Abstract—In this work, we investigate the impact of user clustering on the performance of femtocell-enhanced macrocellular networks. Various degrees of clustering are evaluated: A low degree corresponds to almost uniform user distributions, whereas a high degree relates to densely populated user hot-spots. Clustered user-scenarios are generated by applying a Poisson cluster process. We start out with a plain macrocellular network and cover an increasing amount of these clusters by femtocells. Expressions for the achieved Signal-to-Interference Ratio (SIR) of macrocell- and femtocell users are formulated and evaluated numerically. The efficiency of the femtocell enhancement is quantified by means of sum spectral efficiency and fairness.

Index Terms—Femtocells, Femtocell Deployment, Femtocell Density, User Hot-spot, Spectral Efficiency, Sum Spectral Efficiency, Fairness, Fairness Index

I. INTRODUCTION AND CONTRIBUTIONS

Spatial frequency reuse is one of the key methods to sustain the ever increasing demand for capacity in today’s mobile cellular networks. However, high spatial reuse and thus, small cell sizes can hardly be realized in current macrocellular deployments. This led to the emergence of heterogeneous networks, which contain various types of access nodes and pose new challenges for the design of appropriate system models. The authors of [1–4] propose to apply stochastic geometry for modeling the spatial distribution of base stations and users.

B. Contributions

• We set up a system model based on stochastic geometry, which allows us to explicitly analyze the impact of various degrees of user clustering.
• User clusters are distributed uniformly over the area. We start out with a plain macrocellular network and investigate the effects of covering an increasing amount of randomly chosen clusters by FAPs.
• We stress the importance of a fairness metric, which is often disregarded in literature. Sum spectral efficiency provides only limited view on the user performance, since it conceals the distribution of the users’ individual spectral efficiency values. Severe imbalances can particularly appear in heterogeneous cellular networks, like the one investigated in this paper.

C. Outline

The paper is organized as follows: Section II introduces the system model and presents the generation of user hot-spot scenarios. Analytic expressions for the occurring Signal-to-Interference Ratios (SIRs) are provided in Section III. Then, Section IV sets up a simulation environment and discusses the obtained results for SIR distribution, spectral efficiency and fairness. Section V concludes the work.

II. SYSTEM SETUP

We refer to the system model of [7] and extend it to the sectorized case. The values assigned to the different parameters for simulation are described in Section IV.

A. Cell Geometry (see Figure 1)

• The macrocellular network is composed of a central site and one tier of neighbors. Each site employs three BSs with directional antennas. The main radiation directions are spaced out 120°.
• FAPs are equipped with omni-directional antennas.
• Each FAP is assumed to be surrounded by an indoor area of radius $R_I$. The separation between indoor areas and outdoor environment is modeled by a wall penetration loss $L$.
B. Channel Model

- A power-law path loss model is used. Depending on the propagation environment, one of the following path loss exponents is in force:
  - $\alpha = 4$ . . . Outdoor environment
  - $\beta = 2$ . . . Indoor area.
- The downlink signal experiences path loss and Rayleigh fading.
- Femtocells operate in Open Subscriber Group (OSG) mode. Users are assigned to a F AP or macrocell BS sector according to the highest average received power. Thus, the coverage area of a F AP can exceed the indoor area, as shown in Figure 1 (lower right F AP). On the other hand, indoor areas can partly be covered by nearby macrocell BSs (upper left FAP in Figure 1).
- Interference power typically dominates noise power in heterogeneous networks [4, 8]. Therefore, thermal noise is neglected in this work.

C. User Distribution (see Figure 2)

In order to generate hot-spot scenarios, users are distributed according to a Poisson cluster process [9], which is composed as follows:

1) A parent point process $\Phi$ uniformly spreads $N_c$ circularly shaped areas of radius $R_I$ over the coverage region of a macrocell sector.
2) A set of $N_u$ users is distributed uniformly within each area.

The total amount of users in a macrocell sector is calculated as $N_c \cdot N_u$. By keeping this term constant, the parameters $N_c$ and $N_u$ adjust the degree of clustering (also referred to as extent of clustering, or level of inhomogeneity).

Fig. 1: System model geometry: Macrocell with central macrocell BS and employed F APs. Arrows indicate main radiation direction of the sector antennas, spaced out 120°.

Fig. 2: Partial Femtocell Access Point deployment in clustered user scenario. The positions of users and FAPs are denoted by ’.’ and ‘+’, respectively. Circles depict the indoor area of a femto-covered user-cluster; Outdoor users assigned to a FAP are marked by ‘+’.

D. Notation and Conventions

- Macrocell BSs and FAPs are indexed by $s$ and $f$, respectively. Index ’0’ labels the transmitter of the desired signal.
- The distance between user $u$ and macrocell BS $s$ (F AP $f$) is denoted by $D_{us}$ ($R_{uf}$).
- The signal between user $u$ and macrocell BS $s$ experiences propagation loss:
  $$\ell(D_{us}, \theta_{us}) = G_T(\theta_{us}) D_{us}^{-\alpha}$$
  where $G_T(\cdot)$ denotes the angle-dependent radiation pattern of the employed sector antenna and $\theta_{us}$ is the angle between a user and the antenna’s main radiation direction. The parameters $D_{us}$ and $\alpha$ are the user-to-macrocell BS distance and the experienced path loss exponent, respectively.
- The Rayleigh fading coefficient is denoted as $g_{us}$, when related to macrocell BS sector $s$, and $h_{uf}$, when related to FAP $f$, respectively.

III. Signal-to-Interference Ratio Zones of the Partial Femtocell Deployment

In this section, we provide SIR expressions for a partial FAP deployment: A fraction of the user clusters is covered by FAPs, while the remaining part is under macrocell BS coverage, as depicted in Figure 2.

FAPs are placed at the center of randomly-chosen user clusters. We assume that wherever a FAP is operated, it is
surrounded by an indoor area of size $R_I$ and isolated from the outdoor environment by a wall with penetration loss $L$.

A user experiences one of the following interference situations:

1) It is located inside an indoor area and connected to the Femtocell Access Point. The SIR is obtained as:

$$ \text{SIR}_I = \frac{P_a R_{u0}^{-\beta} h_{u0}}{\sum_{s \in S} P_m \ell(D_{us}, \theta_{us}) g_{us} L + \sum_{f \in F \setminus F_0} P_a R_{uf}^{-\beta} h_{uf} L^2}, $$

where $P_a$ and $P_m$ denote the transmit power of a macrocell BS and a FAP, respectively. The term $L$ is the wall loss, $S$ denotes all transmission sector antennas, $F$ is the set of employed FAPs, and $F_0$ labels the transmitter of the desired signal. The signal from the interfering macrocell BSs experiences wall loss once ($L$), while the FAP interference experiences it twice ($L^2$).

2) Indoor users can also be assigned to a nearby macrocell BS, as indicated by the upper left FAP in Figure 1. The SIR then calculates as:

$$ \text{SIR}_I = \frac{P_m \ell(D_{us}, \theta_{us}) g_{us} L + \sum_{f \in F \setminus F_0} P_a R_{uf}^{-\beta} h_{uf} L^2}{\sum_{s \in S \setminus S_0} P_m \ell(D_{us}, \theta_{us}) g_{us} L + \sum_{f \in F \setminus F_0} P_a R_{uf}^{-\beta} h_{uf} L^2}, $$

with $S_0$ denoting the macrocell BS sector which provides the desired signal.

3) The remaining users are located outdoors. Depending on the strongest receive signal, they either communicate with the macrocell BS:

$$ \text{SIR}_O = \frac{P_m \ell(D_{us}, \theta_{us}) g_{us} L + \sum_{f \in F \setminus F_0} P_a R_{uf}^{-\beta} h_{uf} L^2}{\sum_{s \in S \setminus S_0} P_m \ell(D_{us}, \theta_{us}) g_{us} L + \sum_{f \in F \setminus F_0} P_a R_{uf}^{-\beta} h_{uf} L^2}, $$

or are handed off to a neighboring FAP:

$$ \text{SIR}_O = \frac{P_a R_{u0}^{-\beta} h_{u0} L}{\sum_{s \in S} P_m \ell(D_{us}, \theta_{us}) g_{us} L + \sum_{f \in F \setminus F_0} P_a R_{uf}^{-\beta} h_{uf} L}. $$

In the next section we evaluate the SIR expressions numerically.

IV. NUMERICAL RESULTS

A. Simulation Setup

In this setting, the cell cluster is composed of a center macrocell BS surrounded by six direct neighbors. The neighboring macrocell BSs are located on a hexagonal grid at a distance of 600 m. The antenna radiation pattern is referred from [10] and formulates as:

$$ G_T(\theta) = -\min \left[ 12 \left( \frac{\theta}{\theta_{\text{dB}} } \right)^2 A_m, \right] \ , \ \ -180^\circ \leq \theta \leq 180^\circ, $$

with a 3 dB beamwidth of $\theta_{\text{dB}} = 65^\circ$ and maximum attenuation $A_m = 20$ dB.

A minimum coupling loss of 45 dB between FAP and user, and 70 dB between macrocell BS and user is taken into account [11]. The downlink signal experiences path loss and Rayleigh fading with unit average power. Further simulation parameters are listed in Table I.

Snapshots of a user hot-spot scenario are generated by first distributing $N_c$ cluster centers uniformly over the coverage region of the macrocell BS. Then, $N_u$ users are distributed uniformly around each cluster within a radius $R_I$. Simulation results are evaluated for users of the central macrocell (BS$_0$). The surrounding macrocells serve as interferers.

B. Signal-to-Interference Ratio Distribution

Figure 3 depicts the SIR distribution for various partial FAP deployments. The two extreme cases serve as a reference: (i) Leftmost solid line: No FAPs are employed. All users are assigned to the central macrocell BS and interference is caused only by the surrounding macrocell BSs. (ii) Rightmost solid line: All user-clusters are covered by FAPs. Interference is caused by the macrocell BSs (including the central one) as
C. Sum Spectral Efficiency

Figure 4 depicts the sum spectral efficiency, calculated as

$$\sum_{\text{Users}} \log(1 + \text{SIR}_{\text{User}}),$$

versus the number of employed FAPs. Three scenarios are investigated, ranging from a high to a low degree of user clustering. Clustering is adjusted by varying the number of clusters ($N_c$) and the number of users per cluster ($N_u$), while keeping the total amount of users $N_c \cdot N_u$ constant.

Sum spectral efficiency increases steepest in a user hot-spot scenario (solid line), and lowest in a close to uniform user distribution (dotted line), respectively. The results verify that the efficiency of femtocell operation is considerably dependent on the degree of clustering.

A second important effect is observed: When the number of employed FAPs is increased, the curves flatten due to the additional interference. This indicates a trade-off between femtocell deployment density and gain of sum spectral efficiency.

D. Fairness Index

In this paper, the widely used Jain’s index serves to quantify the balance between macrocell- and femtocell user spectral efficiency. It is formulated as

$$f(t) = \left( \frac{\sum_{i=1}^{N} t_i}{N \sum_{i=1}^{N} t_i^2} \right)^2,$$

where $N$ is the total number of users in the network, and $t_i$ denotes the spectral efficiency achieved by user $i$. Figure 5 depicts the fairness index plotted versus the number of FAPs employed in the macrocell.

In a sparse femtocell deployment (i.e., low number of FAPs), only few users achieve high spectral efficiency values due to their vicinity to the FAP. The remaining users are attached to the macrocell BS and experience additional interference from the femtocells. Therefore, low fairness is achieved. The fairness index increases with the number of employed FAPs and takes on its maximum at full FAP coverage, i.e., all user clusters are covered by a FAP. In this case, the spectral efficiency values of the users are most balanced. The results show that efficient balancing of user spectral efficiency is only possible in scenarios with a high
degree of clustering. In close to uniform distributions, the achieved fairness of a femto-enhancement does not exceed the reference case without FAPs.

V. SUMMARY AND OUTLOOK

In this paper, the efficiency of Femtocell Access Point (FAP) deployments in a macrocellular network was investigated. We focused on the influence of the user distribution, in particular the degree of user-clustering. A sectorized cell layout was used to carry out numerical evaluations.

Results verify that FAPs operates most efficiently in a user hot-spot scenario with high degree of user-clustering. Compared to a scenario without femtocells, a femto-enhancement always improves sum spectral efficiency. However, our findings emphasized that this metric conceals the distribution of individual user performance. Therefore, we introduced Jain’s fairness index. We showed that fairness improvement with regard to the non-femtocell enhanced reference network requires a minimum degree of user-clustering.

Our method can be applied to assess the efficiency of femtocell-enhancements in real world scenarios. The required knowledge about the degree of user-clustering is obtained from metrics as the ones defined in [12, 13].

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