# Inter-Carrier Interference Mitigation by Means of Precoding

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***Abstract*—In this work, we discuss the possibility of using precoding as means to mitigate Inter Carrier Interference (ICI) caused by temporal channel variations in Orthogonal Frequency Division Multiplexing (OFDM) systems. Many different precoding schemes have been introduced in the past. However, the promised gains of these techniques are not achievable nor realistic. In this paper, we introduce a practical low-cost precoding technique to mitigate ICI caused by Doppler spread. The gain in terms of Signal to Interference and Noise Ratio (SINR) is 0.5 dB at a velocity of 500 km/h.**

***Index Terms*—LTE, ICI, OFDM, Precoding.**

## I. Introduction

Current Orthogonal Frequency Division Multiplexing (OFDM) based systems for wireless communication are sensitive to channel variations [1]. The time variant distortion such as the Doppler spread destroys the designed orthogonality between subcarriers and causes Inter Carrier Interference (ICI). Consequently, the performance of such a system is limited.

### A. Related Work

In previous work, one could recognize principally two different techniques of mitigating ICI caused by the Doppler spread. A first type of techniques is based on advanced equalization techniques. In the time-invariant case, it is sufficient to use single tap equalizers. In [2], a so-called $Q$-tap equalizer was introduced. This equalizer considers also channel knowledge of the neighboring subcarriers, which results in better performance. However, this solution has not only higher complexity, but also requires more information about the channel to be estimated. The second type of ICI mitigation techniques is based on precoding. The data vector is spread over different subcarriers which reduces the impact of ICI on the system performance as introduced in [3]. The same data symbols are transmitted on adjacent subcarriers, but rotated by 180 degrees, effectively reducing ICI at the cost of 50% bandwidth reduction. The authors of [4] introduced a precoding method, which does not cause bandwidth reduction. However, a Maximum Likelihood (ML) receiver is required, which significantly increases the complexity. In [5] so called frequency domain Partial Response Coding (PRC) was introduced. This technique can be viewed as special type of precoding, where only adjacent subcarriers are precoded. This technique attracted a lot of interest in the past few years, due to its high theoretical gain in terms of reduction of the ICI power. However, the performance metric utilized by the authors does not necessary reflect performance of a real system. As already argued in [6], PRC as it is introduced in [5], does not improve system's performance, it rather limits it. In [7] a precoding technique is proposed to mitigate ICI utilizing eigendecomposition of the channel matrix. This method is however not of practical relevance due to the requirements on extremely low-delay feedback between receiver and transmitter and its high computational complexity.

### B. Contribution

In this paper, we will show how to use precoding techniques in the frequency domain to mitigate ICI of an OFDM based system.

The main contributions of the paper are:

- We introduce a precoding technique that effectively mitigate ICI caused by channel variation and does not require an ML receiver.
- As cost function we choose the Signal to Interference and Noise Ratio (SINR) that directly reflects performance of a transmission system.
- All data, tools, as well as implementations needed to reproduce the results of this paper can be downloaded from our homepage [8].

The remainder of the paper is organized as follows. In Section II, we describe the mathematical system model. In Section III, we extend our system model for usage of precoding. Furthermore, we show how to design a precoder that maximizes the post-equalization SINR. Finally, we present simulation results in Section IV and conclude our paper in Section V.

## II. System Model

In this section, we introduce a mathematical model of the considered transmission system under time-variant channels.

Consider the following Single Input Single Output (SISO) transmission in the frequency domain

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{n}, \tag{1}$$

where the vector $\mathbf{x}$ denotes the data symbols, the matrix $\mathbf{G}$ the channel matrix including effects of Fast Fourier Transform (FFT) and Cyclic Prefix (CP), $\mathbf{y}$ the received data symbol vector and $\mathbf{n}$ the noise vector. All the vectors have length $N$, which corresponds to the number of subcarriers. The channel matrix $\mathbf{G}$ is thus of size $N \times N$. In case of a time-invariant channel, the matrix is diagonal. If the channel is varying over time, the channel matrix loses its diagonal structure [9].

## III. Precoding

Let us introduce a precoding $\mathbf{W}$ matrix into our system model

$$\mathbf{y} = \mathbf{G}\mathbf{W}\mathbf{x} + \mathbf{n}. \tag{2}$$

The precoding matrix $\mathbf{W}$ is an $N \times N$ matrix. Zhang and Li in their paper [5] introduced a technique called PRC to mitigate influence of the ICI power. Adjacent transmit symbols are mixed together and transmitted over a time-variant channel. This procedure can be fully described by Equation (2), where the precoding matrix $\mathbf{W}$ is a circulant matrix

$$\mathbf{W} = \mathrm{circ}\left(\mathbf{w}\right), \tag{3}$$

in which the vector $\mathbf{w}$ contains the precoding coefficients. In [5] optimal values of the vector $\mathbf{w}$ for various lengths of the vector have been derived. With increasing size of the vector, the ICI power could be dramatically reduced. However, this solution is unrealistic, due to the fact that the chosen vector $\mathbf{w}$ tries to transmit the available transmit power at off-diagonal elements of the channel matrix $\mathbf{G}$, which are rather low and can be used for data transmission only in a very limited way. At the same time, it is necessary to estimate more elements of the channel matrix $\mathbf{G}$.

In the following, we derive how to choose the vector $\mathbf{w}$, while decreasing the ICI power and maintaining a practical low cost single tap equalizer.

For simplicity let us assume, that

- The channel follows a frequency flat Rayleigh distribution.
- Diagonal elements and the elements of the first $K$ off-diagonals of the channel matrix $\mathbf{G}$ are significant. Therefore, we assume, the remaining elements are 0 [1].
- With the first and second assumptions, the channel matrix $\mathbf{G}$ can be fully described by $2K+1$ elements $g_i$ with $-K \leq i \leq K$.
- Consequently, the length of the vector $\mathbf{w}$ is chosen as $2K+1$.

With all these assumptions, a precoded symbol at the $k$-th subcarrier $\tilde{x}_k$ can be written as

$$\tilde{x}_k = \sum_{i=-K}^{K} x_{k-i} w_i. \tag{4}$$

Consequently, a received symbol at the $k$-th subcarrier can be written as

$$\begin{aligned} y_k &= \sum_{j=-K}^{K} \tilde{x}_{k-j} g_j + n_k \\ &= \sum_{j=-K}^{K} g_j \sum_{i=-K}^{K} x_{k-j-i} w_i + n_k. \end{aligned} \tag{5}$$

The above equation can be formulated as a matrix in the following way

$$y_k = \mathbf{w}^{\mathrm{T}} \begin{bmatrix} g_{-K} & \cdots & g_K & & \\ & \ddots & & \ddots & \\ & & g_{-K} & \cdots & g_K \end{bmatrix} \mathbf{x}_k + n_k, \tag{6}$$

with vectors $\tilde{\mathbf{w}}$ and $\mathbf{x}_k$ being defined as

$$\mathbf{w} = \begin{bmatrix} w_{-K} & w_{-K+1} & \cdots & w_{K-1} & w_K \end{bmatrix}^{\mathrm{T}}, \tag{7}$$

$$\mathbf{x}_k = \begin{bmatrix} x_{k-2K} & x_{k-2K+1} & \cdots & x_{k+2K-1} & x_{k+2K} \end{bmatrix}^{\mathrm{T}}. \tag{8}$$

Keeping in mind that we apply a simple single tap equalization at the receiver, one has to choose the vector $\mathbf{w}$ such that the symbol $x_k$ is dominant in $y_k$, which corresponds to the diagonalization of the effective channel matrix $\mathbf{GW}$. Equation (6) can be further modified into

$$y_k = \underbrace{\mathbf{w}^{\mathrm{T}} \tilde{\mathbf{g}} x_k}_{\text{desired signal}} + \underbrace{\mathbf{w}^{\mathrm{T}} \tilde{\mathbf{G}} \mathbf{x}_{\tilde{k}}}_{\text{interference signal}} + n_k, \tag{9}$$

where the vector $\mathbf{x}_{\tilde{k}}$ is the same as the vector $\mathbf{x}_k$ except it does not contain the element $x_k$. The vector $\tilde{\mathbf{g}}$ and matrix $\tilde{\mathbf{G}}$ are defined as follows

$$\tilde{\mathbf{g}} = \begin{bmatrix} g_K & g_{K-1} & \cdots & g_{-K+1} & g_{-K} \end{bmatrix}^{\mathrm{T}} \tag{10}$$

$$\tilde{\mathbf{G}} = \begin{bmatrix} g_{-K} & \cdots & g_{K-1} & & & \\ & \ddots & \vdots & g_K & & \\ & & g_{-K} & \vdots & \ddots & \\ & & & g_{-K+1} & \cdots & g_K \end{bmatrix}. \tag{11}$$

Using above formulated vectors and matrix, the SINR expression can be defined

$$\gamma_k = \frac{\mathbb{E}\left\{\|\mathbf{w}^{\mathrm{T}} \tilde{\mathbf{g}} x_k\|_2^2\right\}}{\mathbb{E}\left\{\|\mathbf{w}^{\mathrm{T}} \tilde{\mathbf{G}} \mathbf{x}_{\tilde{k}}\|_2^2\right\} + \mathbb{E}\left\{\|n_k\|^2\right\}}, \tag{12}$$

which can be further simplified for a transmit power $\mathbb{E}\left\{\|x_k\|^2\right\} = 1$

$$\gamma_k = \frac{\mathbf{w}^{\mathrm{T}} \mathbb{E}\left\{\tilde{\mathbf{g}}\tilde{\mathbf{g}}^{\mathrm{H}}\right\} \tilde{\mathbf{w}}^*}{\mathbf{w}^{\mathrm{T}} \mathbb{E}\left\{\tilde{\mathbf{G}}\tilde{\mathbf{G}}^{\mathrm{H}}\right\} \tilde{\mathbf{w}}^* + \sigma_n^2}, \tag{13}$$

In order to be able to answer question of how to choose the precoding vector $\mathbf{w}$ optimally we need to be able to evaluate the expectation operators in Equation (13). Let us first derive the expression $\mathbb{E}\left\{g_k g_l *\right\}$. The $k$-th channel coefficient $g_k$ is obtained by means of the Fourier transfor

$$g_k = \frac{1}{N}\sum_{n=0}^{N-1} h_n e^{j2\pi\frac{nk}{N}}, \tag{14}$$

where $h_n$ is the channel coefficient at the time sample $n$ in the time domain. Back to the expression $\mathbb{E}\left\{g_k g_l *\right\}$

$$\begin{aligned}&\mathbb{E}\left\{g_k g_l *\right\} = \qquad (15)\\ &= \mathbb{E}\left\{\frac{1}{N}\sum_{n=0}^{N-1} h_n e^{j2\pi\frac{nk}{N}} \frac{1}{N}\sum_{n'=0}^{N-1} h_{n'}^{*} e^{-j2\pi\frac{n'l}{N}}\right\}\\ &= \frac{1}{N^2}\sum_{n=0}^{N-1}\sum_{n'=0}^{N-1} r\left(T_s\omega_d(n'-n)\right) e^{j2\pi\frac{nk}{N}} e^{-j2\pi\frac{n'l}{N}},\end{aligned}$$

where $r(\cdot)$ is the channel autocorrelation function, $T_s$ and $\omega_d$ represents the sampling time and Doppler frequency, respectively. With Equation (15) the expressions for $\mathbb{E}\left\{\tilde{\mathbf{g}}\tilde{\mathbf{g}}^{\mathrm{H}}\right\}$ and $\mathbb{E}\left\{\tilde{\mathbf{G}}\tilde{\mathbf{G}}^{\mathrm{H}}\right\}$ required in the SINR expression can be evaluated. Let us define our problem statement

$$\begin{aligned}&\underset{\mathbf{w}}{\text{maximize}} && \gamma_k \\ &\text{subject to} && \|\mathbf{w}\|_2^2 = 1\end{aligned} \tag{16}$$

At first sight, one could think, that the solution to the Equation (16) is the eigenvector corresponding to the least significant eigenvalue of the matrix $\mathbb{E}\left\{\tilde{\mathbf{G}}\tilde{\mathbf{G}}^{\mathrm{H}}\right\}$. This is however not necessarily true, because this eigenvector only minimizes the ICI power, but does not maximize the desired signal power in the nominator of the SINR expression. For example in case of a user velocity $v = 500$ km/h, the following $\tilde{\mathbf{w}}$ is chosen

$$\mathbf{w} = \left[\begin{array}{ccc} 0.0168 + i0.0004 & 0.9997 & 0.0168 - i0.0004 \end{array}\right]^{\mathrm{T}}. \tag{17}$$

Note, that this is in contrast to the solution from [5], where the vector $\tilde{\mathbf{w}}$ is chosen as

$$\mathbf{w} = \left[\begin{array}{ccc} -0.4775 & 0.7376 & -0.4775 \end{array}\right]^{\mathrm{T}}. \tag{18}$$

From Equation (18), one can see, that much more power is radiated at the adjacent subcarriers in contrast to our proposed solution. This is however not beneficial, because in this way, one tries to transmit information on the off-diagonal elements of the channel matrix, which are in general much lower than the diagonal elements and therefore, simply no or very limited amount of information can be transmitted over such a channel.

## IV. Simulation Results

In this section, we present simulation results and evaluate the performance of different precoding techniques. All data, tools and scripts are available online in order to allow other researchers to reproduce the results as shown in [8, 10, 11].

Table I shows the most important simulator settings. Note, that the rather high number of subframes used in our simulation is chosen in order to obtain very precise estimates of the mean post-equalization SINR and capacity [12].

TABLE I
SIMULATOR SETTINGS FOR FAST FADING SIMULATIONS

| Parameter | Value |
|---|---|
| Bandwidth | 1.4 MHz |
| Carrier frequency | 5.9 GHz |
| Number of transmit antennas | 1 |
| Number of receive antennas | 1 |
| Channel type | Rayleigh fading |
| Number of subframes | 25 000 |
| SNR | 30 dB |
| Equalizer | ZF |

In Figure 1 the post-equalization SINR over user velocity is shown utilizing different precoding schemes. A transmission system utilizing self canceling method [3] shows the best performance in terms of the post-equalization SINR. Notice, that only half of the available bandwidth is effectively utilized in this case. On other hand, a system utilizing PRC [5] shows the worse performance among the compared techniques. As expected the performance of a system not utilizing any precoding scheme and a single tap equalizer is worsening with increasing user velocity. This is caused by increasing ICI power. Once we apply our novel precoding scheme, but at the same time utilize only a single tap equalizer, the performance of such a system is improved compared to a system not using any precoding. The improvement in terms of the post-equalization SINR is around 0.5 dB. This performance gain can be realized with only very small increase of the complexity at the transmitter.

Figure 2 shows the capacity of systems utilizing different precoding schemes over user velocity. Now, one can clearly see, that although the self-canceling method shows the best performance in terms of post-equalization SINR, the real transmission capabilities of the system are limited.

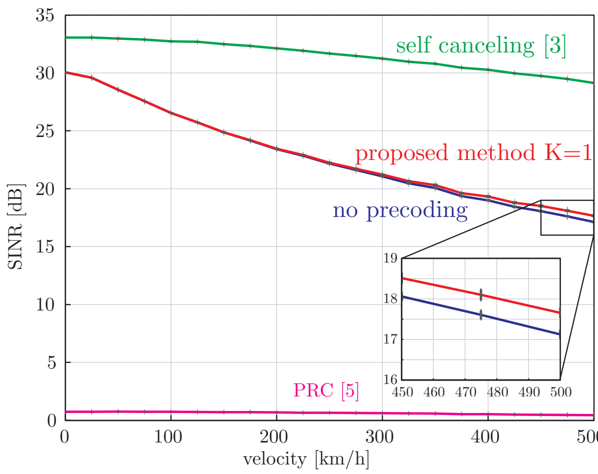


Fig. 1. Post-equalization SINR utilizing different schemes over user velocity.

## V. Conclusion

In this paper we proposed a simple and effective precoding technique to mitigate ICI, to be employed in OFDM based systems. The main features of the precoder are low cost and

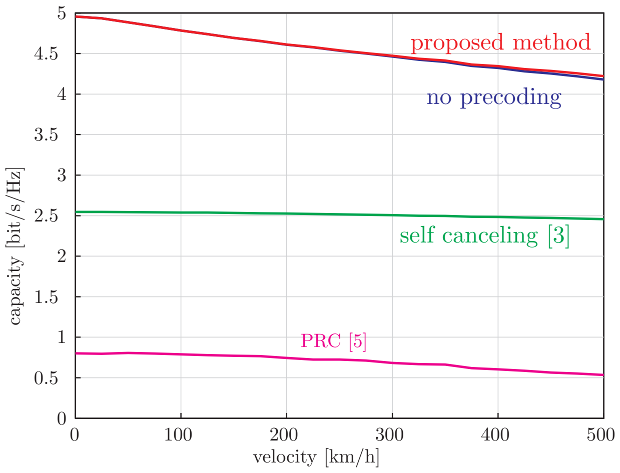


Fig. 2. Capcity utilizing different schemes over user velocity.

the mitigation of ICI in time-varying channels caused by Doppler spread. Experimental results performed in an Long Term Evolution (LTE) compliant simulator confirm the improved performance of the new precoder in terms of signal to interference and noise ratio for mobile speed up to 500 km/h.

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