

RESEARCH ARTICLE

A Comparison of Soft and Hard Coded Relaying

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ABSTRACT

“Amplify and Forward” and “Decode and Forward” are two main relaying functions that have been proposed since the advent of cooperative communications. “Soft Decode and Forward” is a recently introduced relaying principle that employs a Soft-Input Soft-Output (SISO) BCJR encoder in the relay; the intention is to combine the benefits of the classical two relaying algorithms. To compare the performance of a SISO BCJR encoder with a conventional hard convolutional encoder, we establish a suitable framework of two cooperative scenarios employing soft and hard coding in the relay. A novel approach, the mutual information loss due to data processing, is proposed for analysis. We also introduce a new approach to estimate the bit error rate and the equivalent channel SNR for the relaying techniques considered in the paper, involving both the hard and soft encoders. The results reveal new aspects of soft/hard encoding and we discuss the usefulness of SISO BCJR encoding compared with conventional hard encoding.

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1. INTRODUCTION

Spatial transmit diversity by employing multiple antennas at the transmitter is one solution to combat fading in wireless channels. In spite of very promising theoretical results, implementing multiple antennas in the user nodes can be practically infeasible, if not impossible, e.g. due to lack of space. A more recent approach to exploit spatial diversity is cooperation: several users work together to communicate with a common destination or even different destinations, so they can utilize transmit diversity by sharing resources and obtain better performance, i.e., higher throughput or lower error rates [8, 14, 15].

Two well-known relaying functions (e.g. [6]) are “Decode and Forward” (DF) and “Amplify and Forward” (AF). A more recent relaying function is “Soft Decode and Forward” (soft-DF), e.g. [10, 16, 20]. The idea is to combine the benefits of AF and DF and, at the same time, to mitigate the shortcomings of the traditional algorithms. The core of soft-DF is a novel Soft-Input Soft-Output (SISO) BCJR encoder that exploits the trellis structure of the convolutional code. So far, the soft-DF technique has been evaluated in a scenario where distributed turbo coding [19] is applied but, interestingly, there is a lack of literature evaluating recent soft-DF algorithm in simpler scenarios e.g. in which the destination simply employs Maximal Ratio Combining (MRC) instead of advanced iterative decoding algorithms. Studying soft-DF in different application scenarios reveals more details of the characteristics of the SISO BCJR encoder used and

of soft-DF in general. Along with SISO BCJR encoder we will introduce a novel SISO Averaging encoder, which we find has superior performance compared to the SISO BCJR encoder.

This paper is organized as follows: in Section 2 we introduce our system model. In Section 3 we explain the SISO BCJR and the SISO Averaging encoders. In Section 4 we discuss an observed inconsistency regarding the performance of the soft encoding algorithms in certain application scenarios. This motivates our further study of the topic. In Section 5 we present methods to evaluate the performance of relaying system using hard and soft channel encoders at the relay. Simulation results are presented in Section 6. In Section 7 we explain the inconsistency observed in Section 4 and section 8 offers some conclusion remarks.

2. SYSTEM MODEL

We consider a cooperative scenario in which a source node communicates with a destination via an intermediate relay node. In the sequel, we introduce two cases of such a cooperative scenario:

2.1. Case 1

Fig. 1(a) shows the soft-relaying system under consideration. We assume that there is *no* direct link between the source and the destination, which can, e.g., be due to the distance between the source and the destination.

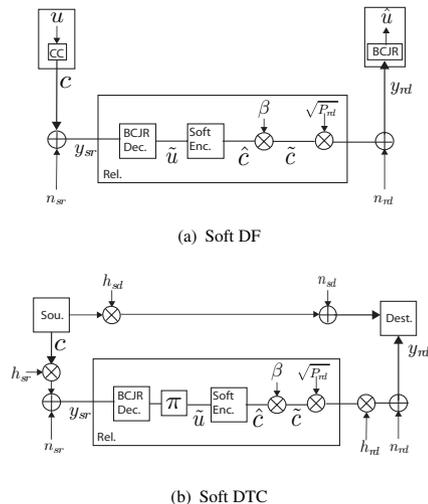


Figure 1. System Model.

In the source node, a block of K data bits is encoded using a k/n convolutional encoder, modulated and transmitted towards the relay. The relay employs a SISO BCJR decoder for decoding the noisy codeword received via the source-relay link. The output of the SISO BCJR decoder is fed into a soft channel encoder. The output of the soft channel encoder is scaled by the factor β to fulfil the power constraint of the relay and transmitted towards the destination. The destination employs the corresponding BCJR decoder for decoding the noisy codeword received via the relay-destination link.

We use two types of soft channel encoders: the SISO BCJR encoder and the SISO Averaging encoder; both soft encoding algorithms will be further explained in Section 3.

Note that we deliberately make the assumption that the destination does not “hear” the source transmission in order to evaluate *separately* how the performance is influenced by the soft information produced by the relay. Thereby, we study the concept of soft channel encoding as such, without mixing the concept with iterative decoding in a distributed Turbo coding scheme (which will be further discussed in *Case 2*). This is also the reason why we use a simple convolutional code, as in this case an optimum symbol-by-symbol decoder (BCJR algorithm) is available [2].

2.2. Case 2

Extending *Case 1* to the more advanced scenario, *Case 2*, we assume that there is also a direct link available between the source and the destination. The intention is to construct a Distributed Turbo Code (DTC) [19] applying soft information relaying [10, 16, 20]. Fig. 1(b) shows the soft-relaying system under consideration. The source functions as in *Case 1*; the difference is that both the relay and the destination overhear the data transmitted from

the source. Since a Turbo code is applied for the overall system, we employ an RSC* encoder in the source. The relay decodes the received noisy codeword as in *Case 1* and interleaves the LLR† values prior to soft encoding. The relay then employs an RSC SISO BCJR encoder for encoding the permuted data symbols coming out of interleaver. As in any parallel concatenated Turbo encoder, the systematic bits of the relay encoder are punctured and only the parity check bits are sent to the destination. Prior to transmission, power related constraints, as explained in *Case 1*, are applied to the parity check symbols. We assume AWGN in the source-relay, the source-destination and the relay-destination links. For simplicity we employ BPSK modulation in the source and the relay, but the magnitude of the transmitted BPSK modulation symbols is weighted according to the power scaling by the soft-encoded magnitudes of the code bits.

For both the *Case 1/2* scenarios, the signals received at the relay (y_{sr}) and the destination (y_{sd}) at each BPSK symbol time instant, respectively, are

$$y_{sr} = \sqrt{P_s} \cdot h_{sr} \cdot c + n_{sr}, \quad c \in \{\pm 1\} \quad (1)$$

$$y_{sd} = \sqrt{P_s} \cdot h_{sd} \cdot c + n_{sd}, \quad c \in \{\pm 1\} \quad (2)$$

and the signal received at the destination equals

$$y_{rd} = \sqrt{P_r} \cdot \beta \cdot h_{rd} \cdot \hat{c} + n_{rd}, \quad (3)$$

where $\beta = 1/\sqrt{|\hat{c}|^2}$, with $|\hat{c}|^2$ the average power of the transmitted channel symbols, averaged over each block of soft-encoded code bits resulting from each block of K data bits. For simplicity, we assume a non-fading scenario in *Case 1*, so $h_{sr} = h_{rd} = 1$ in (1) and (3). For *Case 2* we assume Rayleigh fading channels where h_{sr} , h_{sd} and h_{rd} are channel coefficients drawn independently from zero mean complex Gaussian random variables with variance σ_{sr}^2 , σ_{sd}^2 and σ_{rd}^2 , respectively. The noise components, n_{sr} and n_{rd} , are zero mean complex Gaussian random variables with variance N_0 . The reason for our emphasis on applying Rayleigh fading channel model for *Case 2* (DTC) will be explained in Sections 8.

There is a competing system design for the scenarios of *Case 1/2* using hard decisions for the data bits after soft-input soft-output channel decoding at the relay, prior to hard re-encoding (by a classical convolutional encoder), modulation and transmission to the destination. However, the figures are omitted due to space limits.

The receiver in *Case 1* employs a conventional BCJR decoder corresponding to the encoder of the relay. The receiver in *Case 2* applies iterative turbo decoding for the codeword received partially from the source-destination link and partially from the relay-destination link.

*Recursive Systematic Convolutional

† Log-likelihood ratio

3. SOFT CHANNEL ENCODING

In this section we discuss a commonly adopted scheme for soft channel encoding (Section 3.1) and we propose a different, much simpler scheme (Section 3.2).

3.1. BCJR Soft Channel-Encoder

The concept of the SISO BCJR *encoder* has been stated in the literature, e.g. [10, 16, 20], but we will also briefly explain it, as we will be investigating the characteristics of the soft information generated. We wish to point out here that we follow common practice in the literature (e.g., [10, 16, 20]) when we use a *decoding* algorithm (BCJR) for soft channel *encoding*. The justification is that this soft encoding algorithm has been reported in the literature to achieve much better performance in a distributed Turbo coding scheme than hard encoding at the relay. To the best knowledge of the authors, no theoretical justification has been given in the literature.

Every BCJR component [2] consists of three main parameters: α for the forward recursion, β for the backward recursion, and γ for the state transition probability. Assuming an AWGN channel, the BCJR decoder uses the Gaussian distribution for calculating γ . However, as explained in [2], it is also possible to determine the a-posteriori probabilities (APPs) of all code bits (not only of the data bits), and these APPs of the code bits we will consider as the soft channel-encoder's outputs.

We assume that the outputs of the BCJR decoder (for the source-relay link) in the relay node are the APPs $P(u_k = \pm 1)$ of the *data* bits transmitted from the relay, converted to L-value notation, i.e.

$$\tilde{u}_k \doteq \ln \frac{P(u_k = +1)}{P(u_k = -1)}. \quad (4)$$

In contrast to transmission over an AWGN channel, for which the BCJR decoder can use a Gaussian distribution for calculating the transition probability, γ , the SISO BCJR *encoder* uses the APPs, \tilde{u}_k , of the data bits, u_k , to calculate γ . Therefore the parameters α , β and γ for SISO BCJR encoder are given as

$$\begin{aligned} \gamma_k(s', s) &= P(u_k, s | s') \\ \alpha_k(s) &= \sum_{s'} \alpha_{k-1}(s') \gamma_k(s', s) \\ \beta_{k-1}(s') &= \sum_s \beta_k(s) \gamma_k(s', s), \end{aligned} \quad (5)$$

with the summations carried out over all possible states s', s in the trellis representation of the code and k denoting the time index of the trellis segment considered (details can be found in [2]).

The output of the SISO BCJR encoder is the Log Likelihood Ratio (LLR or L-value) $L(c_{k,i} | \tilde{u}_1, \tilde{u}_2, \dots)$ of the code bits computed from the input L-values (or

corresponding probabilities \tilde{u}_k) of the data bits:

$$L(c_{k,i} | \tilde{u}_1, \tilde{u}_2, \dots) = \ln \frac{\sum_{s' \rightarrow s: c_{k,i}=0} \alpha_{k-1}(s') \gamma_k(s', s) \beta_k(s)}{\sum_{s' \rightarrow s: c_{k,i}=1} \alpha_{k-1}(s') \gamma_k(s', s) \beta_k(s)}. \quad (6)$$

In (6), the code bit $c_{k,i}$ is the i -th code bit of trellis segment k attached to the data bit input \tilde{u}_k .

The L-values of the code bits derived in (6) are then normalized by $\beta \cdot \sqrt{P_r} = \sqrt{P_r} / \sqrt{|\tilde{c}|^2}$ across each soft-encoded channel code word such that the power constraint of the relay is met.

3.2. Averaging Soft Channel-Encoder

The idea of the averaging soft channel encoder is to take the average of the magnitudes of the L-values of those data bits that are involved in a parity-check equation that would be used in a classical hard convolutional encoder. In other words, an averaging soft encoder is, actually, similar to a classical convolutional encoder but the “soft” code bits are generated differently. In contrast to a convolutional encoder, in which a code bit is taken to be the modulo-2 sum of the corresponding data bits, in an averaging soft encoder, the magnitude of an output code bit is the average of the magnitudes of the L-values of the data bits linked to this code bit (by the generator polynomials). The sign is determined by the normal modulo-2 sum on the corresponding data bits, with the “0/1” output bits mapped to “+1/-1” signs for the magnitude determined by averaging.

Although such a soft encoder is rather simple it fulfils some properties that are desirable: if only the signs are considered, the result will be a valid channel code word. Moreover, it is sensible to allocate magnitudes to the coded bits that reflect the significances of the data bits to be encoded by the parity check equations. It wouldn't make sense to allocate large transmit power to data bits at the relay when they have been decoded (at the relay) with small reliabilities. The consequence would be that we communicate to the destination information that is actually very unreliable, but we would be making it strong by using large transmit power. On the other hand, the magnitude should not vanish, when only one of the bits involved in a parity check has a very small magnitude, as this would cancel the protection of all other bits as well. The latter point is interesting, as exactly this will happen when a soft decoding algorithm (such as the one described in Section 3.1) is used as a soft encoder.

We would like to point out that we don't claim the proposed averaging soft channel encoder to be optimal or even “good” in any sense. But, as we will demonstrate below, the averaging soft channel encoder beats the SISO BCJR encoder in bit error performance, which proves that this widely used soft encoder can not be the best choice.

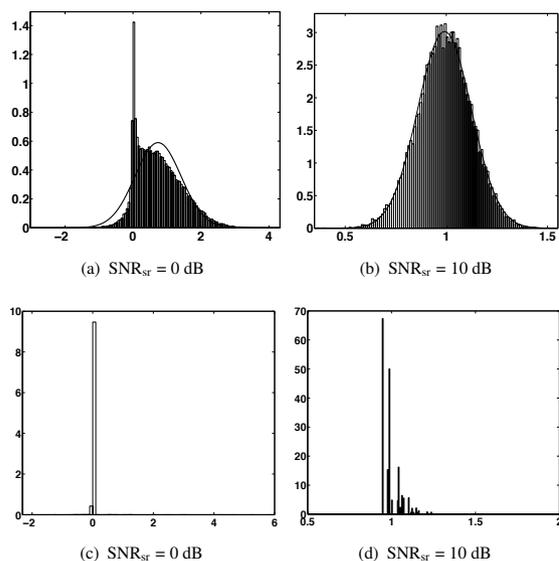


Figure 2. First row: Conditional probability density functions $p(\tilde{c}|c = +1)$ for a (7,5) FF SISO BCJR encoder. Second row: Conditional probability density functions $p(\tilde{c}|c = +1)$ for a (5/7) RSC SISO BCJR encoder. Power normalized to 1.

4. AN OBSERVED INCONSISTENCY

Before we continue to study the performance of the two proposed scenarios, we would like to comment on the motivation of comparing the two cases. The *Case 2* scenario has been widely studied in the literature, e.g. [10, 16, 20]. The works demonstrate that Distributed Turbo Codes (DTCs) with soft information relaying outperforms the hard DTC, which our simulations also confirm (see e.g. Fig. 6 (solid lines); details of the figure will be discussed in the forthcoming sections). However, in spite of the simplicity of the *Case 1* scenario, to the best of our knowledge, there is as yet no paper considering it.

By resorting to the results of soft-DTC, one may be tempted to conclude that, in general, a SISO BCJR encoder outperforms conventional (hard) convolutional encoders. However, by applying a SISO BCJR encoder in the *Case 1* scenario we find that hard-DF achieves better error performance compared to soft-DF (see e.g. Fig. 5). Hence, the results of the two scenarios *Case 1/2* seem to contradict each other. This unexpected behaviour of the SISO BCJR encoder in the two proposed scenarios motivates further analysis. In fact, after observing the results of the *Case 1* scenario, we can not confirm the strict conclusion that “soft coded information relaying is better than hard relaying”. Therefore, a detailed study of the soft coded information relaying under different circumstances is provided in this work.

5. PERFORMANCE EVALUATION

Although turbo codes are appreciated for their exciting error correction performance in point to point communications, but in the case of DTCs, however, where two different nodes (the source and the relay) construct a turbo code, error prone relays can destroy the performance of the turbo code whenever the relay forwards erroneous codewords towards the destination. The problem gets worse during the decoding iterations because of further error propagation by every iteration. Therefore, we evaluate the performance of SISO BCJR and convolutional encoders assuming *Case 1* scenario. The reason is to avoid mixing the performance of SISO BCJR encoder with the effect of error propagation per iteration in a turbo decoder of *Case 2*. We are interested in two parameters:

- The mutual information loss due to soft/hard encoding in the relay.
- The received SNR at destination due to relay transmission.

The statistics of the received signal at the destination corresponding to the relay transmission depends on the (soft) relaying function used. To the best of our knowledge, there is no closed form solution for the probability density function (pdf) of \tilde{c} for non-trivial relaying functions (e.g. SISO BCJR encoding in the relay). Hence, we have measured histograms that describe the conditional pdfs $p(\tilde{c}|c = 1)$. Note that histogram measurement is a common approach used in the literature to analyse the soft-DF technique. As an example, Fig. 2(a) and 2(b) show the pdf $p(\tilde{c}|c = 1)$ when a feed-forward BCJR soft channel encoder is applied in the relay, whereas Fig. 2(c) and 2(d) show the pdf $p(\tilde{c}|c = 1)$ when an RSC BCJR soft channel encoder is applied in the relay; the pdfs $p(\tilde{c}|c = -1)$ would be symmetric. In the literature (e.g. [16, 20]) the pdfs are usually modeled by additive zero mean Gaussian random variables, $n_{\tilde{c}}$, plus a non-zero mean, $\mu_{\tilde{c}}$, i.e.,

$$\tilde{c} = \mu_{\tilde{c}}c + n_{\tilde{c}}, \quad n_{\tilde{c}} \sim \mathcal{N}(0, \sigma_{\tilde{c}}^2), \quad c \in \{+1, -1\} \quad (7)$$

In the remainder of this section we assume that a FF BCJR encoder is employed in the relay. Nevertheless, we will apply the RSC BCJR encoder when considering the *Case 2* scenario in the forthcoming sections*.

We point out that the distribution of \tilde{c} , e.g. Fig. 2(c) or 2(d), not only is not Gaussian but it even does not resemble any standard distribution for which an analytical formula is known. Therefore, an analytical characterization of the SISO BCJR encoders is (at least currently) impossible. Therefore in the remainder of the paper we use histogram

*We emphasize on SISO BCJR encoder because it is the most widely used soft encoder in the literature for soft information relaying. We are not interested in the performance evaluation of SISO Averaging encoder. The SISO Averaging encoder is a competing SISO encoder to show that SISO BCJR encoder is not the optimal SISO encoder. The BER results of SISO Averaging encoder will appear in section 6. Yet, as will be explained, both the SISO encoders have inferior performance compared to convolutional encoder

measurements to investigate the mutual information loss and the received SNR at the destination; such an analysis based on histogram measurements is the only available approach and it is widely used in the literature e.g. [10, 13, 16, 20].

5.1. Mutual Information (Loss)

Mutual information, $I(U; \tilde{U})$, can be used to measure the amount of the information that soft (or hard) data bits, \tilde{u} , at the relay carry about the data symbols, u , transmitted by the source. The two system models, *soft/hard* DF, use two different (*soft/hard*) channel encoders. The intention of calculating mutual information is to measure the mutual information loss, [5], due to employing different channel encoders in the relay. The mutual information $I(U; \tilde{U})$ [5, 18] between the (equally likely binary) transmitted data bits, $u \in \{+1, -1\}$, and the L-values \tilde{u} , assuming \tilde{u} has a Gaussian distribution, is well explained in [18, II-B]. We adopt the concept and, hence, $I(U; \tilde{U})$ is given by

$$I(U; \tilde{U}) = \frac{1}{2} \sum_{u'=\pm 1} \int_{-\infty}^{+\infty} p(\tilde{u} | u = u') \times \log_2 \frac{2p(\tilde{u} | u = u')}{p(\tilde{u} | u = +1) + p(\tilde{u} | u = -1)} d\tilde{u}, \quad (8)$$

with $p(\tilde{u} | u)$ the conditional pdf of the L-values at the relay (see Fig. 1) given the input bits u . Taking similar approach, we have measured the conditional pdf of the code bits, $p(\tilde{c} | c = 1)$, for computing $I(C; \tilde{C})$ from (8), but we have omitted the plots due to lack of space.

To characterize $I(U; \tilde{U})$ associated with hard-DF, we model the source-channel-relay link as a Binary Symmetric Channel (BSC) in which the channel input is a data bit. The output bit of the BSC is flipped with probability q . Hence, mutual information for such a BSC is given (e.g. [5]) by

$$I(U; \tilde{U}) = 1 - H_2(q), \quad (9)$$

with $H_2(q) \doteq -q \cdot \log_2(q) - (1 - q) \cdot \log_2(1 - q)$ the standard binary entropy function. The reason for the BSC assumption is that the conventional hard convolutional encoder at the relay requires binary inputs. Therefore modelling the source-relay link by a BSC is the most natural assumption. The flip probability, q , is the bit error probability at the output of a BCJR decoder [2], which can be determined by measurement from a simulation.

Using the same approach, one can calculate the mutual information $I(C; \tilde{C})$, too. A comparison of the mutual informations $I(U; \tilde{U})$ and $I(C; \tilde{C})$ for both the *hard/soft* DF is useful in measuring mutual information loss due to employing different encoders in the relay. Numerical results will follow in section 6.

5.2. Equivalent Receive SNR at the Destination

In this section we intend to model the source-relay-destination link with an equivalent AWGN channel. Note

that we employ a convolutional encoder at the source and a symbol-by-symbol MAP decoder (BCJR decoder) at the destination; the relay structure has been explained in section 2. We point out that the equivalent receive SNR can be different for another coding/decoding set up.

5.2.1. Hard DF

Due to the error-prone relay, calculating the equivalent receive SNR at the destination for hard DF is somewhat cumbersome. Since the relay decodes and forwards both the correct and erroneous frames, the distribution of the received signal at destination is no longer Gaussian. The common approach to estimate the SNR at the destination is to model the source-relay-destination link as an equivalent AWGN channel with channel SNR_{eq} that depends on both the source-relay and the relay-destination channel qualities.

The total bit error probability is given by

$$P_{\text{tot}}(e | \gamma_{\text{sr}}, \gamma_{\text{rd}}) = P_b(e | \gamma_{\text{sr}})[1 - P_b(e | \gamma_{\text{rd}})] + [1 - P_b(e | \gamma_{\text{sr}})]P_b(e | \gamma_{\text{rd}}), \quad (10)$$

where γ and $P_b(e)$ are the corresponding channel SNR and the bit error probabilities for the two links (source-relay and relay-destination) involved.

Calculating P_{tot} using simulations is straightforward but one can also calculate it using the complementary error function. The bit error probability of convolutional codes under symbol-by-symbol MAP decoding (BCJR decoding) can be approximated by

$$P_b(e) \approx \frac{1}{2} \text{erfc} \left(\sqrt{\frac{\mu_{\text{out}}^2}{2\sigma_{\text{out}}^2}} \right) = \frac{1}{2} \text{erfc} \left(\sqrt{\frac{1}{2} \gamma_{\text{out}}} \right), \quad (11)$$

(e.g. [18]) where μ_{out}^2 and σ_{out}^2 are, respectively, the mean and variance of the data bit L-values at the output of the BCJR decoder, and $\gamma_{\text{out}} = \mu_{\text{out}}^2 / \sigma_{\text{out}}^2$. Similarly, μ_{in}^2 and σ_{in}^2 would define $\gamma_{\text{in}} = \mu_{\text{in}}^2 / \sigma_{\text{in}}^2$ for the input L-values of the BCJR decoder. Regardless of the type of soft or hard encoding in the relay, the output SNR, γ_{out} , at the output of a certain BCJR decoder directly depends on the SNR, γ_{in} , at its input. However, due to the complicated decoding structure of a BCJR decoder, there is no theoretical expression relating γ_{out} to γ_{in} . In [18], the use histogram measurements is proposed to obtain γ_{out} for a certain BCJR decoder at some given γ_{in} . Hence, given number pairs $(\gamma_{\text{in}}, \gamma_{\text{out}})$, one can estimate γ_{out} as a function of γ_{in} using polynomial regression analysis* [12]. For a [7,5] BCJR decoder we obtain

$$\begin{aligned} \gamma_{\text{out}} &= f(\gamma_{\text{in}}) \\ &\approx 3.38 \cdot 10^{-3} \gamma_{\text{in}}^3 - 0.12 \gamma_{\text{in}}^2 + 2.38 \gamma_{\text{in}} + 2.56. \end{aligned} \quad (12)$$

$P_b(e | \gamma_{\text{sr}})$ and $P_b(e | \gamma_{\text{rd}})$ in (10) can be computed using (11) and (12) (with $\gamma_{\text{in}} \in \{\gamma_{\text{sr}}, \gamma_{\text{rd}}\}$). Then, given P_{tot} , the

* Many computational packages such as MATLAB and Mathematica offer simple tools for regression analysis.

equivalent SNR_{out} , $\gamma_{\text{eq-out}}$, follows from

$$\gamma_{\text{eq-out}} = 2 \left(\text{erfc}^{-1}(2P_{\text{tot}}) \right)^2. \quad (13)$$

By substituting (13) into

$$\gamma_{\text{eq}} = f^{-1}(\gamma_{\text{eq-out}}), \quad (14)$$

the equivalent source-relay-destination SNR, γ_{eq} , is computed.

5.2.2. Soft DF

One might exploit the Gaussian assumption of (7) for calculating SNR_{eq} of the soft DF schemes, but, as illustrated by Fig. 2, the Gaussian assumption is not accurate, at all, especially at low SNR. Therefore, in order to calculate SNR_{eq} , we use Monte-Carlo simulations in the destination to determine $\gamma_{\text{eq-out}}$. With $\gamma_{\text{eq-out}}$, calculating γ_{eq} using (14) is straightforward. The estimated BER for soft DF can then be determined by substituting $\gamma_{\text{eq-out}}$ in (11).

6. SIMULATION RESULTS

We start with the simulations for the *Case 1* scenario. Then we will present the results for *Case 2*, and we will continue by explaining the reasons why the two scenarios show very different error performances in spite of employing similar encoders at the relay.

6.1. Case 1

The *Case 1* scenario, explained in section 2.1, has the following characteristics: the source applies a [7,5] convolutional encoder for encoding information frames of length 2000 bits; for simplicity we use BPSK modulation and assume that $h_{\text{sr}} = h_{\text{rd}} = 1$. We assume that the receive signal at the relay is corrupted by zero mean real Gaussian receiver noise with a variance of $N_0 = 1$.

Fig. 3 compares the mutual informations $I(U; \tilde{U})$ of the data bits U and their decoded counterparts \tilde{U} with soft and hard decisions after BCJR decoding at the relay. Fig. 3 also shows the mutual informations $I(C; \tilde{C})$ of the code bits C at the source and the (soft and hard) re-encoded code bits \tilde{C} at the relay. The figure shows that the soft channel encoding scheme actually destroys more information by data processing than the hard encoding algorithm which has less information at its input.

Fig. 4 shows the equivalent source-relay-destination channel SNR for both the hard DF algorithm and the soft DF algorithm using a [7,5] BCJR soft channel encoder. The SNR_{eq} plots are calculated according to the rules introduced in Section 5.2. The SNR_{eq} curve of soft DF merges with the SNR_{eq} curve of hard-DF at high SNR_{sr} , although a small difference remains. The explanation is that the relay usually performs error free decoding at high SNR; therefore the hard-DF algorithm is very close to optimum at high SNR. But for soft-DF, the transmitted

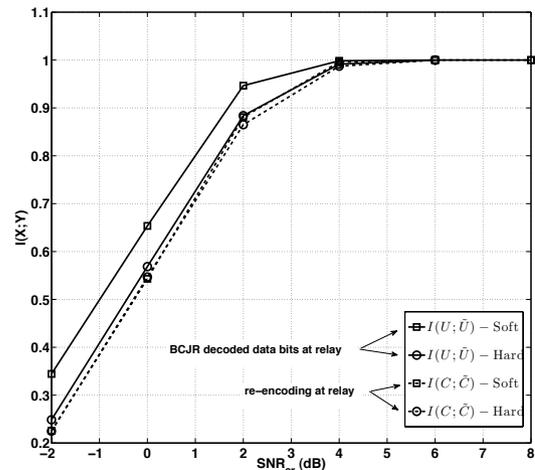


Figure 3. Mutual information loss due to hard and soft channel encoding.

symbol from the relay, \tilde{c} , is Gaussian distributed. Because of the unit power constraint of BPSK modulation that we have to enforce for soft encoding in an average sense as well, we have an average power of $P(\tilde{c}) = \mu_{\tilde{c}}^2 + \sigma_{\tilde{c}}^2 = 1$. Since $\sigma_{\tilde{c}}^2 > 0$, we find that $|\mu_{\tilde{c}}| < 1$ must hold, so some of the transmitted code bits will have smaller instantaneous power than the hard-encoded symbols (which all have “one”): this will cause the slight SNR_{eq} degradation in comparison with hard-DF for large values of SNR_{sr} .

Fig. 5 illustrates the bit error rates of the system for $\text{SNR}_{\text{rd}}=4\text{dB}$. The estimated curves for both the hard and soft DF (for SISO BCJR encoding) confirm the simulation results. It is clear that hard DF considerably outperforms soft DF when L-values of the codebits are transmitted from the relay as soft information. The situation is, however, different for soft DF with the averaging soft channel encoder: although the performance of hard DF is also better, it is only slightly so. Of course, this means that hard DF is still the method of choice, but it also proves that BCJR soft channel encoding is a much worse algorithm than the averaging soft encoder in the given context.

In [10,20] it has been proposed to perform a $\tanh(\hat{c}/2)$ -operation (in Fig. 1) after SISO BCJR encoding and prior to power normalization in the relay. We did not discuss this in the previous sections. However, Fig. 5 illustrates that even though this modification outperforms the scenario where L-values of the codebits are transmitted from the relay, the scheme performs still worse than a hard convolutional encoder.

For the system model of *Case 1*, as no interleaver is employed in the relay, one might propose to perform both the decoding and soft re-encoding operations in one step. This can be realized by classical BCJR decoding using the APPs of the code bits after decoding; details of how to compute those APPs have been given in the classical paper [2]. This scheme has been included in Fig. 5, and

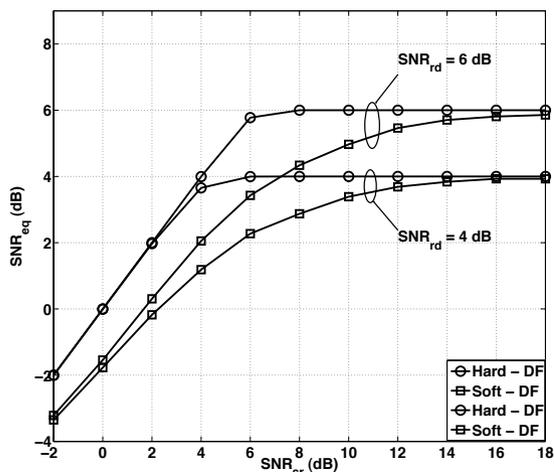


Figure 4. Equivalent source-relay-destination SNR; for soft DF, a (7,5) SISO BCJR channel encoder is used.

we observe that its BER performance still performs much worse than hard encoding in the relay. It is an open question if there is at all a soft coded DF scheme that can perform better than hard DF within the frame work of our system model.

6.2. Case 2

The *Case 2* scenario of section 2.2 has the following characteristics: the frame length, modulation and noise characteristics are as in *Case 1*. A [1, 5/7] RSC convolutional encoder is applied at the source. We assume that all the channels h_{sd} , h_{sr} and h_{rd} are subject to Rayleigh fading with unit variance. Fig. 6 shows the BER performance for the *Case 2* scenario when hard/soft (SISO BCJR) encoding is applied at the relay and $\text{SNR}_{sr} = \text{SNR}_{rd} = 12\text{dB}$. The figure (solid lines) clearly shows that after 5 iterations, soft relaying outperforms hard relaying with a considerable difference of about 10 dB when the relay does not perform CRC*. Such a performance behaviour has been reported in various publications (e.g. [10, 16, 20]). Apparently, the error performance of the *Case 2* scenario contradicts the *Case 1* scenario. In *Case 1*, the hard algorithm outperforms the soft algorithm while in *Case 2* the soft algorithm outperforms the hard algorithm. The open question remaining is why the overall system shows such unexpected performance? This is the topic of the rest of the paper.

7. DISCUSSION

For the analysis of the simulation results of *Case 2* scenario, we assume that the “all-zero” codeword is used. We use a turbo decoder as the one in [11] where BCJR

1 performs based on the source transmission whereas BCJR 2 performs based on both the source transmission (systematic bits) and the relay transmission (parity check bits).

Hard DTC We will first consider the *hard* DTC by considering the distribution of LLR values of information bits at the output of the two BCJR decoders after every iteration [†]. In fact, in the case of decoding failure at the relay, with subsequent re-encoding (of the incorrect data bits), the combination of bits sent over the source-destination and the relay-destination channels will not form a valid codeword of the “overall” channel code. This means that even with zero-noise on the channels linked to the destination we can not assume correct decoding at the destination. This makes it very difficult to analyse such a coding scheme properly by classical techniques that, e.g., involve minimum-distance calculations among valid codewords (e.g. [3, 17]), which as such are impossible to do accurately for advanced channel codes, even in conventional coding application. Hence, we resort to the probability distribution of the LLR values for analysis.

Fig. 7(a) shows the distributions of the received noisy codeword LLRs at the destination as received from the source; the first BCJR decoder (BCJR 1) decodes the message given this information and outputs the L-values of the information bits. The second BCJR decoder (BCJR 2) uses three sets of information for decoding:

1. a priori information of the data bits, calculated by BCJR 1
2. receive signal at the destination, transmitted from the source, (Fig. 7(a)), corresponding to the systematic bits of the codeword
3. receive signal at the destination, transmitted from the relay, (Fig. 7(b)), corresponding to the parity-check bits of the *supposed* codeword that might be based on incorrect decoding at the relay

We expect that decoding will fail in decoder BCJR 2 when the relay fails to decode correctly. Fig. 7(c) shows the distribution of LLR values of the all-zero databits after the first iteration. The BCJR 2 component tries to decode databits given Figs. 7(a), 7(b) and a priori information 7(c) but it fails to decode correctly because of the problem described above (non valid codeword). Decoding failure is evident from Fig. 7(d): the LLR values of data bits reach values as low as -100 , when their signs “should” all be positive to be correct. The erroneous a priori information produced by decoder BCJR 2 will propagate through every iteration, as illustrated in Fig. 7(e) and 7(f). That is the reason why iteration causes performance degradation, as illustrated in Fig. 6 (solid line). Note that decoding failure in the relay for hard DTC of *Case 2* which employs RSC encoder is much severe than decoding failure for hard DF of *Case 1* which employs FF encoder. The reason is that

* Cyclic Redundancy Check

[†] We call one run of each of the BCJR decoders “one iteration”.

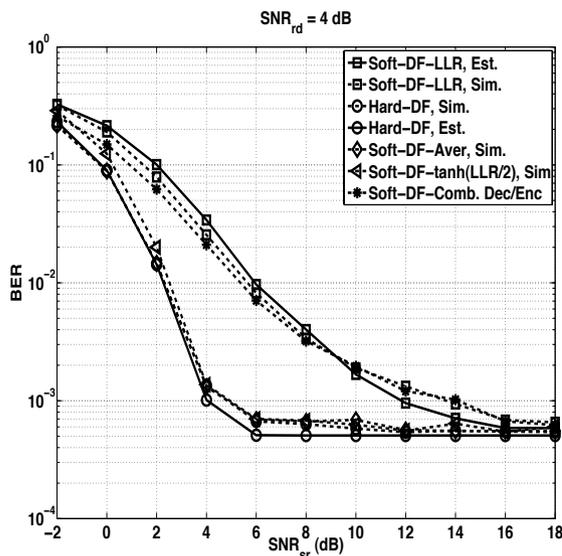


Figure 5. BER for the Case 1 scenario.

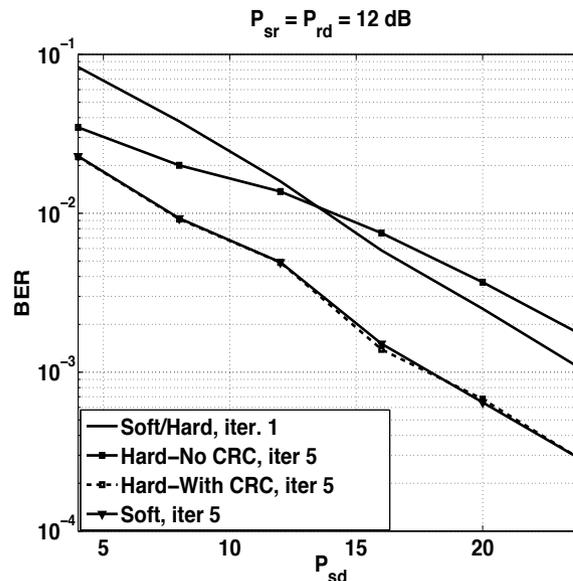


Figure 6. BER for the Case 2 scenario

even one error bit in the relay encoded by an RSC encoder will propagate through the whole frame; and consequently will cause error burst.

Soft DTC The first iteration of the soft turbo decoder (Fig. 8(c)) works like the first iteration of hard turbo decoder (Fig. 7(c)). But in the second iteration of the soft turbo decoder, performance improves – albeit only slightly – because the parity-check bits transmitted from the relay convey some – albeit very little – useful information. But in contrast to hard DTC, in which, highly reliable erroneous information (left hand side information in Fig. 7(b)) confuse iterative decoder of the destination. Hence, with soft-DTC there is no such striking performance degradation by further iterations as the erroneous information from the relay does not appear to be highly reliable. In other words, since the mean of the transmitted soft signal from the relay tends to zero in the “error case”, this set of information will be treated as noise at the destination. Therefore even though BCJR 2 does not considerably improve the error performance, it also does not degrade the performance due to incorrect a priori information, unlike hard-DTC, in which seemingly reliable negative parity-check symbols confuse decoder BCJR 2. Figs. 8(d), 8(e) and 8(f) clearly show that further iterations in the Case2 scenario does not degrade the error performance, considerably. Nevertheless, further iterations, also, does not improve the performance.

Implicit CRC by SISO BCJR Encoding In this sequel we discuss how a SISO BCJR encoder in the relay in combination with a BCJR decoder in the destination (as a part of turbo decoder) performs an implicit CRC. As discussed in previous sections, the pdf of the codeword L-values at the relay at the output of the SISO BCJR

encoder is assumed to be Gaussian. In the case of decoding failure in the relay, the mean ($\mu_{\hat{c}}$) of such a distribution tends to 0, which is clear from Fig. 2(c). The BCJR 2 of the turbo decoder will use the parameters of the assumed distribution for decoding but since the $\mu_{\hat{c}} \rightarrow 0$, the relay bits do not affect the state transition probabilities of BCJR 2. In other word, since $\mu_{\hat{c}}$ of parity check bits corresponding to the second encoder (SISO BCJR encoder in the relay) tends to 0, the BCJR 2 in the turbo decoder at the destination ignores relay transmission. This property can be interpreted as CRC in the relay for soft relaying which means that assigning power to such a frame is just a waste of resources. In fact, better performance of Soft DTC in comparison with hard DTC is not a consequence of optimal SISO encoding but a result of error propagation by hard DTC. We believe that comparing soft-DTC which implicitly performs CRC (and, hence avoids error propagation) with hard DTC which is not protected against error propagation is an unfair comparison.

The arguments of this section are also valid if $\tanh(\hat{c}/2)$ is transmitted instead of the L-values. The reason is that at low SNR_{sr}^* , L-values tend to zero; therefore, $\tanh(\hat{c}/2)$ can be approximated by L-values.

Additional Error Detection In order to establish a fair comparison, let assume that the relay with hard-DTC performs a CRC per frame, before transmission. Based on that, the relay only transmits the parity check bits if decoding has been successful; otherwise, the relay remains silent. Fig. 6 (dashed line) shows the BER performance

*Low SNR_{sr} is the focus of our discussion which occurs repeatedly due to Rayleigh assumption of the channel.

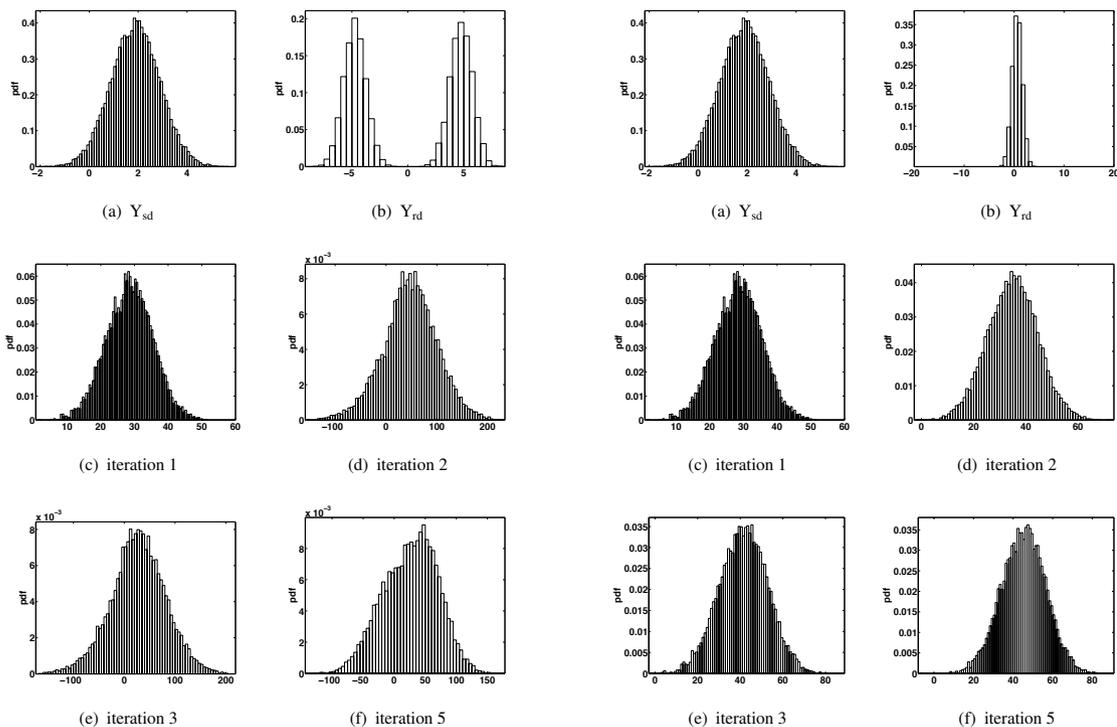


Figure 7. (a) distribution of the received signal at the destination with the “all-zero” codeword transmitted from the source. (b) distribution of the received signal at the destination resulting from RCS convolutional encoding at the relay. (c), (d), (e) and (f) distribution $p(L|c = +1)$ of the information-bit L-values at the destination after the indicated number of iterations.

Figure 8. (a) distribution of the received signal at the destination with the “all-zero” codeword transmitted from the source. (b) distribution of the received signal at the destination resulting from SISO BCJR encoding at the relay. (c), (d), (e) and (f) distribution $p(L|c = +1)$ of the information-bit L-values at the destination after the indicated number of iterations.

of such a scenario (CRC-extended “hard DTC” and unchanged “soft DTC”). The BER curve of hard-DTC has improved dramatically. In fact, the BER curve of hard-DTC performs as it was expected in a turbo decoder: the number of errors now decreases with each iteration unlike in Fig. 6 (solid line) in which iterations degraded the hard DTC’s performance. With the mentioned modification, the BER performance of soft and hard DTC are almost equal; but considering the fact that hard DTC saves power in the case of decoding failure in the relay, one can conclude that hard DTC outperforms soft-DTC.

8. CONCLUSIONS

The only advantage of the SISO BCJR encoder appears in the case of decoding failure at the relay. When the relay fails to decode correctly, error bursts produced by RSC encoder in hard DTC destroy the performance. Using a Rayleigh fading assumption for the channel, we can be sure that, with a certain (non-zero) probability,

there will be channel conditions in which the source-relay link operates at low SNR and, therefore, error bursts will indeed frequently destroy the performance of hard-DTC and it is exactly then when soft-DTC outperforms hard-DTC. Otherwise the SISO BCJR encoder does not outperform the convolutional encoder in the sense of mutual information loss nor SNR enhancement. Hence, employing convolutional encoder in the relay in combination with CRC will be considerably less complex than employing SISO BCJR encoder which, seemingly, implicit CRC is the only advantage of that.

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