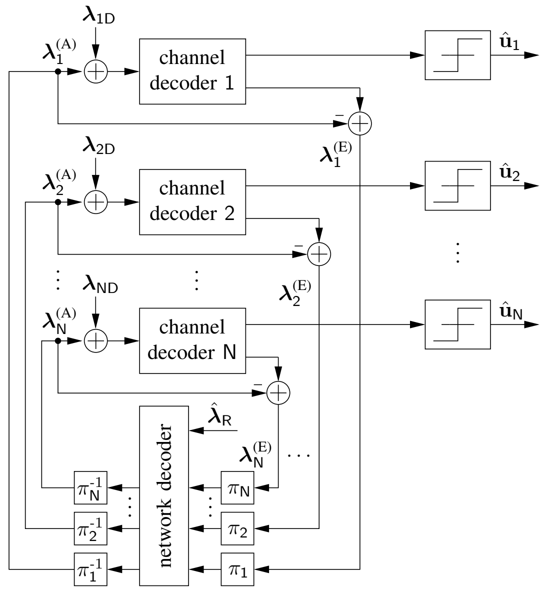

Figure 4: Block diagram of the joint network-channel decoder.

E. Destination

The destination decodes all received signals $\mathbf{y}_{1\mathsf{D}}, \ldots, \mathbf{y}_{\mathsf{ND}}$, $\mathbf{z}$, to obtain detected messages $\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_\mathsf{N}$. All source observations $\mathbf{y}_{i\mathsf{D}}$ are first processed by soft demappers, yielding LLR vectors $\boldsymbol{\lambda}_{i\mathsf{D}}$. Next, we describe the decoding for the cases with and without network code.

Without Network Code. Here, the destination obtains $\mathbf{z}_i$ from $\mathbf{z}$, and it finds the reproduction values $\hat{\boldsymbol{\lambda}}_{i\mathsf{R}}$ corresponding to $\mathbf{z}_i$ (cf. Section IV). Based on $\boldsymbol{\lambda}_{i\mathsf{D}} + \hat{\boldsymbol{\lambda}}_{i\mathsf{R}}$, each source i is separately soft decoded to obtain the detected message $\hat{\mathbf{u}}_i$.

With Network Code. In the network-coded case, the destination first computes the reproduction values $\hat{\boldsymbol{\lambda}}_\mathsf{R}$ from $\mathbf{z}$. The relay processing amounts to an equivalent discrete memoryless channel with transition pmf $p_{\hat{\Lambda}_\mathsf{R}|\mathbf{C}'}(\hat{\lambda}_{\mathsf{R},k}|\mathbf{c}')$, where $\mathbf{c}'$ denotes the vector of interleaved code bits corresponding to $\hat{\lambda}_{\mathsf{R},k}$. Hence, the code bits of all sources are coupled, enabling (iterative) joint network-channel decoding of all source messages. The operation of the joint network-channel decoder (see Fig. 4) is such that the individual channel decoders iteratively exchange extrinsic LLRs via the network decoder.

The network decoder in Fig. 4 computes a priori LLRs $\lambda_{i,k}^{(\mathrm{A})}$ for the ith channel decoder using $N-1$ extrinsic LLRs $\lambda_{j,k}^{(\mathrm{E})}$ (for all $j \neq i$) and the pmf $p_{\hat{\Lambda}_\mathsf{R}|\mathbf{C}'}(\hat{\lambda}_\mathsf{R}|\mathbf{c}')$, where $\hat{\lambda}_\mathsf{R}$ is known due to the transmission of the relay. The sum-product update rule [6] for $\lambda_{i,k}^{(\mathrm{A})}$ ($k = 1, 2, \ldots, L$) is given by

$$\lambda_{i,k}^{(\mathrm{A})} = \log \mu_{p \to c'_{i,k}}(0) - \log \mu_{p \to c'_{i,k}}(1), \quad i = 1, 2, \ldots, N, \quad (2)$$

where

$$\mu_{p \to c'_{i,k}}(c'_{i,k}) = \sum_{\mathbf{c}'_{\sim i}} p_{\hat{\Lambda}_\mathsf{R}|\mathbf{C}'}(\hat{\lambda}_{\mathsf{R},k}|\mathbf{c}') \prod_{j: j \neq i} \mu_{c'_{j,k} \to p}(c'_{j,k}), \quad (3)$$

with

$$\mu_{c'_{j,k} \to p}(c'_{j,k}) = \frac{\exp\big(-c'_{j,k}\lambda_{j,k}^{(\mathrm{E})}\big)}{1 + \exp\big(-\lambda_{j,k}^{(\mathrm{E})}\big)}. \quad (4)$$

Here $\mathbf{c}'_{\sim i}$ denotes the vector obtained by removing the ith element from $\mathbf{c}'$.

The network encoding at the relay is shown below to imply

$$p_{\hat{\Lambda}_{\mathsf{R}}|\mathbf{C}'}(\hat{\lambda}_{\mathsf{R}}|c'_1, c'_2, \ldots, c'_{\mathsf{N}}) = p_{\hat{\Lambda}_{\mathsf{R}}|\mathbf{C}'}\big(\hat{\lambda}_{\mathsf{R}}|\textstyle\sum\oplus_{n=1}^{N} c'_n\big), \quad (5)$$

where $\sum\oplus$ denotes modulo-2 sum. In this case, it can be shown that (2)-(4) simplify to

$$\lambda^{(\mathrm{A})}_{i,k} = \hat{\lambda}_{\mathsf{R},k} \boxplus \sum_{j:j\neq i}\boxplus\, \lambda^{(\mathrm{E})}_{j,k}, \quad i = 1, 2, \ldots, N, \quad (6)$$

where $\sum\boxplus$ denotes a sequence of boxplus operations. We note that (6) is valid for *any* encoding at the relay which couples the code bits of the sources in the form of (5).

One iteration of the joint network-channel decoder consists of N channel decoder invocations and N network decoder invocations. Hence, there are $(2N)!$ possible decoder schedules. We propose to use a serial schedule which in [7] has been shown to perform significantly better than a flooding schedule.

III. Relay Operation

A. Without Network Code

The relay soft demapper computes ($j = 0, 1, \ldots, m-1$)

$$\lambda_{i\mathsf{R},mk+j} = \log \frac{\sum\limits_{a_n:c_{i,mk+j}=0} p_{X_i|Y_{i\mathsf{R}}}(x_{i,k}=a_n|y_{i\mathsf{R},k})}{\sum\limits_{a_n:c_{i,mk+j}=1} p_{X_i|Y_{i\mathsf{R}}}(x_{i,k}=a_n|y_{i\mathsf{R},k})}, \quad (7)$$

where $a_n \in \mathcal{A}$. For a QPSK constellation, (7) amounts to

$$\lambda_{i\mathsf{R},2k} = \frac{2\sqrt{2}}{\sigma^2}\mathrm{Re}\{y_{i\mathsf{R},k}\}, \quad \lambda_{i\mathsf{R},2k+1} = \frac{2\sqrt{2}}{\sigma^2}\mathrm{Im}\{y_{i\mathsf{R},k}\}. \quad (8)$$

The statistics of $\Lambda_{i\mathsf{R},k}$ conditioned on $c_{i,k}$ is required in the vector quantizer design. For QPSK, it follows from (1) and (8) that the conditional LLR distribution is Gaussian with variance $\sigma^2_{\Lambda|C} = 4/\sigma^2$ and mean $\mu_{\Lambda|C} = (1-2c_{i,k})\sigma^2_{\Lambda|C}/2$.

To ensure that the quantization indices can be transmitted to the destination at a rate less than R_0 bpcu, the parameters Q and M of the vector quantizer are chosen such that $N\log_2(Q)/M \leq R_0$. We note that $\log_2(Q)/M$ must not be less than 1 since we need to recover at least the sign. In the case $R_0 < N$ we therefore choose $Q = 2, M = 1$, and let the relay process only a fraction of R_0/N of all LLRs.

B. With Network Code

Here, the demapping is the same as in the previous case. Prior to network encoding, the LLRs are interleaved, using a different interleaver $\pi_i(\cdot)$ for each source, i.e., $\boldsymbol{\lambda}'_{i\mathsf{R}} = \pi_i(\boldsymbol{\lambda}_{i\mathsf{R}})$. As a network code we use the "sign-min operation" $\widetilde{\boxplus}$ with subsequent scalar LLR correction (see below). This allows us to derive closed-form expressions for the (conditional) statistics of the network-coded LLRs which are used for quantizer design without requiring Monte Carlo techniques (this is in contrast to the boxplus operation where no simple closed-form expression for the resulting LLR statistics exists).

The sign-min and boxplus operations are defined as follows:

$$\lambda_1 \,\widetilde{\boxplus}\, \lambda_2 \triangleq \mathrm{sign}(\lambda_1)\mathrm{sign}(\lambda_2)\min\{|\lambda_1|, |\lambda_2|\}, \quad (9)$$

$$\lambda_1 \boxplus \lambda_2 \triangleq \log\frac{1+\exp(\lambda_1+\lambda_2)}{\exp(\lambda_1)+\exp(\lambda_2)}. \quad (10)$$

We note that the sign-min operation (9) can be obtained by applying the max-log approximation to (10) and, hence, "$\widetilde{\boxplus}$" is a good approximation of "$\boxplus$". Furthermore, both operations are associative and commutative. Another important property of (9) and (10) is stated in the following result.

Proposition 1. *Consider independent binary random variables U_i with LLRs Λ_i, $i = 1, 2, \ldots, n$. Let $U = U_1 \oplus U_2 \oplus \cdots \oplus U_n$ and $\Lambda = \Lambda_1 \widetilde{\boxplus} \Lambda_2 \widetilde{\boxplus} \cdots \widetilde{\boxplus} \Lambda_n$. Then,*

$$p_{\Lambda|U_1,\ldots,U_n}(\lambda|u_1, \ldots, u_n) = p_{\Lambda|U}(\lambda|u), \quad (11)$$

i.e., the conditional distribution of Λ depends on $u_1, \ldots, u_n$ only through their modulo-2 sum. The same result applies to the boxplus operation.

Proof: Due to space constraints we only sketch the proof. First, it is sufficient to show (11) for $n = 2$. We compute the conditional cdf of Λ and use the fact that $(-\lambda_1)\widetilde{\boxplus}\lambda_2 = \lambda_1\widetilde{\boxplus}(-\lambda_2)$. Then (11) follows immediately by a change of variables. Along the same lines (11) can be shown to hold also for the boxplus operation. ■

The first step in network encoding at the relay is to apply the sign-min operation to the interleaved LLRs $\lambda'_{i\mathsf{R},k}$, i.e., we compute $\widetilde{\lambda}_{\mathsf{R},k} = \lambda'_{1\mathsf{R},k}\widetilde{\boxplus}\lambda'_{2\mathsf{R},k}\widetilde{\boxplus}\cdots\widetilde{\boxplus}\lambda'_{\mathsf{NR},k}$. We next give a closed-form expression (without proof due to lack of space) for the statistics of the output of the sign-min operation.

Proposition 2. *Let Λ_1, Λ_2 be independent random variables with pdfs $p_{\Lambda_1}(\lambda)$, $p_{\Lambda_2}(\lambda)$. The pdf of $\Lambda = \Lambda_1\widetilde{\boxplus}\Lambda_2$ is given by*

$$\begin{aligned} p_\Lambda(\lambda) = {} & u(\lambda)\big[p_{\Lambda_1}(\lambda)\big(1-F_{\Lambda_2}(\lambda)\big) + p_{\Lambda_2}(\lambda)\big(1-F_{\Lambda_1}(\lambda)\big) \\ & + p_{\Lambda_1}(-\lambda)F_{\Lambda_2}(-\lambda) + p_{\Lambda_2}(-\lambda)F_{\Lambda_1}(-\lambda)\big] \\ & + u(-\lambda)\big[p_{\Lambda_1}(-\lambda)F_{\Lambda_2}(\lambda) + p_{\Lambda_2}(\lambda)\big(1-F_{\Lambda_1}(-\lambda)\big) \\ & + p_{\Lambda_1}(\lambda)\big(1-F_{\Lambda_2}(-\lambda)\big) + p_{\Lambda_2}(-\lambda)F_{\Lambda_1}(\lambda)\big] \\ & - \mathrm{P}\{\Lambda_1 = 0\}\mathrm{P}\{\Lambda_2 = 0\}\delta(\lambda), \end{aligned} \quad (12)$$

where F_{Λ_i} denotes the cdf of Λ_i, $u(\lambda)$ is the unit step function, and $\delta(\lambda)$ is the Dirac delta distribution.

If Λ_i is a discrete or mixed random variable then $F_{\Lambda_i}(\lambda)$ in (12) is to be understood as $F_{\Lambda_i}(\lambda) = \big(\lim_{\xi\to\lambda^-} F_{\Lambda_i}(\xi) + \lim_{\xi\to\lambda^+} F_{\Lambda_i}(\xi)\big)/2$. Since "$\widetilde{\boxplus}$" is associative, the pdf of $\Lambda = \Lambda_1\widetilde{\boxplus}\Lambda_2\widetilde{\boxplus}\cdots\widetilde{\boxplus}\Lambda_n$ can be computed by iterating (12) $n-1$ times in arbitrary order.

Using (12) we can analytically compute the (conditional) pdf of $\widetilde{\Lambda}_{\mathsf{R},k}$. The conditional pdf of an LLR Λ satisfies the following consistency condition:

$$p_{\Lambda|U}(\lambda|0) = p_{\Lambda|U}(\lambda|1)\exp(\lambda). \quad (13)$$

For two LLRs Λ_1, Λ_2 satisfying the above condition, the boxplus output $\Lambda = \Lambda_1\boxplus\Lambda_2$ also satisfies (13), whereas $\widetilde{\Lambda} = \Lambda_1\widetilde{\boxplus}\Lambda_2$ generally violates (13). Therefore we perform

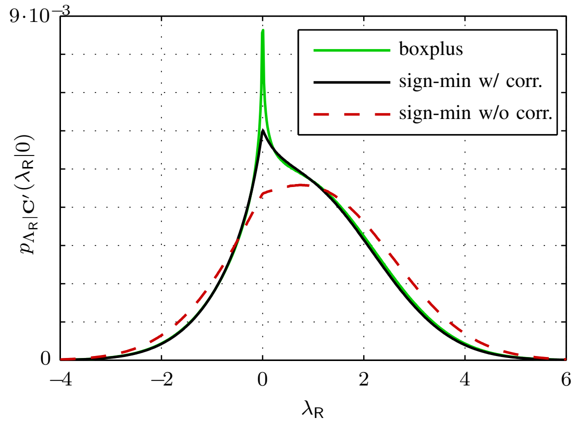

Figure 5: Comparison of LLR statistics at $\gamma_{i\mathsf{R}} = 0\,\text{dB}$.

scalar LLR correction (cf. [8], [9]) to obtain accurate bit reliability information. We define $\phi : \mathbb{R} \to \mathbb{R}$ to be a scalar function which maps λ to an LLR

$$\phi(\lambda) \triangleq \log \frac{p_{U|\Lambda}(U = 0|\Lambda = \lambda)}{p_{U|\Lambda}(U = 1|\Lambda = \lambda)}, \tag{14}$$

whose conditional distribution satisfies (13). Furthermore, for symmetric channels and uniform distribution of U, $\phi(\lambda)$ is odd. In our setting (Gaussian noise and sign-min operation with exact LLRs as input), we have $|\phi(\lambda)| \leq |\lambda|$ since the sign-min operation overestimates the boxplus result. We note that LLR correction becomes more important as the SNR decreases. Fig. 5 shows the conditional LLR statistics $p_{\Lambda_\mathsf{R}|\mathbf{C}'}(\lambda_\mathsf{R}|0)$ for $N = 2$ sources at $\gamma_{i\mathsf{R}} = 0\,\text{dB}$. It can be seen that the pdf of the corrected LLRs is much closer to the pdf of the boxplus operation than the sign-min operation without correction. Therefore, the sign-min operation with LLR correction yields an excellent approximation to the boxplus operation.

In a second step, using (14) and (12), we can compute the final (corrected) network-coded LLRs $\lambda_{\mathsf{R},k} = \phi(\widetilde{\lambda}_{\mathsf{R},k})$ analytically. Assuming that ϕ is invertible on $\mathbb{R}$, the statistics of the corrected LLRs is given by

$$p_{\Lambda_\mathsf{R}}(\lambda_{\mathsf{R},k}) = p_{\widetilde{\Lambda}_\mathsf{R}}\left(\phi^{-1}(\lambda_{\mathsf{R},k})\right) / \left|\phi'\left(\phi^{-1}(\lambda_{\mathsf{R},k})\right)\right|, \tag{15}$$

which can be calculated numerically. We note that ϕ is invertible in our setting. We thus have found expressions which allow us to compute the network-coded LLRs analytically. Furthermore, we can compute the statistics of the network-coded LLRs numerically, enabling online quantizer design.

The parameters Q and M of the vector quantizer are chosen such that $\log_2(Q)/M \leq R_0$, ensuring that the quantization indices can be received by the destination. Again, the quantization rate per sample, $\log_2(Q)/M$, must not be less than 1. Therefore, if $R_0 < 1$ we set $Q = 2, M = 1$ and the relay processes only a fraction of R_0 of all LLRs.

IV. Vector Quantizer Design

The relay uses vector quantization with Q levels to map an M dimensional vector of real-valued LLRs $\boldsymbol{\lambda}$ to an integer-valued index z. The design of the LLR vector quantizers is critical for the performance of the system. We aim at maximizing the mutual information $I(\mathbf{C}; Z)$ between the quantizer index z and the M dimensional vector of code bits $\mathbf{c}$ corresponding to the LLRs $\boldsymbol{\lambda}$. For a fixed number of quantization levels Q we want to solve

$$p^\star_{Z|\boldsymbol{\Lambda}}(z|\boldsymbol{\lambda}) = \underset{p_{Z|\boldsymbol{\Lambda}}(z|\boldsymbol{\lambda}) \in \{0,1\}}{\arg\max} \; I(\mathbf{C}; Z). \tag{16}$$

The deterministic quantizer $\mathsf{Q}(\cdot)$ is described by the mapping $p^\star_{Z|\boldsymbol{\Lambda}}(z|\boldsymbol{\lambda}) \in \{0,1\}$. Given the joint distribution $p_{\mathbf{C},\boldsymbol{\Lambda}}(\mathbf{c},\boldsymbol{\lambda}) = \prod_{i=1}^{M} p_{\Lambda_i|C_i}(\lambda_i|c_i) p_{C_i}(c_i)$, a suitably modified version of the information bottleneck algorithm proposed in [10] allows us to find a locally optimal solution of (16).

Since the joint distribution $p_{\mathbf{C},\boldsymbol{\Lambda}}(\mathbf{c},\boldsymbol{\lambda})$ can be computed both with and without network code, quantizer design can be performed online according to the current source-relay SNRs and thus there is no need to store the quantizers at the relay and destination nodes. In order to make lookup-based encoding feasible (for relatively small M), we pre-quantize the LLRs. We have verified that this pre-quantization does not deteriorate the value of the objective function in (16) at the (locally) optimal point $p^\star_{Z|\boldsymbol{\Lambda}}(z|\boldsymbol{\lambda})$.

The mutual information $I(\mathbf{C}; Z)$ in (16) depends only on the conditional distribution $p_{Z|\boldsymbol{\Lambda}}(z|\boldsymbol{\lambda})$ of z but not on the corresponding reproducer vector $\hat{\boldsymbol{\lambda}}$. Therefore, following [9], we propose to compute the reproducer values as

$$\hat{\lambda}_k = \log \frac{\sum_{\mathbf{c}:c_k=0} p_{\mathbf{C}|Z}(\mathbf{c}|z)}{\sum_{\mathbf{c}:c_k=1} p_{\mathbf{C}|Z}(\mathbf{c}|z)}, \quad k = 1, 2, \ldots, M. \tag{17}$$

We note that (17) ensures that the conditional pdf of $\hat{\Lambda}$ satisfies the consistency condition (13) as we would expect from an LLR distribution. The Q reproduction vectors $\hat{\boldsymbol{\lambda}}_j$, $j = 1, 2, \ldots, Q$, have to be available at the destination in the network decoder (6). One strategy to achieve this is to signal the SNRs on the source-relay channels to the destination. Then, the destination can run the same quantizer design as the relay does and thus the destination knows the reproduction vectors. In any case, we assume that the vectors $\hat{\boldsymbol{\lambda}}_j$ are available at the destination and neglect the signaling overhead.

Finally, we note that by increasing the dimension M while keeping the rate $\log_2(Q)/M$ constant we could not increase the system performance. Clearly, due to the independence of the LLRs we can only obtain a (small) "packing gain" by increasing M. Hence, for a given rate constraint R_0 we choose to find parameters Q and M such that the compression rate is close to R_0 and M is as small as possible, thereby reducing the encoding complexity.

V. Numerical Results

In this section we analyze the SNR required to achieve a bit error rate (BER) of 10^{-4} for transmission with and without network code.

General Setup. Each source transmits $K = 1024$ information bits which are encoded by a rate-1/2 recursive, systematic convolutional code with generator polynomial $[1\ 13/15]_8$ (in octal notation). The sources use a QPSK signal constellation.

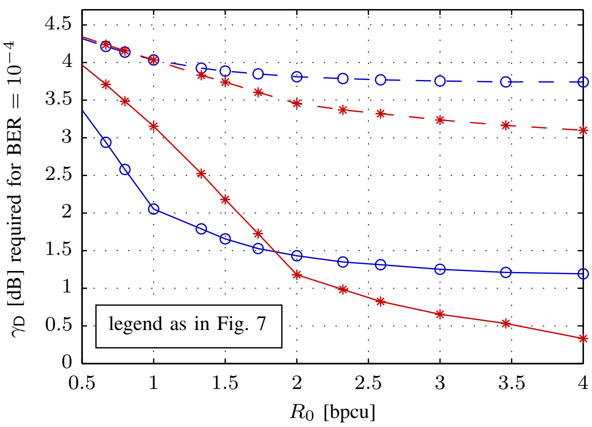

Figure 6: Comparison of transmission strategies for $N = 2$.

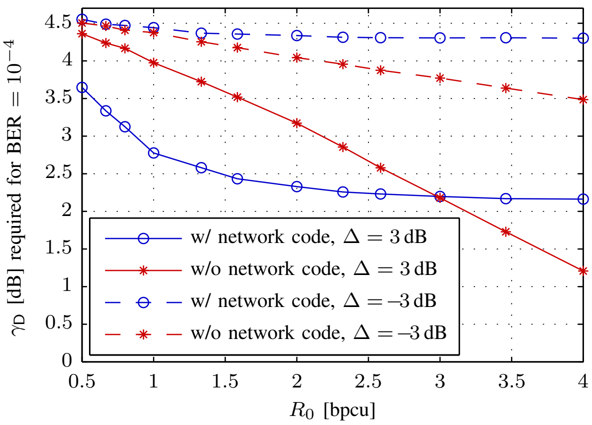

Figure 7: Comparison of transmission strategies for $N = 4$.

The joint network-channel decoder at the destination performs 5 iterations and uses a serial schedule. We have chosen the values of R_0 such that there is a configuration in which $R_0 = \log_2(Q_1)/M_1 = N \log_2(Q_2)/M_2$, where we require $2 \leq Q_1, Q_2 \leq 16$ and $1 \leq M_1, M_2 \leq 6$. If there are multiple choices for M_1 and M_2 we chose the smallest possible values.

The path-loss exponent in (1) is chosen as $n = 3.52$. To allow for a concise presentation of the results we restrict to the symmetric MARC, i.e., $d_\mathsf{D} \triangleq d_{i\mathsf{D}}$, $d_\mathsf{R} \triangleq d_{i\mathsf{R}}$, $\gamma_\mathsf{D} \triangleq \gamma_{i\mathsf{D}}$, $\gamma_\mathsf{R} \triangleq \gamma_{i\mathsf{R}}$. We analyze two MARC geometries, namely $d_\mathsf{R} = 0.8213 \cdot d_\mathsf{D}$ ($[\gamma_\mathsf{R}]_\mathrm{dB} = [\gamma_\mathsf{D}]_\mathrm{dB} + 3\,\mathrm{dB}$) and $d_\mathsf{R} = 1.2176 \cdot d_\mathsf{D}$ ($[\gamma_\mathsf{R}]_\mathrm{dB} = [\gamma_\mathsf{D}]_\mathrm{dB} - 3\,\mathrm{dB}$). In the following let $\Delta = \gamma_\mathsf{R}/\gamma_\mathsf{D}$. We note that in this scenario, a transmission without relay requires $\gamma_\mathsf{D} = 4.7\,\mathrm{dB}$ for a BER of 10^{-4}.

Results for $N = 2$ and $N = 4$. Fig. 6 shows γ_D required to achieve BER $= 10^{-4}$ versus R_0 for $N = 2$. We observe that for $\Delta = 3\,\mathrm{dB}$, a transmission with network code is preferable for $R_0 \leq 1.8$ bpcu. In case $\Delta = -3\,\mathrm{dB}$ and $R_0 \leq 1$ bpcu, the scheme with network code performs marginally better than a transmission without network code. We note that weak source-relay channels have a higher performance impact on transmissions with network code because the probability of bit errors (sign flips) in λ_R is $p'_e = N p_e + \mathcal{O}(p_e^2)$, where p_e denotes the bit error probability for each $\lambda_{i\mathsf{R}}$. Hence, there are on average N times as many bit errors in the network-coded LLRs than in the uncoded LLRs.

We further observe that the scheme with network code does not need more than ~ 2.6 bpcu to transmit the network-coded LLRs with sufficient accuracy since its performance saturates. In general the performance degradation with decreasing R_0 is graceful for both schemes as long as the relay can process all LLRs. If $R_0 < N$ for transmissions without network code and if $R_0 < 1$ in the network-coded case, the performance deteriorates quickly due to the puncturing at the relay.

Fig. 7 shows simulation results for $N = 4$. In case $\Delta = 3\,\mathrm{dB}$, network-coded transmission outperforms the transmission without network for the larger rate regime $R_0 \leq 3$ bpcu. Here, twice the amount of data has to be transmitted without network code compared to $N = 2$. If $\Delta = -3\,\mathrm{dB}$, the network-coded scheme is always worse than the non-cooperative scheme, due to the larger number of bit errors in the network-coded data.

VI. Conclusions

We have studied the impact of the available rate on the relay-destination link on the performance of CF schemes with and without network coding. For the design of the optimal vector quantizers we have derived closed-form expressions for the LLR statistics in the network-coded case. These expressions have been shown to provide an excellent approximation to the statistics of the boxplus operation. Our numerical results show that network coding is preferable if R_0 is small or if N is large. This will likely be the case, e.g., for relays in mobile environments. On the other hand, if the relay is fixed and its link to the destination allows for high data rates, a transmission scheme without network coding will most likely be preferable.

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