Iterative Channel Estimation in LTE Systems

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***Abstract*—In this paper, we discuss iterative channel estimation for the Long Term Evolution (LTE) downlink. The LTE standard utilizes coherent detection and therefore to achieve a high data rate over mobile radio channels, it is essential to obtain high accurate channel state information at the receiver side. For channel estimation purposes, the LTE standard provides training data known as pilot symbols. The open question is whether the accuracy of channel estimation based on pilot symbols is sufficient to achieve such high data rate transmission. The quality of the channel estimates can be further enhanced if soft or hard information bits, which are available after the decoding stage, are utilized by the channel estimator to re-estimate the channel. In order to quantify the gain provided by the iterative channel estimation, several scenarios have been considered in this work.**

***Index Terms*—LTE, OFDM, Channel Estimation, Iterative Receiver.**

I. Introduction

Channel estimation plays a crucial role in the performance of wireless communication systems, since its knowledge is utilized to detect the data symbols [1]. With enhancing the channel estimation performance, also the performance of the entire system is improved [2]. Many investigations have been conducted on channel estimation in LTE [3], [4], [5]. The majority of them investigate non-iterative channel estimation algorithms based only on the known pilot symbols provided by the LTE standard. In recent years, receivers with iterative channel estimation have become more and more attractive because of their superb performance [6]. Hard or soft information bits from the decoding stage are utilized to estimate the data symbols and then applied to further improve the channel estimation [7], [8]. In this paper we discuss and analyse the performance of different iterative channel estimation algorithms for the block fading case.

The main contributions of the paper are:

- We calculate the soft estimated data symbols for the LTE standard (according to [9]) for 4-Quadrature Amplitude Modulation (QAM), 16-QAM and 64-QAM constellations symbols.
- We derive various iterative channel estimation algorithms with different complexities and performance behaviours.
- We investigate the impact of processing the soft (a-posteriori or extrinsic) and hard feedback information for channel estimation.

The remainder of the paper is organized as follows. In Section II, the system model is described. Various iterative channel estimation algorithms are presented in Section III. In Section IV, we evaluate the performance of the channel estimators in terms of data throughput and Mean Square Error (MSE). Finally, Section V concludes the paper with a summary of most important points obtained in this work.

II. System Model

In this section, we briefly give an overview of the pilot symbol structure defined in the LTE standard and discuss the structure of an iterative receiver. Afterwards, we introduce a signal model and we show how the soft symbols are calculated.

A. Overview

In LTE systems, the initial channel estimates can be obtained based on pilot symbols, which are placed in the Orthogonal Frequency Division Multiplexing (OFDM) time-frequency grid. In Figure 1, the pilot symbol structure for one subframe corresponding to normal cyclic prefix configuration for up to four transmit antenna ports is shown (for different antenna ports different color boxes) [9]. Pilot symbols transmitted from multiple antenna ports are orthogonal to each other. That is, when one antenna port transmits a pilot symbol, the other antenna ports do not transmit. For the iterative channel estimation we take advantage of hard or soft estimation of data symbols (white boxes), available after the decoding stage, which in addition to the pilot symbols are utilized to re-estimate the channel after the initial estimation.

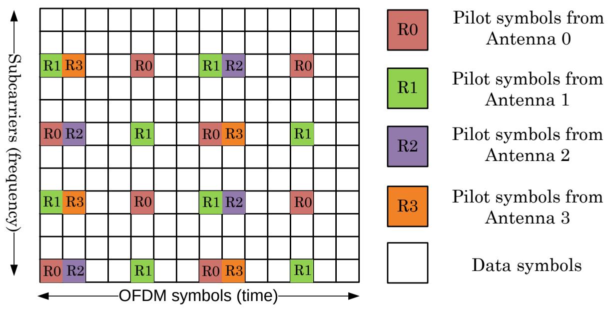

Fig. 1. Pilot symbol structure.

B. Iterative Receiver

Figure 2 depicts the iterative receiver structure. The channel state information knowledge, derived from the pilot symbols, is utilized for demodulation and soft de-mapping of the OFDM symbols. Afterwards, the detector provides to the decoding unit soft information in terms of Log Likelihood Ratios (LLRs) for each transmitted information coded bit. In order to take advantage of the feedback information, the decoding unit has to be modified and new components such as the soft symbol mapper has to be added to the receiver (the iterative receiver part in Figure 2). The modified decoding unit, besides hard decoded bits, outputs also soft estimated coded bits. This feedback information is utilized by the (soft) symbol mapper to calculate the estimated data symbols, which in turn, together with the pilot symbols are applied by the channel estimator to re-estimate the channel. The process is performed again and again and new channel estimates are obtained. Through the successive iterations, the overall system performance can be improved.

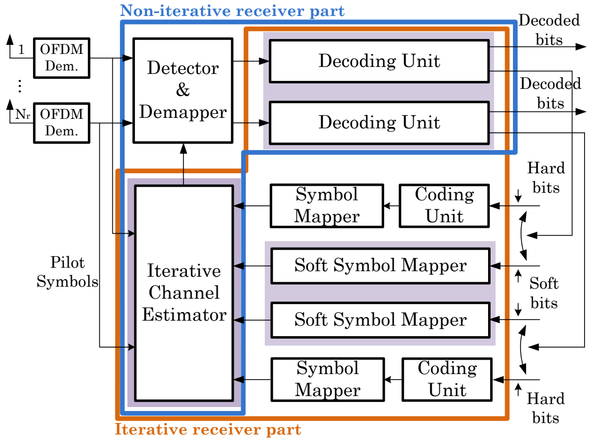

Fig. 2. Iterative receiver structure.

C. Signal Model

The received frequency domain signal $\mathbf{r}_{n_\mathrm{r},n_\mathrm{s}}$ (after OFDM demodulation) at the receive antenna port n_r and OFDM symbol n_s is obtained as

$$\mathbf{r}_{n_\mathrm{r},n_\mathrm{s}} = \sum_{n_\mathrm{t}=1}^{N_T} \mathbf{S}_{n_\mathrm{t},n_\mathrm{s}}\, \mathbf{h}_{n_\mathrm{t},n_\mathrm{r}} + \mathbf{n}_{n_\mathrm{r},n_\mathrm{s}}, \tag{1}$$

where $\mathbf{S}_{n_\mathrm{t},n_\mathrm{s}} = \mathrm{diag}(\mathbf{s}_{n_\mathrm{t},n_\mathrm{s}})$ is a diagonal matrix and $\mathbf{s}_{n_\mathrm{t},n_\mathrm{s}}$ is the transmit signal vector at the transmit antenna port n_t ($n_\mathrm{t} = 1 \ldots N_T$) and OFDM symbol n_s, which comprises the pilots $s_{p,n_\mathrm{t},n_\mathrm{s},n_\mathrm{k}}$ and data subcarriers $s_{d,n_\mathrm{t},n_\mathrm{s},n_\mathrm{k}}$, where n_k is the subcarrier index. The vector $\mathbf{h}_{n_\mathrm{t},n_\mathrm{r}}$ is the channel vector in the frequency domain and $\mathbf{n}_{n_\mathrm{r},n_\mathrm{s}}$ is the additive zero mean white Gaussian noise vector with the variance σ_n^2. The length of vectors $\mathbf{s}_{n_\mathrm{t},n_\mathrm{s}}$, $\mathbf{h}_{n_\mathrm{t},n_\mathrm{r}}$ and $\mathbf{n}_{n_\mathrm{r},n_\mathrm{s}}$ is N_K, which corresponds to the number of subcarriers within an OFDM symbol n_s. If we concatenate Equation (1) for N_S OFDM symbols, we obtain

$$\mathbf{r}_{n_\mathrm{r}} = \sum_{n_\mathrm{t}=1}^{N_T} \mathbf{S}_{n_\mathrm{t}}\, \mathbf{h}_{n_\mathrm{t},n_\mathrm{r}} + \mathbf{n}_{n_\mathrm{r}}, \tag{2}$$

where $\mathbf{S}_{n_\mathrm{t}}$ is of size $(N_K\, N_S) \times N_K$, and $\mathbf{r}_{n_\mathrm{r}}$ and $\mathbf{n}_{n_\mathrm{r}}$ are of size $(N_K\, N_S) \times 1$. On the one hand, for initial channel estimation the sum in Equation (2) disappears because if there is a pilot symbol located at the n_t-th transmit antenna port within the n_s-th OFDM symbol on the n_k-th subcarrier, symbols at the remaining transmit antenna ports at the same positions are zero. Therefore, Equation (2) extracted at pilot symbols p is simply $\mathbf{r}_{p,n_\mathrm{r}} = \mathbf{S}_{p,n_\mathrm{t}}\mathbf{h}_{p,n_\mathrm{t},n_\mathrm{r}} + \mathbf{n}_{p,n_\mathrm{r}}$. On the other hand, for iterative channel estimation, the matrix $\mathbf{S}_{n_\mathrm{t},n_\mathrm{s}}$ from Equation (1) is replaced by the matrix $\tilde{\mathbf{S}}_{n_\mathrm{t},n_\mathrm{s}}$, in which the data symbols are replaced by the estimated soft or hard data symbols. Thus, Equation (2) for the iterative case can be written as

$$\mathbf{r}_{n_\mathrm{r}} = \tilde{\mathbf{S}}\mathbf{h}_{n_\mathrm{r}} + \mathbf{n}_{n_\mathrm{r}}, \tag{3}$$

with $\tilde{\mathbf{S}}$, of size $(N_K\, N_S) \times (N_K\, N_T)$, comprising the pilot symbols and the soft or hard estimated data symbols.

D. Soft Symbol Calculation

The a-posteriori or extrinsic LLRs of coded bits from the output of the decoding unit are considered as a-priori information and utilized by the soft symbol mapper to calculate the soft estimated data symbols as [10]

$$\tilde{s}_d = \mathbb{E}\left[s_d\right] = \sum_{i=1}^{M} s_{d,i} \cdot \prod_{n_b=1}^{N_C} \mathrm{P}\Big(f_{n_b}(s_{d,i})\Big), \tag{4}$$

where $s_{d,i}$ is one of the constellation points that is formed by the information bits $(f_1, f_2, \ldots, f_{N_C})$, M is the number of symbols in the symbol alphabet and $N_C = \log_2 M$ is the number of bits per symbol. $\mathrm{P}(f_{n_b}(s_{d,i}))$ is the probability of the bit f_{n_b} that corresponds to the given symbol constellation point $s_{d,i}$. For notational simplicity we have omitted the transmit antenna port n_t, OFDM symbol n_s and subcarrier n_k indexes from $\tilde{s}_{d,n_\mathrm{t},n_\mathrm{s},n_\mathrm{k}}$. The LLR of the coded bit f_{n_b} is computed by the log-ratio of the probability of the information bit being 1 over the probability of the information bit being 0

$$\mathrm{L}(f_{n_b}) = \ln\frac{\mathrm{P}(f_{n_b} = 1)}{\mathrm{P}(f_{n_b} = 0)}. \tag{5}$$

We know that $\mathrm{P}(f_{n_b} = 1) + \mathrm{P}(f_{n_b} = 0) = 1$. Using the definitions of LLRs above, we find the probability of the information bit being 1(0) as

$$\mathrm{P}\big(f_{n_b} = 1(0)\big) = \frac{e^{1(0)\times \mathrm{L}(f_{n_b})}}{1 + e^{\mathrm{L}(f_{n_b})}}. \tag{6}$$

Furthermore, Equation (6) can be written in terms of tangent hyperbolic functions

$$\mathrm{P}\big(f_{n_b} = 1(0)\big) = \frac{1}{2}\big(1 + (-)\tanh\left(\mathrm{L}(f_{n_b})/2\right)\big). \tag{7}$$

Any Gray-mapped QAM constellation symbol can be divided into real and imaginary parts $s_d = \mathrm{Re}(s_d) + j\mathrm{Im}(s_d)$ [10]. This implies that the real and imaginary components of each soft symbol are independent. For the real $\mathrm{Re}(\tilde{s}_d)$ and imaginary $\mathrm{Im}(\tilde{s}_d)$ part of the soft estimated symbol $\tilde{s}_d$, by applying Equation (7) in Equation (4), for 4-QAM, 16-QAM and 64-QAM (for the LTE standard defined modulation alphabet according to [9]) we find

- 4-QAM

$$\mathrm{Re}(\tilde{s}_d) = -a_1 \text{ and } \mathrm{Im}(\tilde{s}_d) = -a_2. \tag{8}$$

- 16-QAM

$$\mathrm{Re}(\tilde{s}_d) = -a_1 a_3 - 2a_1 \text{ and } \mathrm{Im}(\tilde{s}_d) = -a_2 a_4 - 2a_2. \quad (9)$$

- 64-QAM

$$\mathrm{Re}(\tilde{s}_d) = -4a_1 - 2a_1 a_3 - a_1 a_3 a_5 \text{ and} \quad (10)$$

$$\mathrm{Im}(\tilde{s}_d) = -4a_2 - 2a_2 a_4 - a_2 a_4 a_6, \quad (11)$$

where $a_{n_b} = \tanh\big(\mathrm{L}(f_{n_b})/2\big)$ is the tangent hyperbolic function as a function of the bit LLR $\mathrm{L}(f_{n_b})$. The soft estimated symbols are then obtained as $\tilde{s}_d = \mathrm{Re}(\tilde{s}_d) + j\mathrm{Im}(\tilde{s}_d)$.

III. Iterative Channel Estimation

In this section, different iterative channel estimation algorithms considered in this paper are derived.

A. Iterative Least Square (LS) channel estimator

Iterative LS channel estimate tries to find a solution $\hat{\mathbf{h}}^{(i)}_{\mathrm{ILS},n_\mathrm{r}}$ by minimizing the Euclidean norm squared $J(\hat{\mathbf{h}}^{(i)}_{\mathrm{ILS},n_\mathrm{r}}) = \left\| \tilde{\mathbf{S}}\, \hat{\mathbf{h}}^{(i)}_{\mathrm{ILS},n_\mathrm{r}} - \mathbf{r}_{n_\mathrm{r}} \right\|_2^2$. The minimum is found by computing the gradient of the cost function $\nabla J(\hat{\mathbf{h}}^{(i)}_{\mathrm{ILS},n_\mathrm{r}})$ and equating it to the zero vector. Thus, the iterative LS channel estimator is derived as [11]

$$\hat{\mathbf{h}}^{(i)}_{\mathrm{ILS},n_\mathrm{r}} = (\tilde{\mathbf{S}}^{\mathrm{H}}\tilde{\mathbf{S}})^{-1}\tilde{\mathbf{S}}^{\mathrm{H}}\mathbf{r}_{n_\mathrm{r}}, \quad (12)$$

under the condition that $\mathrm{rank}(\tilde{\mathbf{S}}) = N_K N_T$ [12], where i is the iteration index number. For notational simplicity, we have omitted the iteration index i in $\tilde{\mathbf{S}}^{(i)}$.

B. Iterative Linear Minimum Mean Square Error (LMMSE) channel estimator

The iterative LMMSE channel estimator provides estimated channel coefficients by minimizing the MSE

$$\epsilon = \mathbb{E}\left\{ \left\| \mathbf{h}_{n_\mathrm{r}} - \hat{\mathbf{h}}^{(i)}_{\mathrm{ILMMSE},n_\mathrm{r}} \right\|_2^2 \right\}. \quad (13)$$

Utilizing the second order statistics of the channel the iterative LMMSE estimator is derived, which improves the performance of the iterative LS estimator by filtering its estimate as [11]

$$\hat{\mathbf{h}}^{(i)}_{\mathrm{ILMMSE},n_\mathrm{r}} = \mathbf{R}_{\mathbf{h}_{n_\mathrm{r}},\mathbf{h}_{n_\mathrm{r}}}(\mathbf{R}_{\mathbf{h}_{n_\mathrm{r}},\mathbf{h}_{n_\mathrm{r}}} + \sigma_n^2(\tilde{\mathbf{S}}^{\mathrm{H}}\tilde{\mathbf{S}})^{-1})^{-1}\hat{\mathbf{h}}^{(i)}_{\mathrm{ILS},n_\mathrm{r}}, \quad (14)$$

where $\mathbf{R}_{\mathbf{h}_{n_\mathrm{r}},\mathbf{h}_{n_\mathrm{r}}}$ is the autocorrelation matrix of the channel vector $\mathbf{h}_{n_\mathrm{r}}$, which is assumed to be perfectly known.

C. Iterative Approximate LMMSE (ALMMSE) channel estimator

The computational complexity that comes with iterative LMMSE is very high because of the matrix inversion involved in Equation (14). Therefore, its use in a real-time implementation is limited and restricted to a low complexity implementation. In this subsection, we investigate an approximate algorithm which reduces the complexity of the iterative LMMSE and preserves its performance. This estimator is presented in [3] for LTE and [13] for WiMAX but derived only for initial channel estimation. Therefore, we will modify this estimator for the iterative case. As presented in [3] and [13], the main idea of ALMMSE estimator is to approximate the correlation matrix $\mathbf{R}_{\mathbf{h},\mathbf{h}}$ by utilizing only the correlation of the L closest subcarriers instead of using the full correlation between all subcarriers as in LMMSE case. Assuming that the correlation is frequency independent, the approximated $L \times L$ correlation matrix $\hat{\mathbf{R}}^{(L)}_{\mathbf{h},\mathbf{h}}$ is obtained by averaging over the correlation matrices of sizes $L \times L$ [14]. As in [3], first the number of L closest subcarriers is defined, over which the autocorrelation matrix $\hat{\mathbf{R}}^{(L)}_{\mathbf{h},\mathbf{h}}$ is approximated. Next, the channel vector $\hat{\mathbf{h}}^{(i)}_{n_t,n_\mathrm{r}}$ is defined over the interval of these L consecutive subcarriers as

$$\begin{cases} \left[\hat{h}^{(i)}_{n_\mathrm{t},n_\mathrm{r},1} \dots \hat{h}^{(i)}_{n_\mathrm{t},n_\mathrm{r},L}\right]^{\mathrm{T}} & ; k \le \frac{L+1}{2} \\ \left[\hat{h}^{(i)}_{n_\mathrm{t},n_\mathrm{r},k-\lfloor\frac{L-1}{2}\rfloor} \dots \hat{h}^{(i)}_{n_\mathrm{t},n_\mathrm{r},k+\lceil\frac{L-1}{2}\rceil}\right]^{\mathrm{T}} & ; \text{otherwise} \\ \left[\hat{h}^{(i)}_{n_\mathrm{t},n_\mathrm{r},N_K-L+1} \dots \hat{h}^{(i)}_{n_\mathrm{t},n_\mathrm{r},N_K}\right]^{\mathrm{T}} & ; k \ge N_K - \frac{L-1}{2} \end{cases} \quad (15)$$

where k is the subcarrier index. The output of the estimator for the given interval of L subcarriers becomes $\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{n_\mathrm{r}} = \left[\hat{\mathbf{h}}^{(i),(\mathrm{L}),\mathrm{T}}_{1,n_\mathrm{r}} \dots \hat{\mathbf{h}}^{(i),(\mathrm{L}),\mathrm{T}}_{N_T,n_\mathrm{r}}\right]^{\mathrm{T}}$. Furthermore, we derive iterative LS $\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{\mathrm{ILS},n_\mathrm{r}}$ over all subcarriers and define it over the interval of L consecutive subcarriers as above

$$\begin{cases} \left[[\hat{\mathbf{h}}^{(i),\mathrm{ILS}}_{1,n_\mathrm{r}}]^{\mathrm{T}}_{1:L} \dots [\hat{\mathbf{h}}^{(i),\mathrm{ILS}}_{N_T,n_\mathrm{r}}]^{\mathrm{T}}_{1:L}\right]^{\mathrm{T}} \\ \left[[\hat{\mathbf{h}}^{(i),\mathrm{ILS}}_{1,n_\mathrm{r}}]^{\mathrm{T}}_{k-\lfloor\frac{L-1}{2}\rfloor:k+\lceil\frac{L-1}{2}\rceil} \dots [\hat{\mathbf{h}}^{(i),\mathrm{ILS}}_{N_T,n_\mathrm{r}}]^{\mathrm{T}}_{k-\lfloor\frac{L-1}{2}\rfloor:k+\lceil\frac{L-1}{2}\rceil}\right]^{\mathrm{T}} \\ \left[[\hat{\mathbf{h}}^{(i),\mathrm{ILS}}_{1,n_\mathrm{r}}]^{\mathrm{T}}_{N_K-L+1:N_K} \dots [\hat{\mathbf{h}}^{(i),\mathrm{ILS}}_{N_T,n_\mathrm{r}}]^{\mathrm{T}}_{N_K-L+1:N_K}\right]^{\mathrm{T}} \end{cases} \quad (16)$$

Finally, we calculate the iterative ALMMSE channel estimate by filtering the iterative LS estimate $\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{\mathrm{ILS},n_\mathrm{r}}$ with the filtering matrix $\mathbf{A}^{(\mathrm{L})}_{\mathrm{IALMMSE}}$ as follows $\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{n_\mathrm{r}} = \mathbf{A}^{(\mathrm{L})}_{\mathrm{IALMMSE}}\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{\mathrm{ILS},n_\mathrm{r}}$, where the filtering matrix is given by

$$\mathbf{A}^{(\mathrm{L})}_{\mathrm{IALMMSE}} = \hat{\mathbf{R}}^{(\mathrm{L})}_{\mathbf{h}_{n_\mathrm{r}},\mathbf{h}_{n_\mathrm{r}}}\left(\hat{\mathbf{R}}^{(\mathrm{L})}_{\mathbf{h}_{n_\mathrm{r}},\mathbf{h}_{n_\mathrm{r}}} + \sigma_n^2\Big((\tilde{\mathbf{S}}^{\mathrm{H}}\tilde{\mathbf{S}})^{-1}\Big)^{(\mathrm{L})}\right)^{-1} \quad (17)$$

Then, we select the k-th subcarrier for the estimated channel vector $\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{n_\mathrm{r}}$

$$\hat{h}^{(i)}_{n_\mathrm{r},k} = \begin{cases} \left[[\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{1,n_\mathrm{r}}]^{\mathrm{T}}_{k} \dots [\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{N_T,n_\mathrm{r}}]^{\mathrm{T}}_{k}\right]^{\mathrm{T}} \\ \left[[\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{1,n_\mathrm{r}}]^{\mathrm{T}}_{\lceil\frac{L+1}{2}\rceil} \dots [\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{N_T,n_\mathrm{r}}]^{\mathrm{T}}_{\lceil\frac{L+1}{2}\rceil}\right]^{\mathrm{T}} \\ \left[[\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{1,n_\mathrm{r}}]^{\mathrm{T}}_{L+k-N_K} \dots [\hat{\mathbf{h}}^{(i),(\mathrm{L})}_{N_T,n_\mathrm{r}}]^{\mathrm{T}}_{L+k-N_K}\right]^{\mathrm{T}} \end{cases} \quad (18)$$

IV. Simulation Results

In this section, we discuss the performance of the iterative channel estimation algorithms based on simulation results. We compare the iterative approaches with the pilot based approaches. Naturally, the question whether to apply a-posteriori, extrinsic or hard estimated data symbols in the feedback loop arises. In the following, we investigate the impact of processing this feedback information for channel estimation. We distinguish between initial (pilot based), a-posteriori, extrinsic, hard and perfect cases. In the initial case, in the plots denoted by It.1, the channel is estimated based only on pilot symbols. When the system knows the channel state information perfectly then this is recognized as the perfect case and denoted as PERFECT. The utilization of the a-posteriori, extrinsic and hard feedback information in the feedback loop is denoted (in the front of the abbreviations of the channel estimation algorithms) by app, ext and $hard$, respectively. The channel estimators are compared with each other in terms of MSE and throughput versus Signal to Noise Ratio (SNR). All results are obtained using the LTE Link Level Simulator version "r1089" [15], [16]. Table I presents the most important simulation settings.

Parameter	Value
Bandwidth	1.4 MHz
Number of subframes	1000
Number of iterations	5
Number of closest subcarrier L	12 (for ALMMSE)
Number of transmit antennas	2
Number of receive antennas	2
Channel Quality Indicator (CQI)	4,7,10
Modulation order	4-,16-, and 64-QAM
Transmission mode	Transmit Diversity (TxD)
Channel type	ITU PedB
Detector	SSD

TABLE I
SIMULATION SETTINGS

Until otherwise stated, for an iterative ALMMSE estimator we chose to use L=12 closest subcarrier for the following simulations. Figure 3 depicts the throughput versus the number of iterations for the three channel estimation algorithms when a-posteriori feedback information is utilized. The highest performance gain is achieved from the first to the second iteration. Then, from the second iteration on the throughput is increased slightly and after the fifth iteration the performance is improved negligibly. Therefore, in the following simulations only five iterations between the channel estimator and the decoding unit are considered. For further analysis of the iterative channel estimation algorithms, we consider only the 4-QAM modulation order. The extension to other modulation orders is straightforward. For comparison purposes, in Figure 4, the throughput versus SNR for 4-QAM, 16-QAM and 64-QAM modulation orders, when a-posteriori feedback information is utilized, is plotted. As for 4-QAM, approximately the same performance pattern are also observed for 16-QAM and 64-QAM.

A. A-posteriori, extrinsic and hard feedback information

In this subsection, we analyse the performance of the system when the a-posteriori, extrinsic and hard feedback information are utilized in the feedback loop. Figure 5 depicts the throughput (analysed at throughput 0.4 Mbit/s) versus SNR, where the channel estimation algorithms (initial and iterative case) for the above mentioned feedback information after the fifth iteration are compared to each other. We observe that the best system performance is achieved when a-posteriori feedback information is utilized in the feedback loop. The improvement of the a-posteriori case with respect to the extrinsic case is approximately 0.1 dB for the three iterative channel estimation algorithms. On the other hand, if we observe the case when hard feedback information is utilized we see that the iterative hard LS estimator does not improve with respect to the initial (pilot based) case, iterative hard ALMMSE estimator obtains approximately 0.3 dB SNR gain and iterative hard LMMSE estimator 0.15 dB gain compared to their initial cases. Clearly, the soft (a-posteriori or extrinsic) case outperforms the hard case. A-posteriori LS estimator obtains approximately 0.35 dB SNR gain, a-posteriori ALMMSE estimator 0.8 dB and a-posteriori LMMSE estimator 0.4 dB gain with respect to the hard case. Obviously, the major drawback utilizing hard feedback information is the error propagation which is more severe compared to the soft case. Since the highest system performance is achieved when a-posteriori information is utilized, in the following, we compare the channel estimation algorithms to each other for the a-posteriori feedback information case.

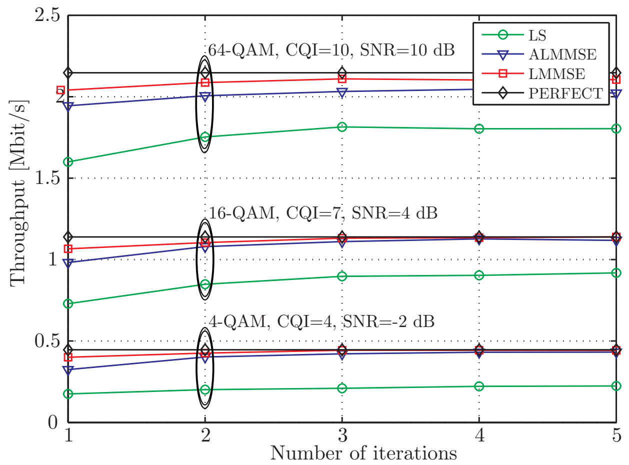

Fig. 3. Throughput versus the number of iterations for a-posteriori feedback information.

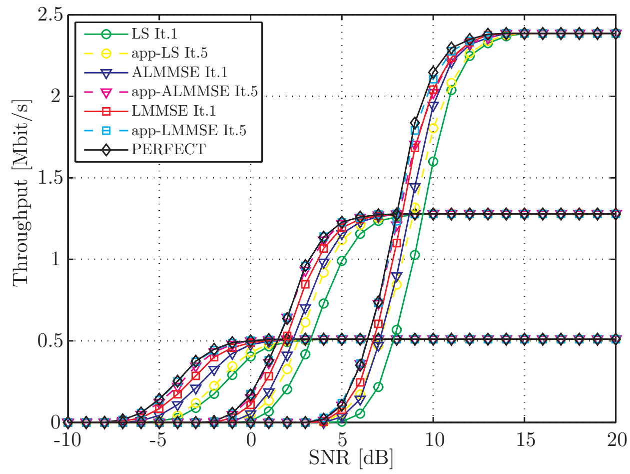

Fig. 4. Throughput for 4-,16- and 64-QAM modulation orders for a-posteriori feedback information.

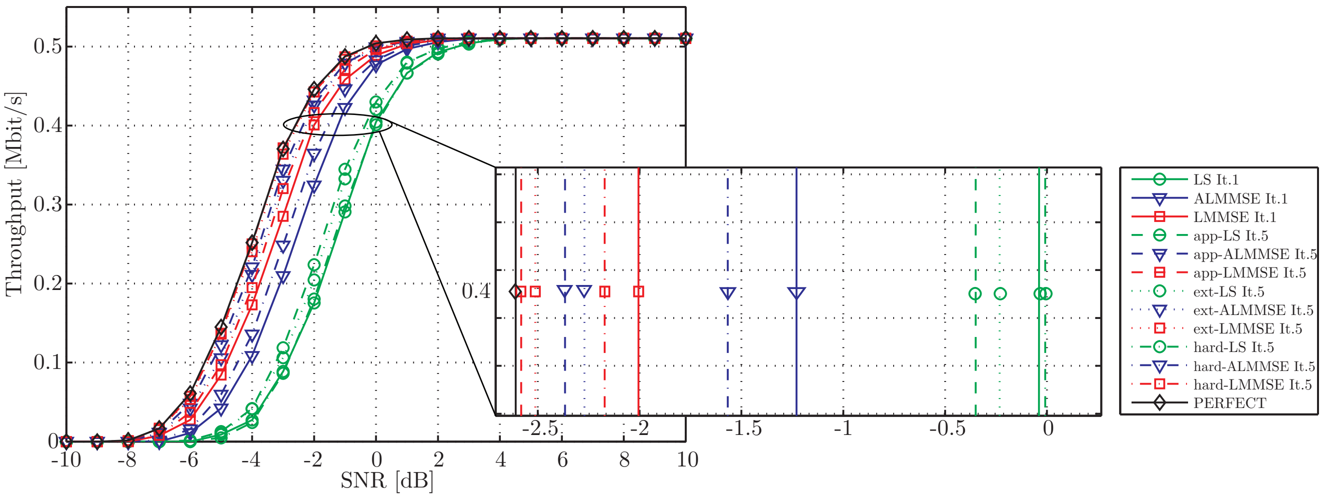

Fig. 5. Comparison for a-posteriori, extrinsic and hard feedback information for the channel estimators in terms of throughput.

B. Comparison of the channel estimation algorithms for a-posteriori feedback information

First, we investigate the ALMMSE estimator in dependence of the closest subcarrier index L. In Figure 6 and Figure 7, the performance of the iterative (for the fifth iteration) ALMMSE estimator for different values of L, and in comparison to LS and LMMSE estimators, is plotted. It can be observed that with increasing L the MSE decreases while the throughput increases. For $L = N_K$, the iterative ALMMSE estimator performance is equal to the iterative LMMSE estimator. It is obvious that with increasing L also the complexity of the estimator increases. This fact allows adjusting the performance and complexity of the estimator to achieve a good trade-off. We see that even for L=2 consecutive subcarriers the iterative ALMMSE estimator performs better than the iterative LS estimator. Increasing the number L more than 12 the performance of the system is increased negligibly as compared from L=2 to L=12. Therefore, we conclude that the choice L=12 is a good trade-off between complexity and performance.

Figure 8 depicts the MSE versus SNR for the three iterative algorithms (after the fifth iteration) in comparison to their initial cases. For the three iterative channel estimation algorithms, the MSE considerably decreases after five iterations compared to the initial case. The MSE decreases most rapidly for the LMMSE, followed by the ALMMSE, and then the LS. Figure 9 depicts the throughput versus SNR. For iterative LS we observe approximately 0.3 dB SNR gain, for iterative ALMMSE 1.1 dB and for iterative LMMSE 0.6 dB with respect to their initial cases. Iterative ALMMSE outperforms the initial LMMSE for about 0.35 dB and loses 0.25 dB compared to iterative LMMSE. On the other hand, iterative LMMSE channel estimator performs approximately equal to the system with perfect channel knowledge, with an SNR loss of 0.05 dB. Table II and III summarize in more detail the above SNR gain and loss values, measured at throughput of 0.4 Mbit/s. Each iterative estimator algorithm is compared with its initial estimator as well as with the other iterative estimators and also with the system with perfect channel knowledge.

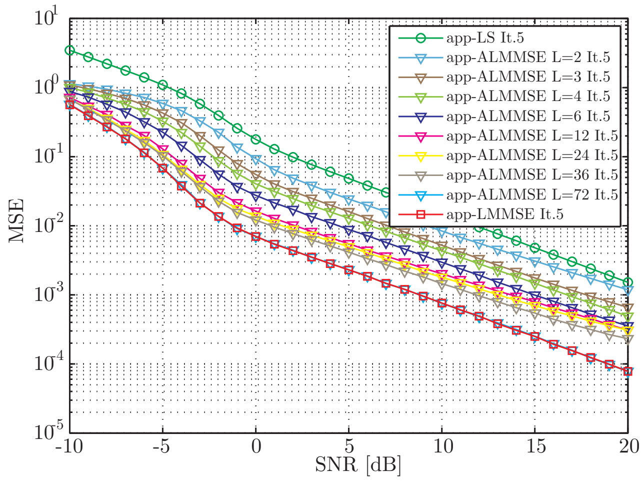

Fig. 6. Iterative ALMMSE estimator for different values of L compared to iterative LS and LMMSE estimators in terms of MSE.

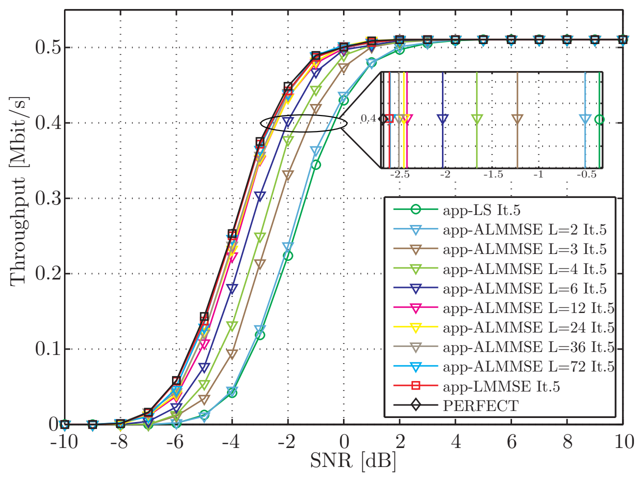

Fig. 7. Iterative ALMMSE estimator for different values of L compared to iterative LS and LMMSE estimators in terms of throughput.

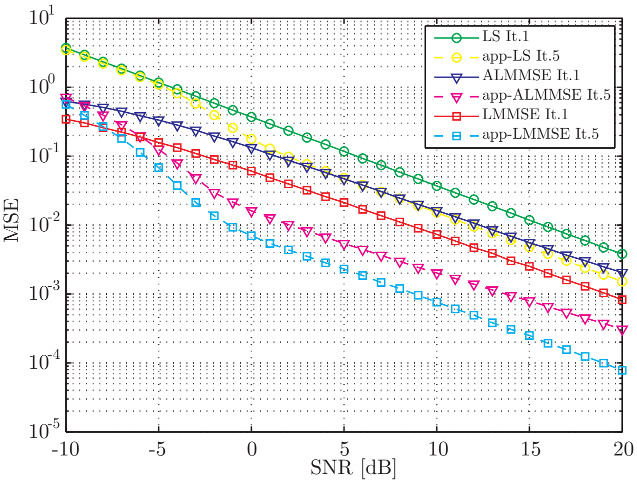

Fig. 8. Comparison of initial (pilot based) and iterative LS, ALMMSE and LMMSE estimators in terms of MSE for a-posteriori feedback information.

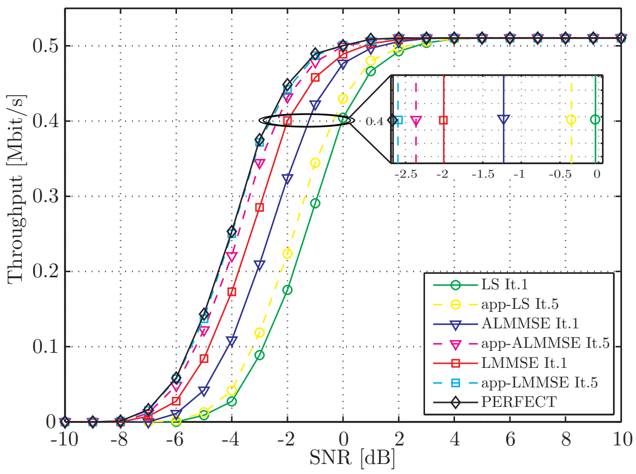

Fig. 9. Comparison of initial (pilot based) and iterative LS, ALMMSE and LMMSE estimators in terms of throughput for a-posteriori feedback information.

SNR gain values			
Iter. versus Init.	Init. LS	Init. ALMMSE	Init. LMMSE
Iter. LS	0.3 dB	-	-
Iter. ALMMSE	2.3 dB	1.1 dB	0.35 dB
Iter. LMMSE	2.5 dB	1.35 dB	0.6 dB

TABLE II
SUMMARY OF SNR GAIN VALUES OF THE ITERATIVE OVER THE INITIAL (PILOT BASED) ESTIMATORS

SNR loss values				
Iter. versus Iter.	Iter. LS	Iter. ALMMSE	Iter. LMMSE	Perfect
Iter. LS	-	2 dB	2.25 dB	2.3 dB
Iter. ALMMSE	-	-	0.25 dB	0.3 dB
Iter. LMMSE	-	-	-	0.05 dB

TABLE III
SUMMARY OF SNR LOSS VALUES OF THE ITERATIVE ESTIMATORS OVER EACH OTHER AND OVER THE PERFECT CASE

V. CONCLUSION

We showed the calculation procedure of the soft estimated data symbols for the LTE standard. Furthermore, we derived different iterative channel estimation algorithms with different performance behaviours and complexities. We investigated the impact of processing the a-posteriori, extrinsic and hard feedback information for the channel estimation. The system with iterative channel estimation achieves a considerable performance improvement in terms of MSE and throughput compared to the system with non-iterative channel estimation. The best performance is achieved when a-posteriori feedback information is utilized. Among the estimators presented in this paper, the iterative LS estimator achieves the lowest performance. The iterative LMMSE estimator achieves approximately the performance of the system with perfect channel knowledge with an SNR loss of 0.05 dB. Despite the fact that the performance of the iterative LMMSE estimator is excellent, its complexity is very high for a real-time implementation. Therefore, the goal of iterative ALMMSE estimator is to reduce the complexity of the iterative LMMSE estimator and in turn to preserve its performance. The iterative ALMMSE estimator exploits only the correlation between the L closest subcarriers. Thus, there is a trade-off between complexity and performance by varying the parameter L. Iterative ALMMSE estimator with L=12 obtains approximately 1.1 dB SNR gain with respect to the initial case and loses approximately 0.25 dB compared to the iterative LMMSE estimator.

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