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# CONVEX OPTIMIZATION FOR RECEIVE ANTENNA SELECTION IN MULTI-POLARIZED MIMO TRANSMISSIONS

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## ABSTRACT

Diversity and achievable rate in wireless transmissions can be enhanced by using multiple-input multiple-output (MIMO) systems. This directly increases the overall cost due to additional Radio Frequency (RF) chains for each antenna element. The issue can be resolved with antenna selection techniques at the transmitter or the receiver by using a small number of RF chains. These RF chains are multiplexed between larger number of transmitter/receiver antenna elements. Dual and triple polarized antenna structures are a very good solution for realizing compact devices and also robust against many imperfections as compared to spatially separated antenna structures. We model such polarized antenna systems and then apply convex optimization theory for selecting the best possible antennas in terms of capacity maximization. Channel parameters like transmit and receive correlations, cross-polarization discrimination (XPD) are taken into consideration while modeling polarized systems. We compare our results with the Spatially Separated (SP) MIMO with and without selection by performing extensive Monte-Carlo simulations. We found that by using convex optimization algorithm, the performance of multiple polarized systems can be significantly enhanced. For certain channel conditions we observe that triple polarized systems increase the performance significantly compared to dual-polarized and spatially separated systems. We observed that applying selection at the receiver only boosts the performance in Non Line of sight (NLOS) channels compared to Line of Sight (LOS) channels.

***Index Terms—*** Antenna Selection, dual polarization, triple polarization, spatial correlation, angular correlation, cross polarization discrimination (XPD), convex optimization.

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## 1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems have received increased attention because they significantly improve wireless link performance through capacity and diversity gains [1]. A major limiting factor in the deployment of MIMO systems is the cost of multiple analog chains (such as low noise amplifiers, mixers and analog-to-digital converters) at the receiver end. Antenna selection at the transmitter/receiver is a powerful technique that reduces the number of analog chains required, yet preserving the diversity benefits obtained from the full MIMO system. With antenna selection, a limited number of transmit/receive chains are dynamically multiplexed between several transmit/receive antennas. MIMO antenna selection techniques have been extensively studied, and there are several antenna selection criteria. The earliest works on antenna selection have been in the context of Single-Input Multiple-Output (SIMO) systems. For example, selection diversity, where the receiver only selects the strongest antenna signal has long been used in SIMO systems [2]. Receive antenna selection in MIMO systems offer more degrees of freedom than in SIMO systems. For full-diversity space-time codes, a subset of available antennas can be selected to maximize the channel norm [3]. For spatial-multiplexing systems, antennas can be selected to minimize the error ratios [4]. A useful tutorial paper on antenna selection can be found in [5]. Various selection algorithms applied to MIMO OFDM systems can be found in [6]. Exhaustive search based on maximum output SNR is proposed in [7], when the system uses linear receivers. Since exhaustive search is computationally expensive for large MIMO systems, several sub-optimal algorithms with lower complexity are derived at the expense of performance. A selection algorithm based on accurate approximation of the conditional error probability of quasi-static MIMO systems is derived in [8]. In [9], the authors formulate the receive antenna selection problem as a combinatorial optimization problem and relax it to a convex optimization problem. They employ an interior point algorithm based on the barrier method, to solve a relaxed convex problem. However, they treat only the case of capacity maximization. An alternative approach to receive antenna selection for capacity maximization that offers near optimal performance at a complexity, significantly lower than the schemes in [10] but marginally greater than the schemes in [11], is described in [12].

Our approach is based on formulating the selection problem as a combinatorial optimization problem and relaxing it to obtain a problem with a concave objective function and convex constraints. We follow the lines of [9][12], and apply this to the system of arrays with Dual-Polarized (DP) and Triple-Polarized (TP) antenna structures. Application of receive antenna selection on polarized array can be found in [13][14]. We first model the DP and TP systems with respect to many channel characteristics, e.g, K-factor, channel correlations and cross polarization discrimination (XPD). A good investigation on the modeling of DP MIMO channels in [15]. In [16] the author models TP systems and presents the performance in terms of outage probabilities. We then compare the results with the Spatially Separated-Single Polarized (SS-SP) systems with the same channel characteristics. We extend our DP and TP systems to Spatially-Separated Dual-Polarized (SS-DP) and Triple Polarized (SS-TP) systems. These systems are a combination of both spatial and polarization domain.

The remainder of this paper is organized as follows: Section 2 details the generic model and the structure of polarization diverse arrays. We detail all the important channel parameters and their influence on the MIMO channel matrix. In Section 3, the performance metric to optimize with receive antenna selection, is outlined with details. Description of the performance by applying the optimization approach is presented in 4. Simulation results for multiple polarized systems (SS-DP, SS-TP) and comparison with the performance of SS-SP are discussed in Section 5. We conclude our work in Section 6.

## 2. CHANNEL MODEL

The channel is modeled as a Ricean fading channel, i.e, the channel matrix can be composed of a fixed (possibly line-of-sight) part and a random (fast fading) part according to

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\tilde{\mathbf{H}} \qquad (1)$$

where $K$ is the Ricean $K$-factor, $\bar{\mathbf{H}}$ is a deterministic matrix and $\tilde{\mathbf{H}}$ is a random matrix. The random matrix $\tilde{\mathbf{H}}$ consists of complex Gaussian entries which are independent from one channel realization to the next. In other words, if $K = 0$ then $\mathbf{H}$ models a pure Rayleigh fading channel and if $K = \infty$ then it models a static channel. For a DP system, the channel matrix is described in V and H polarizations, i.e., its elements represent the input-output relation from V to V, V to H, H to H, and H to V polarized waves [15][17],

$$\mathbf{H}_{DP} = \begin{bmatrix} \tilde{h}^{VV} & \tilde{h}^{VH} \\ \tilde{h}^{HV} & \tilde{h}^{HH} \end{bmatrix}, \qquad (2)$$

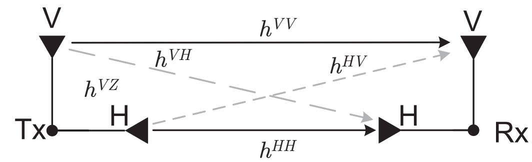


**Fig. 1**. Configuration of dual-polarized system.

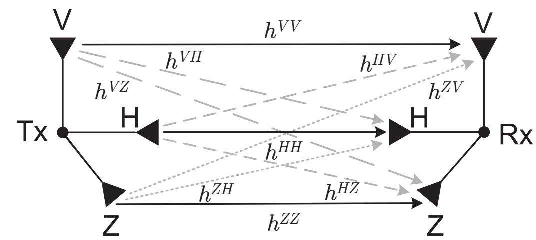


**Fig. 2**. Configuration of triple-polarized system.

and that for $3 \times 3$ triple-polarized channels represented as [18][19],

$$\mathbf{H}_{TP} = \begin{bmatrix} \tilde{h}^{VV} & \tilde{h}^{VH} & \tilde{h}^{VZ} \\ \tilde{h}^{HV} & \tilde{h}^{HH} & \tilde{h}^{HZ} \\ \tilde{h}^{ZV} & \tilde{h}^{ZH} & \tilde{h}^{ZZ} \end{bmatrix}, \qquad (3)$$

A $4 \times 4$ MIMO channel with two spatially separated DP antennas on each side can for example be written as,

$$\mathbf{H} = \begin{bmatrix} \tilde{h}^{1V,1V} & \tilde{h}^{1V,1H} & \tilde{h}^{1V,2V} & \tilde{h}^{1V,2H} \\ \tilde{h}^{1H,1V} & \tilde{h}^{1H,1H} & \tilde{h}^{1H,2V} & \tilde{h}^{1H,2H} \\ \tilde{h}^{2V,1V} & \tilde{h}^{2V,1H} & \tilde{h}^{2V,2V} & \tilde{h}^{2V,2H} \\ \tilde{h}^{2H,1V} & \tilde{h}^{2H,1H} & \tilde{h}^{2H,2V} & \tilde{h}^{2H,2H} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix}, \qquad (4)$$

where the scalar channel between the $i^{\text{th}}$ transmit antenna and the $j^{\text{th}}$ receive antenna is denoted by $\tilde{h}_{jV,iV}$ for the vertical component and $\tilde{h}_{jH,iH}$ for the horizontal component. The cross-components are denoted by $\tilde{h}_{jV,iH}$ and $\tilde{h}_{jH,iV}$, respectively.

### 2.1. Channel XPD

An electromagnetic wave can change its polarization state due to reflections, as it propagates through a channel. A common way of describing a channel's ability to separate V and H polarizations is the so called channel XPD factor which is for the fast fading part of the channel and is defined as,

$$\begin{aligned} X_V &= E\left\{\left|\tilde{h}^{VV}\right|^2\right\}/E\left\{\left|\tilde{h}^{HV}\right|^2\right\} \\ X_H &= E\left\{\left|\tilde{h}^{HH}\right|^2\right\}/E\left\{\left|\tilde{h}^{VH}\right|^2\right\} \end{aligned} . \qquad (5)$$

Similarly for TP channels we have the following XPD definitions as,

$$\begin{aligned} X_{VH} &= E\left\{\left|\tilde{h}^{VV}\right|^2\right\}/E\left\{\left|\tilde{h}^{HV}\right|^2\right\} \\ X_{HV} &= E\left\{\left|\tilde{h}^{HH}\right|^2\right\}/E\left\{\left|\tilde{h}^{VH}\right|^2\right\} \\ X_{ZV} &= E\left\{\left|\tilde{h}^{ZZ}\right|^2\right\}/E\left\{\left|\tilde{h}^{VZ}\right|^2\right\} \\ X_{VZ} &= E\left\{\left|\tilde{h}^{VV}\right|^2\right\}/E\left\{\left|\tilde{h}^{ZV}\right|^2\right\} \\ X_{HZ} &= E\left\{\left|\tilde{h}^{HH}\right|^2\right\}/E\left\{\left|\tilde{h}^{ZH}\right|^2\right\} \\ X_{ZH} &= E\left\{\left|\tilde{h}^{ZZ}\right|^2\right\}/E\left\{\left|\tilde{h}^{HZ}\right|^2\right\} \end{aligned}, \quad (6)$$

where a symmetric leakage is assumed. The same "symmetry" assumption was made for V/H polarized waves in [17] and it is also motivated by the measurements reported in [20] where the leakage from V to H and H to V have the same power on average. We also define the variable $\alpha$, $0 < \alpha \leq 1$, which corresponds to the part of the radiated power that is coupled from V to H and vice versa. There is perfect discrimination between the V and H polarized components as $\alpha \to 0$, and a "leakage" between polarizations when $0 < \alpha \leq 1$. the relation between the channel XPD and $\alpha$ is, thus, given by,

$$\text{XPD} = \frac{1-\alpha}{\alpha}, \quad 0 < \alpha \leq 1, \quad (7)$$

where we have used the following normalizations,

$$E\left\{\left|\tilde{h}^{VV}\right|^2\right\} = E\left\{\left|\tilde{h}^{HH}\right|^2\right\} = 1 - \alpha \quad (8)$$

$$E\left\{\left|\tilde{h}^{HV}\right|^2\right\} = E\left\{\left|\tilde{h}^{VH}\right|^2\right\} = \alpha. \quad (9)$$

Similarly for TP array we have some additional normalizations as follows,

$$E\left\{\left|\tilde{h}^{VV}\right|^2\right\} = E\left\{\left|\tilde{h}^{HH}\right|^2\right\} = E\left\{\left|\tilde{h}^{ZZ}\right|^2\right\} = 1 - (\alpha_1 + \alpha_2). \quad (10)$$

$$E\left\{\left|\tilde{h}^{VH}\right|^2\right\} = E\left\{\left|\tilde{h}^{ZV}\right|^2\right\} = E\left\{\left|\tilde{h}^{HZ}\right|^2\right\} = \alpha_1 \quad (11)$$

$$E\left\{\left|\tilde{h}^{HV}\right|^2\right\} = E\left\{\left|\tilde{h}^{VZ}\right|^2\right\} = E\left\{\left|\tilde{h}^{ZH}\right|^2\right\} = \alpha_2. \quad (12)$$

The above normalizations are motivated by power or energy conservation arguments. That is, the channel cannot introduce more energy to the transmitted signal and with this normalization the power is conserved by subtracting from the co-polarized component the corresponding amount of power $\alpha$ that has leaked into the cross-polarized component. This normalization is of great importance when comparing DP to SP systems. The XPD for TP channel can then be represented by,

$$\text{XPD} = \frac{1-(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}, \quad 0 < (\alpha_1+\alpha_2) \leq 1, \quad (13)$$

Similarly, we define the channel XPD for the fixed part of the DP channel as,

$$\text{XPD}_f = \frac{1-\alpha_f}{\alpha_f}, \quad 0 < \alpha_f \leq 1. \quad (14)$$

with the following normalizations

$$\left|\bar{h}^{VV}\right|^2 = \left|\bar{h}^{HH}\right|^2 = 1 - \alpha_f. \quad (15)$$

$$\left|\bar{h}^{HV}\right|^2 = \left|\bar{h}^{VH}\right|^2 = \alpha_f. \quad (16)$$

Similarly for triple polarized array we have some additional normalizations as follows,

$$\left|\bar{h}^{VV}\right|^2 = \left|\bar{h}^{HH}\right|^2 = \left|\bar{h}^{ZZ}\right|^2 = 1 - (\alpha_{1f} + \alpha_{2f}). \quad (17)$$

$$\left|\bar{h}^{VH}\right|^2 = \left|\bar{h}^{ZV}\right|^2 = \left|\bar{h}^{HZ}\right|^2 = \alpha_{1f} \quad (18)$$

$$\left|\bar{h}^{HV}\right|^2 = \left|\bar{h}^{VZ}\right|^2 = \left|\bar{h}^{ZH}\right|^2 = \alpha_{2f}. \quad (19)$$

$$(20)$$

The channel XPD for the fixed part of the TP channel is given by,

$$\text{XPD}_f = \frac{1-(\alpha_{1f}+\alpha_{2f})}{\alpha_{1f}+\alpha_{2f}}, \quad 0 < (\alpha_{1f}+\alpha_{2f}) \leq 1. \quad (21)$$

### 2.2. Channel Correlation

The elements of the Spatially Separated-Single Polarized (SS-SP) MIMO channel matrix will be correlated, when the channel is not rich enough, i.e., when there is not enough scattering to decorrelate the elements of the channel matrix and/or when the antenna spacing is too small. We define the transmit, $t_s$, and receive, $r_s$, spatial and co-polarized correlation coefficients as,

$$t_s = \frac{E\left\{\tilde{h}_{iV,iV}\tilde{h}^*_{iV,jV}\right\}}{1-\alpha} = \frac{E\left\{\tilde{h}_{iH,jH}\tilde{h}^*_{iH,iH}\right\}}{1-\alpha}, i \neq j, \quad (22)$$

$$r_s = \frac{E\left\{\tilde{h}_{iV,iV}\tilde{h}^*_{jV,iV}\right\}}{1-\alpha} = \frac{E\left\{\tilde{h}_{iH,jH}\tilde{h}^*_{iH,iH}\right\}}{1-\alpha}, i \neq j. \quad (23)$$

Similarly, we define the transmit, $t_p$, and receive, $r_p$, polarization correlation coefficients as,

$$t_p = \frac{E\left\{\tilde{h}_{iV,iV}\tilde{h}^*_{iV,iH}\right\}}{\sqrt{\alpha(1-\alpha)}} = \frac{E\left\{\tilde{h}_{iH,iV}\tilde{h}^*_{iH,iH}\right\}}{\sqrt{\alpha(1-\alpha)}}, \quad (24)$$

$$r_p = \frac{E\left\{\tilde{h}_{iV,iV}\tilde{h}^*_{iH,iV}\right\}}{\sqrt{\alpha(1-\alpha)}} = \frac{E\left\{\tilde{h}_{iV,iH}\tilde{h}^*_{iH,iH}\right\}}{\sqrt{\alpha(1-\alpha)}}. \quad (25)$$

For example, the measurements reported in [21] showed that the average envelope correlations (worst case) were all less than 0.2, and, in fact, all of the reported measurements in [20] showed that $t_p \approx r_p \approx 0$. The correlations between elements of TP structures can be shown in a straight forward manner as above.

### 2.3. Total Channel

A $2\times2$ dual-polarized MIMO channel is expressed as follows,

$$\tilde{\mathbf{H}}_{DP} = \mathbf{\Sigma}_{DP} \odot \left(\mathbf{C}_{r_p}^{1/2}\mathbf{W}_{2\times2}\mathbf{C}_{t_p}^{1/2}\right), \tag{26}$$

$$\mathbf{\Sigma}_{DP} = \begin{bmatrix} \sqrt{1-\alpha} & \sqrt{\alpha} \\ \sqrt{\alpha} & \sqrt{1-\alpha} \end{bmatrix}, \tag{27}$$

$$\mathbf{C}_{r_p} = \begin{bmatrix} 1 & r_p \\ r_p^* & 1 \end{bmatrix};\ \mathbf{C}_{t_p} = \begin{bmatrix} 1 & t_p \\ t_p^* & 1 \end{bmatrix}, \tag{28}$$

are the polarization leakage, receive and transmit correlation matrices and $\mathbf{W} \in \mathcal{N}_C(0,1)$ is i.i.d complex-valued Gaussian matrix. A $3\times3$ triple-polarized MIMO channel is expressed as follows,

$$\tilde{\mathbf{H}}_{TP} = \mathbf{\Sigma}_{TP} \odot \left(\mathbf{C}_{r_p}^{1/2}\mathbf{W}_{3\times3}\mathbf{C}_{t_p}^{1/2}\right), \tag{29}$$

where

$$\mathbf{\Sigma}_{TP} = \begin{bmatrix} \sqrt{1-\beta} & \sqrt{\alpha_1} & \sqrt{\alpha_2} \\ \sqrt{\alpha_2} & \sqrt{1-\beta} & \sqrt{\alpha_1} \\ \sqrt{\alpha_1} & \sqrt{\alpha_2} & \sqrt{1-\beta} \end{bmatrix}, \tag{30}$$

where $\beta = (\alpha_1+\alpha_2)$ and the condition for "symmetry" is that $0 \le \beta \le 1$.

$$\mathbf{C}_{r_p} = \begin{bmatrix} 1 & r_p & r_p \\ r_p^* & 1 & r_p \\ r_p^* & r_p^* & 1 \end{bmatrix};\ \mathbf{C}_{t_p} = \begin{bmatrix} 1 & t_p & t_p \\ t_p^* & 1 & t_p \\ t_p^* & t_p^* & 1 \end{bmatrix}, \tag{31}$$

are the polarization leakage, receive and transmit correlation matrices. Here we assume that the correlation values for each pair of polarization, in a TP structure are equal. Extension to arrays of multiple SS-DP and SS-TP antenna arrays are straight forward and shown below as,

$$\tilde{\mathbf{H}}_{DP} = \mathbf{1}_{M_R/2\times N_T/2} \otimes \mathbf{\Sigma}_{DP} \odot \left(\mathbf{C}_r^{1/2}\mathbf{W}_{M_R\times N_T}\mathbf{C}_t^{1/2}\right), \tag{32}$$

where $M_R$ and $N_T$ are the number of receive and transmit antennas respectively. They should always be multiples of two for the DP case.

$$\tilde{\mathbf{H}}_{TP} = \mathbf{1}_{M_R/3\times N_T/3} \otimes \mathbf{\Sigma}_{TP} \odot \left(\mathbf{C}_r^{1/2}\mathbf{W}_{M_R\times N_T}\mathbf{C}_t^{1/2}\right), \tag{33}$$

where $M_R$ and $N_T$ should always be multiples of three for the TP case. The $\mathbf{C}_r = \mathbf{C}_{r_s} \otimes \mathbf{C}_{r_p}$ and $\mathbf{C}_t = \mathbf{C}_{t_s} \otimes \mathbf{C}_{t_p}$ are the receive correlation and transmit correlation matrices of the $M_R \times M_R$ MIMO channel with $M_R$ spatially separated dual-polarized and triple-polarized antennas on each side. The matrix $\mathbf{1}_{M_R/2\times N_T/2}$ and $\mathbf{1}_{M_R/3\times N_T/3}$ are representing matrices of all elements to be one respectively. The spatial correlation matrices are given, for example for a $2\times2$ SS system, as follows,

$$\mathbf{C}_{r_s} = \begin{bmatrix} 1 & r_s \\ r_s^* & 1 \end{bmatrix};\ \mathbf{C}_{t_s} = \begin{bmatrix} 1 & t_s \\ t_s^* & 1 \end{bmatrix}. \tag{34}$$

## 3. RECEIVE ANTENNA SELECTION IN MIMO

We focus here on receive antenna selection for capacity maximization. The capacity of the MIMO system described in Section II is given by the well known formula,

$$C(\mathbf{H}) = \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{H}^H\mathbf{R}_{ss}\mathbf{H}\right) \tag{35}$$

$$= \log_2\det\left(\mathbf{I}_{N_T} + \frac{\gamma}{N_T}\mathbf{R}_{ss}\mathbf{H}\mathbf{H}^H\right) \tag{36}$$

due to the matrix identity $\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA})$, where $\gamma = E_s/N_0$, $\mathbf{R}_{ss} = E\left(\mathbf{s}(k)\mathbf{s}(k)^H\right)$ is the covariance matrix of the transmitted signals with $\mathrm{trace}(\mathbf{R}_{ss}) = 1$. The determinant is denoted by $\det(\cdot)$ and $\mathbf{I}_{N_T}$ represents the $N_T \times N_T$ identity matrix. However, when only $M_R^{'} < M_R$ receive antennas are used, the capacity becomes a function of the antennas chosen, where $M_R^{'}$ is the number of antennas selected at the receiver. If we represent the indices of the selected antennas by $\mathbf{r} = [r_1, \ldots, r_{M_R^{'}}]$, the effective channel matrix is $\mathbf{H}$ with those rows only corresponding to these indices. Denoting the resulting $M_R^{'} \times N_T$ matrix by $\mathbf{H}_r$, the channel capacity with antenna selection is given by,

$$C_r(\mathbf{H}_r) = \log_2\det\left(\mathbf{I}_{N_T} + \frac{\gamma}{N_T}\mathbf{R}_{ss}\mathbf{H}_r\mathbf{H}_r^H\right). \tag{37}$$

In the absence of Channel State Information (CSI) at the transmitter, $\mathbf{R}_{ss}$ is chosen as $\mathbf{I}_{N_T}$. Our goal is to choose the index set $\mathbf{r}$ such that the capacity in (37) is maximized. A closed form characterization of the optimal solution is difficult. We propose a possible selection method in the next section.

## 4. OPTIMIZATION ALGORITHM FOR ANTENNA SELECTION

We formulate the problem of receive antenna selection as a constrained convex optimization problem [22] that can be solved efficiently using numerical methods such as interior-point algorithms [23]. Similar to [12], the $\Delta_i (i = 1, \ldots, M_R)$ is defined such that,

$$\Delta_i = \begin{cases} 1, & i^{th} \text{ receive antenna selected} \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

By definition, $\Delta_i = 1$ if $r_i \in \mathbf{r}$, and 0 else. Now, consider an $M_R \times M_R$ diagonal matrix $\mathbf{\Delta}$ that has $\Delta_i$ as its diagonal entries. Thus, the achievable MIMO channel capacity with antenna selection can be re-written as

$$C_r(\mathbf{\Delta}) = \log_2\det\left(\mathbf{I}_{N_T} + \frac{\gamma}{N_T}\mathbf{H}^H\mathbf{\Delta}\mathbf{H}\right), \tag{39}$$

$$= \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{\Delta}\mathbf{H}\mathbf{H}^H\right). \tag{40}$$

The capacity expression given by $C_r(\mathbf{\Delta})$ is concave in $\mathbf{\Delta}$. The proof follows from the following facts: The function $f(\mathbf{X}) = \log_2\det(\mathbf{X})$ is concave in the entries of $\mathbf{X}$ if $\mathbf{X}$ is a positive definite matrix, and the concavity of a function is preserved under an affine transformation [22]. The variables $\Delta_i$ are binary valued $(0$ or $1)$ integer variables, thereby rendering the selection problem NP-hard. We seek a simplification by relaxing the binary integer constraints and allowing $\Delta_i \in [0,1]$. Thus, the problem of receive antenna subset selection for capacity maximization is approximated by the constrained convex relaxation plus rounding schemes.

$$\text{maximize} \;\; \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{\Delta}\mathbf{H}\mathbf{H}^H\right) \tag{41a}$$

subject to

$$0 \le \Delta_i \le 1, \;\; i = 1, \ldots, \tag{41b}$$

$$\text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M_R} \Delta_i = M_R^{'}. \tag{41c}$$

where $\mathbf{H}$ is given by (26) for DP and (29) for TP antenna systems.

## 5. RESULTS AND DISCUSSION

In this section we evaluate the capacity for different channel scenarios depending on parameters like correlation, XPD and K-factor. For all Ricean fading examples the fixed $2\times 2$ channel components are given by to,

$$\bar{\mathbf{H}}_{DP} = \begin{bmatrix} \sqrt{1-\alpha_f} & \sqrt{\alpha_f} \\ \sqrt{\alpha_f} & \sqrt{1-\alpha_f} \end{bmatrix}. \tag{42}$$

Similarly for the triple-polarized case we have,

$$\bar{\mathbf{H}}_{TP} = \begin{bmatrix} \sqrt{1-\beta_f} & \sqrt{\alpha_{1f}} & \sqrt{\alpha_{2f}} \\ \sqrt{\alpha_{2f}} & \sqrt{1-\beta_f} & \sqrt{\alpha_{1f}} \\ \sqrt{\alpha_{1f}} & \sqrt{\alpha_{2f}} & \sqrt{1-\beta_f} \end{bmatrix}, \tag{43}$$

where $\beta_f = (\alpha_{1f} + \alpha_{2f})$. A nominal value of $\text{XPD}_f = \frac{1-\beta_f}{\beta_f} = 15\text{dB}$ is chosen for simulations [15]. For the SS-SP systems, we have the following matrices with fixed channel (see Eq. (1)).

$$\bar{\mathbf{H}}_{2SS-SP} = \begin{bmatrix} \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \\ \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \end{bmatrix}. \tag{44}$$

Similarly for three SS-SP case we have,

$$\bar{\mathbf{H}}_{3SS-SP} = \begin{bmatrix} \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \\ \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \\ \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \end{bmatrix}. \tag{45}$$

Throughout all simulations we used typical correlation values $t_p = r_p = 0.3$ and $t_s = r_s = 0.5$ [20][21]. We compute ergodic capacity by averaging over 100 instantaneous capacity values, varying the matrix $\mathbf{W} \in \mathcal{N}_C(0,1)$ as i.i.d complex-valued Gaussian. We compare Antenna Selection (AS) methods by selecting $M_R'$ out of $M_R$ antennas against Non Antenna Selection (NAS) by utilizing all $M_R' = M_R$ antennas. As selection method we apply (41).

From Figure 3 we observe that with the given channel parameters, the performance of 2SS-DP and 3SS-SP systems is almost the same for all $M_R^{'}$. The 3SS-TP systems has a better performance with selection for values of $M_R^{'} > 4$ as compared to 2SS-DP. We also observe in the figure that all the systems with antenna selection perform better compared to non Antenna Selection (NAS) systems. The 3SS-TP system has the best overall performance. In Figure 4, we show the impact of the XPD parameter on the ergodic capacity of the polarized systems with and without selection. We use the case of $M_R^{'} = 6$ as an example for both DP and TP systems. For a fair comparison of DP and TP systems we used $\text{XPD}_f = 15\text{dB}$ for both systems. In the simulations we used $\beta_f = (\alpha_{1f} + \alpha_{2f})$. We assumed $\alpha_{1f} = \alpha_{2f}$ in our simulations (see Eq. (13)). The same condition is applied for the varying XPD from (30) and the condition of symmetry is taken as $\beta = (\alpha_1 + \alpha_2)$. Again $\alpha_1 = \alpha_2$ is assumed for simulations (see Eq. (21)). We observe that with the given channel parameters, SS-SP systems are not effected by the $\alpha$ values. We observe that 3SS-TP without selection has the best performance. A selection within 3SS-TP systems is far better than a selection within 2SS-DP. Figure 5 shows the performance in terms of Ricean K-factor. We observe that the performance gets worse when the LOS component K increases. We also observe that DP systems with or without antenna selection are effected more by the K-factor compared to TP systems. We also extract from the figure that TP systems with selection perform a lot better than all other systems except full complexity TP systems. In Figure 6 we compared the CO antenna selection method to the well known Capacity Maximization (CM) method based on exhaustive search for the maximum norm of the selected sub-channels [5][7]. For CM the average was taken over $10^5$ channel realizations for the matrix $\mathbf{W}$. We observe that the CO method performs better compared to the CM method for DP systems specially for $M_R^{'} = 2, ..., 5$. The gain acquired is almost 3.2 bit/s/Hz at $M_R^{'} = 5$. For TP systems we also see a significant gain achieved by using this method. A gain of 2.5 bit/s/Hz can be observed at $M_R^{'} = 3$. We also observe a gain of 0.5 bit/s/Hz at $M_R^{'} = 5$ for 3SS-SP systems while comparing the CO to the CM method.

## 6. CONCLUSIONS

In this work we investigated a model for dual and triple polarized MIMO channels. We used convex optimization to optimize the performance of such systems for maximizing ergodic capacity. We used the relaxation of a binary integer constraint to have a convex optimization algorithm and solved it, using disciplined convex programming. The optimization

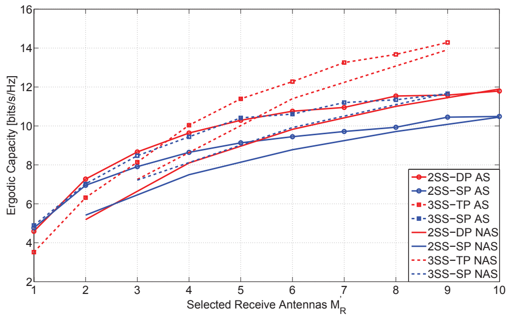


**Fig. 3**. Capacity vs antennas selected for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, Rayleigh fading $K = 0$ and XPD $= 10$dB.

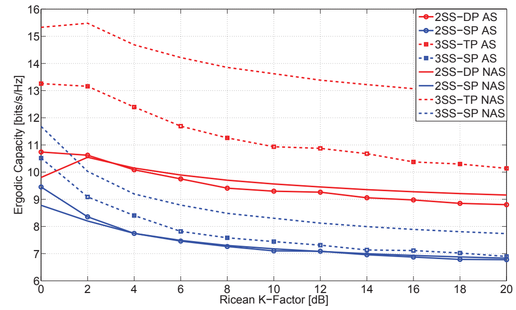


**Fig. 5**. Capacity vs K-factor for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with $M_R^{'} = 6$, SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, and XPD $= 10$dB, XPD$_f = 15$dB.

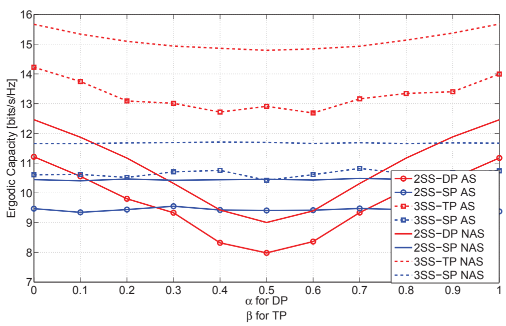


**Fig. 4**. Capacity vs XPD for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with $M_R^{'} = 6$, SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, Rayleigh fading $K = 0$.

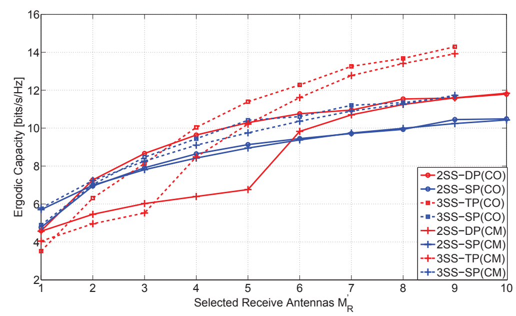


**Fig. 6**. Capacity vs antennas selected for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, Rayleigh fading $K = 0$ and XPD $= 10$dB. Comparison between Convex Optimization (CO) and Capacity Maximization (CM) based selection).

algorithm finds the best antennas for selection. We also compared the results with an array consisting of spatially separated single polarized array of linear elements. We found that by using an optimization algorithm, the performance of multiple polarized systems can be significantly enhanced. For certain channel conditions we see that triple polarized systems increase the performance significantly compared to spatially separated systems. We also observe that applying selection at the receiver only boosts the performance in NLOS channels compared to LOS channels. A comparison with the exhaustive search method of capacity maximization for selection shows that convex optimization based search method performs better for polarized MIMO systems with antenna selection.

## 7. REFERENCES

[1] A. J. Paulraj, R. Nabar, and D. A. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2nd edition, 2003.

[2] J. Proakis, *Digital Communications*, McGraw Hill, 4 edition, 2000.

[3] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2580–2588, 2002.

[4] Jr. Heath, R. W. and A. Paulraj, "Antenna selection for spatial multiplexing systems based on minimum er-

ror rate," in *Proc. IEEE Int. Conf. Communications (ICC'01)*, 2001, vol. 7, pp. 2276–2280.

[5] A. F. Molisch and M. Z. Win, "MIMO systems with antenna selection," *IEEE Microwave Magazine*, vol. 5, no. 1, pp. 46–56, 2004.

[6] A. Habib, C. Mehlführer, and M. Rupp, "Performance comparison of antenna selection algorithms in WiMAX with link adaptation," in *Proc. 4th Int. Conf. Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM'09)*, 2009, pp. 1–5.

[7] Jr. Heath, R. W., S. Sandhu, and A. Paulraj, "Antenna selection for spatial multiplexing systems with linear receivers," *IEEE Communications Letters*, vol. 5, no. 4, pp. 142–144, 2001.

[8] F. Kharrat-Kammoun, S. Fontenelle, S. Rouquette, and J. J. Boutros, "Antenna selection for MIMO systems based on an accurate approximation of QAM error probability," in *Proc. IEEE 61st Vehicular Technology Conf. (VTC Spring'05)*, 2005, vol. 1, pp. 206–210.

[9] FengGang Sun, Ju Liu, HongJi Xu, and Peng Lan, "Receive antenna selection using convex optimization for MIMO systems," in *Proc. Third Int. Conf. Communications and Networking in China (ChinaCom'08)*, 2008, pp. 426–430.

[10] A. Gorokhov, D. A. Gore, and A. J. Paulraj, "Receive antenna selection for MIMO spatial multiplexing: theory and algorithms," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2796–2807, 2003.

[11] Guowei Zhang, Lan Tian, and Lan Peng, "Fast antenna subset selection for MIMO wireless systems," in *Proc. Punta del Este Information Theory Workshop (ITW '06)*, 2006, pp. 493–496.

[12] A. Dua, K. Medepalli, and A. J. Paulraj, "Receive antenna selection in MIMO systems using convex optimization," *IEEE Transactions on Wireless Communications*, vol. 5, no. 9, pp. 2353–2357, 2006.

[13] A. Habib, C. Mehlführer, and M. Rupp, "Receive antenna selection for polarized antennas," in *Proceedings of 18th IEEE International Conference on Systems, Signals and Image Processing*, Sarajevo, Bosnia, June 2011.

[14] A. Habib, C. Mehlführer, and M. Rupp, "Performance of compact antenna arrays with receive selection," in *Proceedings of 7th IEEE International Conference on Wireless Advanced 2011*, London, U.K, June 2011.

[15] M. Coldrey, "Modeling and capacity of polarized MIMO channels," in *Proc. IEEE Vehicular Technology Conf. (VTC Spring'08)*, 2008, pp. 440–444.

[16] A. Habib, "Multiple polarized MIMO with antenna selection," in *18th IEEE Symposium on Communications and Vehicular Technology (SCVT'11), Belgium, Nov. 2011.*

[17] C. Oestges, B. Clerckx, M. Guillaud, and M. Debbah, "Dual-polarized wireless communications: from propagation models to system performance evaluation," *IEEE Transactions on Wireless Communications*, vol. 7, no. 10, pp. 4019–4031, 2008.

[18] F. Quitin, C. Oestges, F. Horlin, and P. De Doncker, "Multipolarized mimo channel characteristics: Analytical study and experimental results," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 9, pp. 2739–2745, 2009.

[19] F. Quitin, F. Bellens, A. Panahandeh, J.-M. Dricot, F. Dossin, F. Horlin, C. Oestges, and P. De Doncker, "A time-variant statistical channel model for tri-polarized antenna systems," in *Proc. IEEE 21st Int Personal Indoor and Mobile Radio Communications (PIMRC) Symp*, 2010, pp. 64–69.

[20] H. Asplund, J.-E. Berg, F. Harrysson, J. Medbo, and M. Riback, "Propagation characteristics of polarized radio waves in cellular communications," in *Proc. 66th Fall Vehicular Technology Conf. (VTC Fall'07)*, 2007, pp. 839–843.

[21] V. Erceg, P. Soma, D. S. Baum, and S. Catreux, "Multiple-input multiple-output fixed wireless radio channel measurements and modeling using dual-polarized antennas at 2.5 GHz," *IEEE Transactions on Wireless Communications*, vol. 3, no. 6, pp. 2288–2298, 2004.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[23] A. Habib, B. Krasniqi, and M. Rupp, "Antenna selection in polarization diverse MIMO transmissions with convex optimization," in *18th IEEE Symposium on Communications and Vehicular Technology (SCVT'11), Belgium, Nov. 2011.*