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Gas exchange of almost leak-tight display cases

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A gas exchange model based on diffusion and stack pressure driven convection for display cases will be discussed and evaluated experimentally.

1 Introduction

A requirement for display cases for art objects is to be leak tight. However, due to design constraints there a are tiny gaps resulting in a gas exchange between the interior of the showcase and its environment. Usually the leak-tightness is being quantified by the so-called air exchange rate, the ratio of the gas volume exchanged between the display-case and its environment per unit time and the volume of the display-case. Commonly the concentration of a tracer gas in the display-case is monitored to calculate the air exchange rate. For this purpose usually a linear gas exchange-law is assumed, see [1]. However, the gas exchange is influenced by the stack pressure [3] and temperature differences limiting the applicability of the linear exchange law.

Thus, taking hydro-static pressure differences due to temperature and concentration differences of the tracer gas and diffusion into account a non linear exchange model is derived and validated experimentally.

2 The gas exchange

We consider a showcase of height h with two horizontal gaps of width b and length l and cross section A_{gap}. One gap is located near the bottom of the showcase ($z = -h/2$) and the other close to the top of the showcase ($z = h/2$). Then the equation for the time dependent behavior of the mole fraction $x^{(i)}(t)$ of the tracer gas, usually CO_2, in the showcase is given by

$$\frac{\mathrm{d}x^{(i)}}{\mathrm{d}t} = -\frac{x^{(i)} - x^{(e)}}{t_{\mathrm{ref}}} f_H(Pe), \qquad f_H(Pe) = \frac{Pe}{2}\frac{e^{Pe}+1}{e^{Pe}-1}, \qquad Pe = C_s\frac{\Delta\rho}{\rho}, \tag{1}$$

where the reference time, $t_{\mathrm{ref}} = Vl/2A_{\mathrm{gap}}D$, which can be interpreted as the reciprocal value of the air exchange rate and the stack pressure parameter $C_S = \rho g h b^2/24\nu D$ are two parameters describing the leakage behavior. Note the ρ and ν are the density and kinematic viscosity of air, D is the diffusion constant of the tracer gas in air and $x^{(e)}$ is mole fraction of the trace gas in the environment. Deriving this model it has been assumed that the gas in the showcase is well mixed and that the mean pressures inside and outside the showcase are equal [2], [4] .

The difference between the density in the showcase $\rho^{(i)}$ and the density in the environment $\rho^{(e)}$ depends on the temperature difference [3] and the difference of the mole fraction of the tracer gas inside and outside the showcase

$$\frac{\Delta\rho}{\rho} = -\sum_s \beta_s\left(x_s^{(i)} - x_s^{(e)}\right) - \beta_T\left(T^{(i)} - T^{(e)}\right), \tag{2}$$

where $\beta_T = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p = \frac{1}{T}$, $\beta_s = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial x_s}\right)_p = \frac{\mathcal{M}_a - \mathcal{M}_s}{\mathcal{M}_a}$ are the thermal expansion coefficient and the expansion coefficient due to composition. Here $T^{(i)}$ and $T^{(e)}$ denote the temperatures in the interior of the showcase and in the environment, respectively.

In case of two vertical gaps a similar evolution equation for the mole fraction $x^{(i)}$ as in the case of two horizontal gaps holds. In eq. (1) the function $f_H(Pe)$ has to be replaced by $f_V(Pe) = \int_0^1 f_H(\xi Pe)\,\mathrm{d}\xi$.

3 Evaluation of the gas exchange model

In Figure 1 the prediction of the gas exchange model for the case of a constant temperature difference is shown. The red curve is for the isothermal case. Since the tracer gas has a higher density than the air outside the showcase the outflow is

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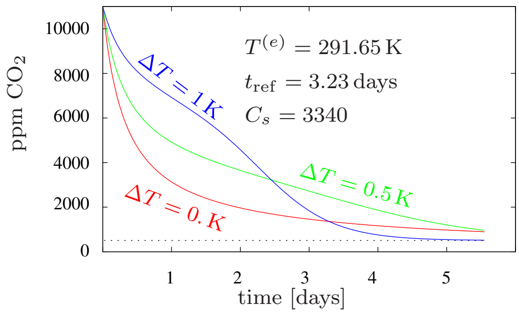

Fig. 1: Influence of a temperature difference onto the exchange of the tracer gas

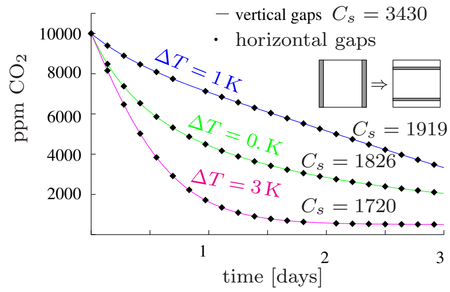

Fig. 2: Gas exchange through vertical gaps ($t_{\mathrm{ref}} = 3.23\,\mathrm{d}$, $C_s = 3430$ (solid line) approximated by a pair of horizontal gaps (dots), $t_{\mathrm{ref}} = 3.23$, C_s best fit

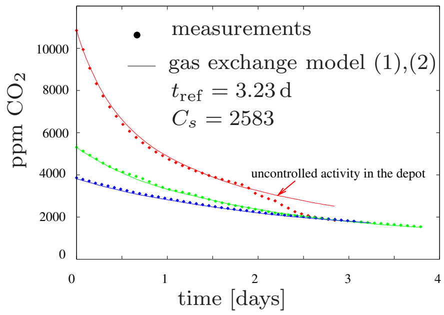

Fig. 3: Decay of the mole fraction of the tracer gas CO_2 in an air conditioned environment. prediction solid line, measurement: dots

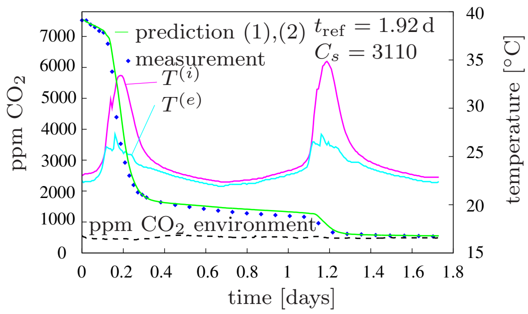

Fig. 4: Decay of the mole fraction of the tracer gas under museum conditions The internal and external temperatures are measured and used as input data. prediction: solid line measured data dots

through the lower gap. Increasing the temperature inside the showcase reduces the density of the gas. As a consequence the stack pressure is reduced and the gas exchange is reduced. If the mole fraction of the heavier tracer gas decreases the density difference will eventually vanish and only diffusion will be responsible for the gas exchange. After a further decrease of the mole fraction of the tracer gas the gas in the showcase becomes lighter than the ambient air. Thus the flow direction changes, see blue and green line in Figure 1. For a real showcase the form of the gaps is not known. Thus we want to investigate if the two horizontal gap model (1) can describe the gas exchange through an arbitrary gap. As a test case we choose a showcase with two vertical gaps. In Figure 2 the comparison of the simulated tracer gas concentration using the vertical gap model is approximated by the horizontal gap model using appropriate model constants t_{ref} and C_S. By inspection we conclude that the horizontal gap model can reproduce the vertical gap model sufficiently well. In Figure 3 a comparison of the measurements at a test showcase in an air conditioned environment (art depot) with the predictions by the model (1) is shown. We observe a reasonable agreement with the exception of the red curve. Here the measurement have been disturbed by an uncontrolled activity in the depot. Furthermore the test showcase has been placed in the Ephesos museum Vienna near a window where the showcase is exposed to sun light in the late after noon. Additionally to the mole fraction of the tracer gas the temperature inside and outside the showcase has been recorded. Using the measured temperature difference as an input into the gas exchange model (1),(2) we get a perfect agreement between the model prediction and the measurement, see figure 4.

4 Conclusions

A non-linear Gas exchange model has been derived. Besides the the characteristic reference time t_{ref}, which is the inverse of the volume exchange rate a second parameter C_S measuring the influence of the stack pressure has been introduced. Reasonable agreement with experimental measurements has been obtained. Moreover, it has been shown that the linear model can be used only for small concentrations of the tracer gas under well defined temperature conditions. Otherwise non-linear (stack pressure) effects have to be taken into account.

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