Joint Network-Channel Coded Multi-Way Relaying

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***Abstract*—We propose a joint network-channel coding scheme for multi-way relay networks, generalizing existing methods for the two-way relay channel. With our scheme, the relay performs log-likelihood ratio quantization and subsequent network encoding, resulting in low relay complexity. We design the compression function at the relay using the information bottleneck framework. To optimize the performance of our scheme, the transmission of network-coded soft information from the relay to the terminals is carefully designed. The individual terminals perform iterative joint decoding of the messages transmitted by all other terminals. Numerical simulations demonstrate the effectiveness of the proposed relaying protocol, showing that our scheme scales well with the number of terminals and is suitable for asymmetric channel conditions.**

***Index Terms*—cooperative systems, relaying, network coding, quantization, iterative decoding**

I. Introduction

We consider the *multi-way relay channel* (MWRC) with $N \geq 2$ terminals and one relay (cf. Fig. 1). We focus on the half-duplex case with orthogonal transmissions which is simple to implement in practice and allows the terminals to overhear each others transmissions. With the help of the relay, the terminals perform *full data exchange*, i.e., each terminal wants to decode the $N-1$ other messages. This setting allows us to model information exchange for, e.g., distributed computing and social networks.

Information-theoretic limits are not available in literature for our model if $N > 2$. However, various protocols for the two-way case ($N = 2$) have been analyzed in [1]. The Gaussian full-duplex MWRC without direct links between the terminals has been considered in [2] and [3]. While most practical transmission schemes are limited to $N = 2$ (e.g., [4]) or $N = 3$ (e.g., [5]), we aim for a protocol which allows a single relay to support significantly more terminals and yet has reasonable complexity.

In this paper, we propose a scheme in which the relay performs "noisy" *network coding* (NC) [6] by jointly compressing its received signals. Using a carefully designed transmit signal, the relay forwards the compressed representation to the terminals which iteratively decode all messages. We note that supporting multiple terminals with a single relay not only reduces deployment cost, but also increases spectral efficiency since the rate loss induced by the relay hop decreases as N increases. More specifically, our contributions are as follows:

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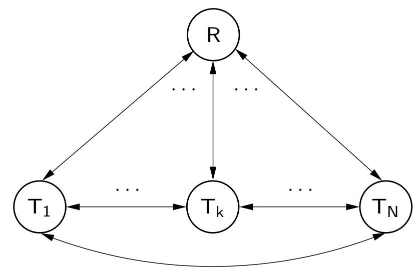


Figure 1: The MWRC with N terminals.

- We propose a scalable transmission scheme for full exchange in the MWRC which is well suited also for asymmetric channel conditions.
- We use the *information bottleneck* (IB) framework [7] to design the relay's compression function which consists of a scalar quantization step and a network encoding step.
- We carefully design square QAM signal constellations for the transmission from the relay to the terminals in order to optimize the performance of our scheme.
- We provide numerical simulation results which demonstrate the effectiveness of the proposed transmission strategy.

The remainder of this paper is organized as follows. Sec. II introduces the system model and describes the basic operation of all network nodes. In Sec. III, the processing at the relay and the transmit signal design is explained in detail. In Sec. IV we describe the iterative joint network-channel decoder used at the terminals. Numerical results are presented in Sec. V and conclusions are provided in Sec. VI.

II. System Model

A. MWRC Model

We consider the time-division MWRC with N terminals, $\mathsf{T}_1, \ldots, \mathsf{T}_\mathsf{N}$, and one relay R as depicted in Fig. 1. In this model, the terminals perform full data exchange and overhear each others transmissions. The terminals consecutively transmit their independent messages in the first N time slots. In the $(N+1)$th time slot, the relay forwards to the terminals a jointly compressed representation of the data it has received in the previous N time slots. Each transmission is a broadcast transmission to the $N-1$ other nodes. Finally, each terminal jointly decodes all messages using the side information obtained via the direct links together with the information from the relay. We assume that the total transmission time is shared equally among all nodes, i.e., each node has M channel uses per time slot and, hence, each transmission consists of $(N+1)M$ channel uses in total.

B. Channel Model

We model the $N^2 + N$ individual channels as Gaussian channels. For the transmissions of the terminals we have (all vectors in this expression are of length M)

$$\mathbf{y}_{ij} = \mathbf{x}_i + \mathbf{w}_{ij}, \tag{1}$$

where $\mathbf{x}_i$ is the signal transmitted by terminal T_i, $\mathbf{y}_{ij}$ is the corresponding receive signal at node j (i.e., another terminal or the relay), and $\mathbf{w}_{ij} \sim \mathcal{CN}(\mathbf{0}, \sigma_{ij}^2 \mathbf{I})$ is white Gaussian noise (assumed zero-mean and circularly symmetric). Similarly, for the transmission of the relay we have

$$\tilde{\mathbf{z}}_j = \mathbf{x}_{\mathsf{R}} + \mathbf{w}_j. \tag{2}$$

We impose a transmit energy constraint at each node, i.e., we fix $E_s \triangleq \mathrm{E}\big[\|\mathbf{x}_i\|_2^2\big]$, where $\mathrm{E}[\cdot]$ and $\|\cdot\|_2$ denote expectation and ℓ_2-norm. The signal-to-noise ratio (SNR) for the link between node i and node j is given by $\gamma_{ij} = E_s/(M\sigma_{ij}^2)$. The MWRC is called *asymmetric* if $\gamma_{i_1\mathsf{R}} \neq \gamma_{i_2\mathsf{R}}$ for any i_1, i_2. We assume that the channels are reciprocal and that each node has receive *channel state information* (CSI), i.e., $\gamma_{ij} = \gamma_{ji}$ is known at node j. This implies that the relay has no CSI of the direct links between the terminals.

C. Terminals

Each terminal consists of an encoder and a decoder.

Encoder: The message at terminal i $(i \in \{1, 2, \ldots, \mathsf{N}\})$ is a length-K_i sequence $\mathbf{u}_i \in \{0,1\}^{K_i}$ of independent and equally likely bits. The sequence $\mathbf{u}_i$ is channel encoded using a linear binary code $\mathcal{C}_i$ of rate $R_i = K_i/L_i$, yielding a length-L_i sequence $\mathbf{c}_i \in \{0,1\}^{L_i}$ of code bits. Next, the code bits are mapped to a signal constellation $\mathcal{A}_i$ of cardinality $|\mathcal{A}_i| = 2^{m_i}$, yielding length-$M$ sequences $\mathbf{x}_i \in \mathcal{A}_i^M$ of transmit symbols. The code rates are chosen as $R_i = K_i/(m_i M)$. For simplicity, we assume $K \triangleq K_i$, $L \triangleq L_i$ and, hence, $R \triangleq R_i$, $m \triangleq m_i$. The sum rate is then given by $R_s = mRN/(N+1)$.

Decoder: The decoder at terminal j jointly decodes the received signals $\mathbf{y}_{1j}, \ldots, \mathbf{y}_{\mathsf{N}j}$, and $\tilde{\mathbf{z}}_j$ to obtain all messages of the other terminals. To this end, iterative joint network-channel decoding on the factor graph of the overall code (cf. Fig. 4) is performed via the sum-product algorithm [8]. This corresponds to invoking the network and channel decoders according to a given schedule and exchanging extrinsic information between the individual decoders. We note that the terminals may use any channel code which permits soft-input soft-output (SISO) decoding. Sec. IV describes the decoder in more detail.

D. Relay

Fig. 2 shows a block diagram of the relay. First, the relay performs soft demapping of each received signal, yielding a vector of log-likelihood ratios (LLRs) which can optionally be processed by a SISO channel decoder. This allows us to trade-off performance against computational complexity (soft decoding at the relay improves performance). Next, the LLRs $\boldsymbol{\lambda}_i$ for the code bits $\mathbf{c}_i$ are processed by a scalar quantizer $\mathsf{Q}_i(\cdot)$ with Q_i levels (cf. Sec. III-A), i.e., $\mathbf{q}_i \in \mathcal{Q}_i^L$, where $\mathcal{Q}_i \triangleq \{1, 2, \ldots, Q_i\}$. The quantizer indices $\mathbf{q}_i$ are then interleaved, $\mathbf{q}_i' = \pi_i(\mathbf{q}_i)$, to avoid short cycles in the factor graph of the overall code. We note that one of the interleavers π_i can be omitted. Next, the network encoder (cf. Sec. III-B) performs an element-wise mapping of all sequences $\mathbf{q}_i'$ to $\mathbf{z} \in \mathcal{Z}^L$, where $Z \triangleq |\mathcal{Z}|$ determines the compression ratio. Finally, the output of the network encoder is mapped to the transmit signal $\mathbf{x}_{\mathsf{R}} = \phi(\mathbf{z})$ which is broadcast to all terminals. The optimization of the mapping $\phi(\cdot)$ for square QAM signal constellations is discussed in Sec. III-C.

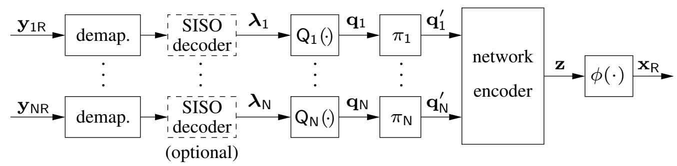


Figure 2: Relay block diagram.

III. Relay Operation

A. LLR Quantization

The relay uses N scalar quantizers $\mathsf{Q}_i(\cdot)$ to map the LLRs $\boldsymbol{\lambda}_i \in \mathbb{R}^L$ to index vectors $\mathbf{q}_i \in \mathcal{Q}_i^L$, where $q_{i,k} = \mathsf{Q}_i(\lambda_{i,k})$. The design of the LLR quantizers is critical for the performance of the system. We aim at maximizing the mutual information $I(c_{i,k}; q_{i,k})$ between the quantizer output $q_{i,k}$ and the corresponding code bit $c_{i,k}$ for a fixed number of quantization levels Q_i, i.e., we want to solve[1]

$$p^\star(q_i|\lambda_i) = \underset{p(q_i|\lambda_i)\in\{0,1\}}{\arg\max}\ I(c_i; q_i). \tag{3}$$

Here, the deterministic quantizer $\mathsf{Q}_i(\cdot)$ is described by $p^\star(q_i|\lambda_i) \in \{0,1\}$. Given the joint distribution $p(c_i, \lambda_i)$, the modified IB algorithm proposed in [9] allows us to find a locally optimal solution of (3).

The relay stores a single set of quantizers for a sufficiently wide range of SNRs and then uses for each channel the quantizer that has been optimized for the corresponding SNR $\gamma_{i\mathsf{R}}$. Finally, we note that the quantizer design described above is superior to other quantizer designs, e.g., mean-square error optimal (Lloyd-Max) quantization and uniform quantization.

B. Network Encoding

Let $\mathbf{q}^{(k)} = (q_{1,k}' \ q_{2,k}' \ \ldots \ q_{\mathsf{N},k}')^T$, $k = 1, 2, \ldots, L$, denote a length-N vector consisting of the kth elements of the interleaved quantization index sequences $\mathbf{q}_i'$. Similarly, $\mathbf{c}^{(k)} = (c_{1,k}' \ c_{2,k}' \ \ldots \ c_{\mathsf{N},k}')^T$ denotes the vector of code bits corresponding to the quantization indices in $\mathbf{q}^{(k)}$. Furthermore,

[1]For the sake of notational clarity, the bit position index k is suppressed since the operations on all bit position are identical.

let $\mathbf{a}_{\sim i}$ denote the vector obtained by removing the ith element from $\mathbf{a}$.

Network encoding is performed at the relay using a deterministic function $g : \mathcal{Q}_1 \times \mathcal{Q}_2 \times \ldots \times \mathcal{Q}_{\mathsf{N}} \mapsto \mathcal{Z}$ which maps the kth elements of all $\mathbf{q}'_i$ to an integer $z_k = g(\mathbf{q}^{(k)}) \in \mathcal{Z}$, i.e., $\mathbf{z} = (z_1\ z_2\ \ldots\ z_L)^T$. The design of the NC function is motivated by the iterative decoding procedure at the destination (see Sec. IV for details). To maximize the information exchange between the individual channel decoders we seek to maximize $I(c'_{i,k}; z_k|\mathbf{c}^{(k)}_{\sim i})$ for each $i \in \{1, 2, \ldots, \mathsf{N}\}$. Loosely speaking, given perfect a priori information $\mathbf{c}^{(k)}_{\sim i}$, z_k should contain as much information about $c'_{i,k}$ as possible. However, since $I(c'_{i,k}; z_k|\mathbf{c}^{(k)}_{\sim i})$ cannot be maximized for each i independently, we resort to maximizing a function of these mutual information expressions. As previously proposed in [10], we use (we again drop the bit index k)

$$I_{\text{rel}} = \sum_{i=1}^{N} I(c'_i; z|\mathbf{c}_{\sim i}) \tag{4}$$

as the relevant information to be maximized. For fixed Z, the maximization of (4) with respect to $g(\cdot)$ can be written as

$$p^\star(z|\mathbf{q}) = \underset{p(z|\mathbf{q})\in\{0,1\}}{\arg\min} \Big\{ N\mathrm{E}\big[\mathrm{D}\big(p(\mathbf{c}|\mathbf{q})\|p(\mathbf{c}|z)\big)\big] - \sum_{i=1}^{N} \mathrm{E}\big[\mathrm{D}\big(p(\mathbf{c}_{\sim i}|\mathbf{q}_{\sim i})\|p(\mathbf{c}_{\sim i}|z)\big)\big] \Big\}, \tag{5}$$

where $\mathrm{D}(\cdot\|\cdot)$ denotes the Kullback-Leibler divergence. Here, g is specified by the deterministic mapping $p^\star(z|\mathbf{q}) \in \{0, 1\}$. We note that randomized NC cannot improve performance since it can be shown that optimizing with respect to all $p(z|\mathbf{q}) \in [0, 1]$ in (5) yields the same minimizer $p^\star(z|\mathbf{q}) \in \{0, 1\}$. The problem in (5) can be recognized as an instance of the IB problem, and the algorithm presented in [10] allows us to find a locally optimal solution.

Finally, we note that the choice of the relevant information in (4) ensures that the data of the individual terminals has large (little) impact on z if the respective SNR $\gamma_{i\mathsf{R}}$ is high (low). Hence, user selection is of minor importance in our scheme.

C. Signal Design

We next discuss how to map the output of the network encoder to an optimized square Z^2-QAM constellation. Two consecutive network-coded symbols are mapped to one constellation symbol and, hence, there are $Z^2!$ possible mappings. However, due to the circular symmetry of the channel we can treat the in-phase and quadrature components independently. We can therefore restrict our attention to one-dimensional Z-ary PAM constellations with $Z!$ possible mappings. The kth relay transmit symbol is thus $x_{\mathsf{R},k} = \phi(z_{2k-1}, z_{2k}) = \varphi(z_{2k-1}) + j\varphi(z_{2k})$, where ϕ denotes the square QAM mapping constructed from the PAM mapping φ.

As objective function for our optimization we use

$$\tilde{I}_{\text{rel}} = \frac{1}{N} \sum_{j=1}^{N} \sum_{i=1}^{N} I(c'_i; \tilde{z}_j|\mathbf{c}_{\sim i}), \tag{6}$$

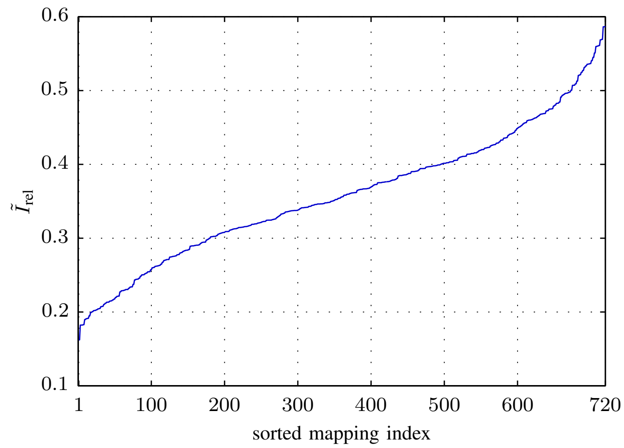


Figure 3: Impact of ϕ on $\tilde{I}_{\text{rel}}$ for $Z = 6$ ($\gamma_{i\mathsf{R}} = 3\,\text{dB}$).

i.e., the average of the relevant information at the terminals. We note that the mapping enters in (6) through $\tilde{\mathbf{z}}_j = \phi(\mathbf{z}) + \mathbf{w}_j$. The choice of the mapping is important because it makes a difference which network-coded symbols are more or less likely to get confused at the terminals. The optimal mapping depends on the SNRs $\gamma_{\mathsf{R}j}$ and is given by

$$\varphi^\star = \arg\max_{\varphi} \tilde{I}_{\text{rel}}. \tag{7}$$

$\tilde{I}_{\text{rel}}$ is evaluated using the pdfs

$$p(\tilde{z}_j|\mathbf{c}) = \sum_{z\in\mathcal{Z}} p(\tilde{z}_j|z)p(z|\mathbf{c}), \tag{8}$$

where $p(z|\mathbf{c})$ is obtained from the design of the network encoder and $p(\tilde{z}_j|z)$ is determined by (2). We note that if the channel is noise-free, i.e., if $p(\tilde{z}_j|z) = \delta_{\tilde{z}_j,z}$, we have $\tilde{I}_{\text{rel}} = I_{\text{rel}}$ and the mapping is irrelevant.

For values of Z which are of interest, it is feasible to solve (7) using an exhaustive search over all $Z!$ mappings. Fig. 3 shows the impact of the mapping on $\tilde{I}_{\text{rel}}$ for $Z = 6$ and $\gamma_{i\mathsf{R}} = 3\,\text{dB}$, $i = 1, 2, \ldots, N$ (all other system parameters are as in Sec. V-A). We observe a difference of more than a factor of three in $\tilde{I}_{\text{rel}}$ between the best and the worst mapping. This difference also translates to the bit error rate (BER) performance of the system (cf. Sec. V-D) and, hence, it is crucial to use the optimal mapping.

IV. Joint Network-Channel Decoder

The processing at the relay couples the code bits transmitted by the terminals, enabling iterative joint decoding of all messages. In what follows, we describe the decoder at terminal j (the other terminals decode in an analogous manner). Fig. 4 shows the factor graph of the overall code including the messages as seen by terminal j.

The operation of the joint network-channel decoder is such that the individual channel decoders iteratively exchange extrinsic LLRs via the network decoder (9)-(11). The network decoder computes a priori LLRs $\lambda^{(\mathrm{A})}_{i,k}$ for the ith channel

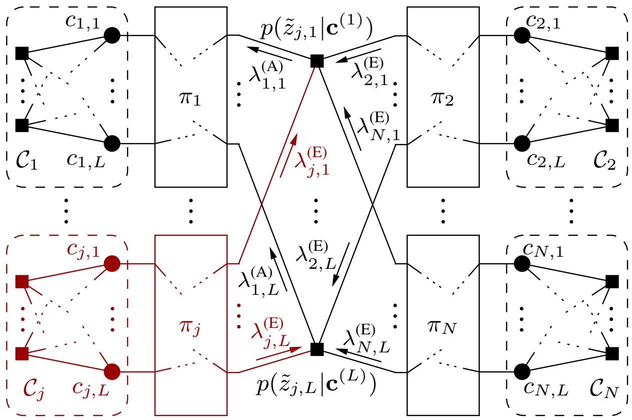


Figure 4: Factor graph of overall code as seen by terminal j.

decoder using $N-1$ extrinsic LLRs $\lambda_{n,k}^{(E)}$ (for all $n \neq i$) and the local function $p(\tilde{z}_{j,k}|\mathbf{c}^{(k)})$ which is given by (8). The sum-product update rule for the a priori LLRs $\lambda_{i,k}^{(A)}$ ($k = 1, 2, \ldots, L$) is given by

$$\lambda_{i,k}^{(A)} = \log \frac{\mu_{p\to c'_{i,k}}(c'_{i,k}=0)}{\mu_{p\to c'_{i,k}}(c'_{i,k}=1)}, \quad i = 1, \ldots, j-1, j+1, \ldots, N, \tag{9}$$

where

$$\mu_{p\to c'_{i,k}}(c'_{i,k}) = \sum_{\mathbf{c}_{\sim i}^{(k)}} p(\tilde{z}_{j,k}|\mathbf{c}^{(k)}) \prod_{n:n\neq i} \mu_{c'_{n,k}\to p}(c'_{n,k}), \tag{10}$$

and

$$\mu_{c'_{n,k}\to p}(c'_{n,k}=b) = \frac{\exp\left(-b\lambda_{n,k}^{(E)}\right)}{1+\exp\left(-\lambda_{n,k}^{(E)}\right)}, \quad b \in \{0,1\}. \tag{11}$$

Since the codeword $\mathbf{c}_j$ is known at terminal j, the a priori LLRs $\lambda_{j,k}^{(A)}$ need not be computed and the extrinsic LLRs are given by

$$\lambda_{j,k}^{(E)} = \begin{cases} \infty, & \text{if } c'_{j,k} = 0 \\ -\infty, & \text{if } c'_{j,k} = 1 \end{cases}. \tag{12}$$

One iteration of the joint network-channel decoder consists of $N-1$ channel decoder invocations and $N-1$ network decoder invocations. Hence, there are $(2N-2)!$ possible decoder schedules. We propose to use a serial schedule which in [10] has been shown to perform significantly better than a flooding schedule.

V. Numerical Results

In this section we assess the performance of our proposed scheme by Monte Carlo simulations. We analyze the BER for the transmission over the symmetric and the asymmetric MWRC, respectively. Moreover we study the performance impact of the mapping ϕ.

A. General Setup

Each terminal transmits $K = 1024$ information bits and we fix the sum-rate to $R_s = 1$. The terminals transmit using a QPSK signal constellation ($m = 2$) normalized to power P_s. The channel code is a recursive, systematic convolutional code with generator polynomial $[1\ 13/15]_8$ (in octal notation), punctured to a code rate of $R = (N+1)/(2N)$. We set $E_s = 2K/3$ and, hence, $P_s = 2(N+1)/(3N)$, which corresponds to $P_s = 1$ for $N = 2$ terminals. We assume that the direct channels between the terminals are symmetric with respect to the SNR, i.e., $\gamma \triangleq \gamma_{ij}$, $i, j = 1, 2, \ldots, N$. The relay performs SISO decoding, the resulting LLRs are quantized with 5 levels, and the NC function is designed with $Z = 5$. The joint network-channel decoder at the destination performs 4 iterations and uses a serial schedule. We have used random interleavers with depths equal to the block length $L = K/R$.

B. Symmetric Case

We first study the performance of our proposed scheme in the symmetric MWRC. For the SNR on the terminal to relay channels we assume $[\gamma_{i\mathrm{R}}]_{\mathrm{dB}} = [\gamma]_{\mathrm{dB}} + 6\,\mathrm{dB}$. Fig. 5a shows the BER performance of the proposed scheme for 2 to 7 terminals. For comparison the performance without relay (at the same sum rate) is also given.

We observe a graceful performance degradation as we add additional terminals. The SNR penalty is approximately constant for each additional terminal. At a BER of 10^{-3} the system with 7 terminals loses $1.3\,\mathrm{dB}$ over the system with 2 terminals and gains about $1.8\,\mathrm{dB}$ compared to a transmission without relay. We note that this behavior is observed for a wide range of sum rates.

C. Asymmetric Case

We now turn our attention to the asymmetric MWRC with 4 terminals. The SNRs are related as follows:

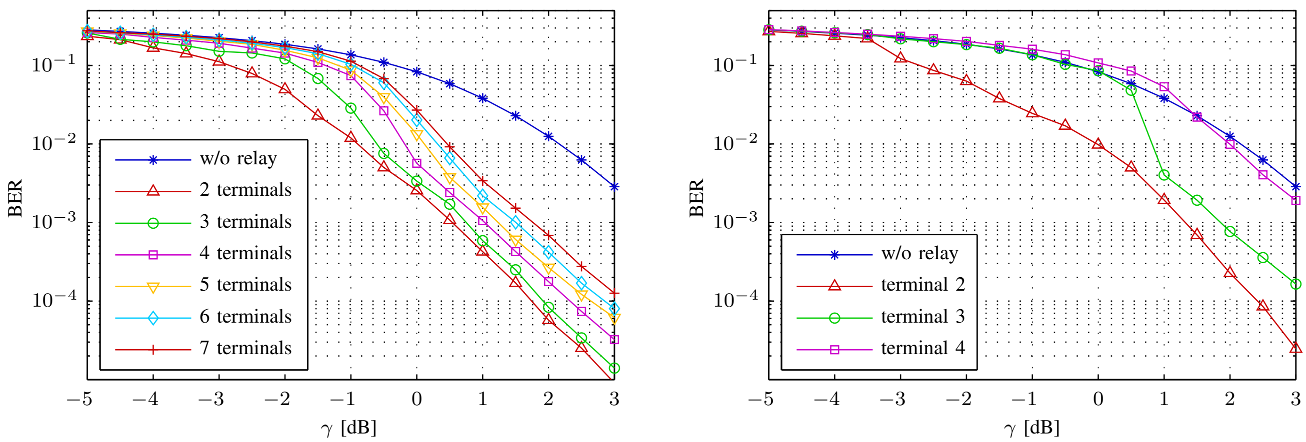
$$[\gamma_{1\mathrm{R}}]_{\mathrm{dB}} = [\gamma]_{\mathrm{dB}} + 6\,\mathrm{dB},$$
$$[\gamma_{2\mathrm{R}}]_{\mathrm{dB}} = [\gamma]_{\mathrm{dB}} + 13/3\,\mathrm{dB},$$
$$[\gamma_{3\mathrm{R}}]_{\mathrm{dB}} = [\gamma]_{\mathrm{dB}} + 8/3\,\mathrm{dB},$$
$$[\gamma_{4\mathrm{R}}]_{\mathrm{dB}} = [\gamma]_{\mathrm{dB}} + 1\,\mathrm{dB}.$$

Fig. 5b shows the BER performance achieved by the decoder at terminal 1. For comparison the performance without relay (at the same sum rate) is also given.

We observe that our proposed scheme is able to handle asymmetric channel conditions very well. Specifically, the performance is not dominated by the terminal with the worst link to the relay. In this case, the decoding of terminals 2 and 3 benefits from the relay, while for terminal 4 the decoder achieves essentially the same BER as without relay. We note that an advantage of the proposed scheme is that the relay need not be able to fully decode all messages. Since the probability of successful decoding at the relay (for finite blocklength) decreases with increasing N, protocols which require the relay to decode will generally perform poorly for large N.

D. Performance Impact of Mapping ϕ

Finally, we study the performance impact of the mapping ϕ which is chosen at the relay. The results in Fig. 5 have been obtained using the optimal mapping according to (6)-(8). For two terminals and SNRs as in Sec. V-B, Fig. 6



(a) (b)

Figure 5: BER performance in the (a) symmetric case, and (b) asymmetric case (decoding at terminal 1).

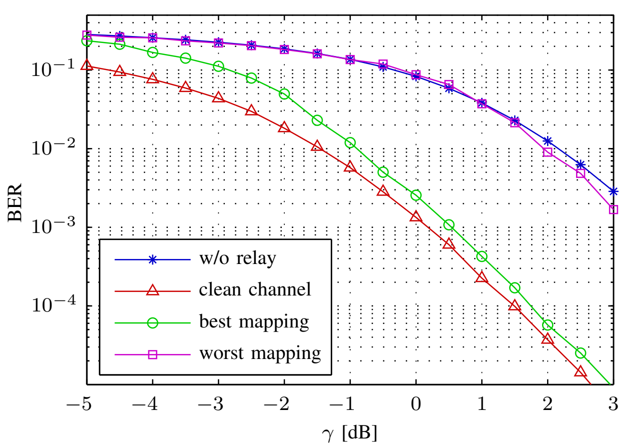


Figure 6: BER performance impact of the mapping ϕ ($N = 2$).

shows the BER performance achieved with the best and the worst possible mapping, respectively. Additionally, we give the performance for the case of "clean", i.e., noise-free, relay to terminal channels. The noise-free case constitutes a lower bound in terms of the achievable BER.

We observe that using the optimal mapping, we are less than $0.5\,\text{dB}$ away from the noise-free case for BER values of interest. On the other hand, the performance using the worst mapping is inferior; it is essentially equal to the performance without relay (at the same sum rate). For a wide range of BER values the SNR gap between the mappings is about $3\,\text{dB}$. This confirms the usefulness of the signal design proposed in Sec. III-C.

VI. Conclusions

We have proposed a scalable transmission scheme for full data exchange in the MWRC which supports significantly more than two terminals. Our scheme performs LLR quantization and subsequent network encoding at the relay. The compression function is designed using the IB framework. Each terminal decodes the messages of all other terminals using an iterative message-passing decoder. We have shown that increasing the number of terminals yields only a minor degradation of the BER. The proposed scheme also performs well in asymmetric channel conditions. Moreover we have discussed the optimization of square QAM signal constellations for the transmission of the network-coded data. Finally, simulation results confirm that the optimal QAM mapping closely approaches the performance in the noise-free case.

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