# Effective HARQ Code Rate Modeling for LTE

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In this paper we propose an intuitive model for the effective code rate of HARQ-coded LTE transmissions. The model is derived from the circular structure of the rate matching buffer employed in LTE. Comparison with prior results, which were obtained by empirically extracting the code rates from the rate matcher, show the accuracy of the presented, entirely analytical, model.

*Introduction:* As initially stated in [1], and extended to a more practical LTE scenario in [2], the decoding probability of a data packed encoded with HARQ can be effectively modeled by jointly considering the set of retransmitted packets as a single encoded packed, coded with an inner code of rate $r_n$ and an outer repetition code of rate $1/N_n$. However, the $r_n$ and $N_n$ code rates are dependent on the specific HARQ type and implementation a transmission system implements.

In this work, we focus on the case of LTE, where a circular buffer [3] is employed for joint rate matching and generation of different retransmission versions of the data for HARQ (re-)transmissions [4].

Prior work [1, 2] did not focus in how to analytically obtain the values of $r_n$ and $1/N_n$, but rather on the modeling of HARQ on a mutual information level and the modeling of the LTE HARQ, respectively.

In the specific case of [2], the equivalent puncturing matrices were extracted from a standards-compliant implementation [5]. Thus, these values were valid only for a specific set of modulation and coding schemes and retransmission order. With a generalized model such as the one proposed here, the values can be analytically derived, which enables HARQ models to employ any arbitrary retransmission ordering, as well as, if necessary, flexibly allowing any arbitrary code rate in terms of $r_n$ and $N_n$.

*HARQ modeling:* According to [2], it is possible to model the channel coding and rate matching procedure for the $n$-th HARQ transmission $(n=0,1,\ldots,N)$ using a combination of an inner channel code processing unrepeated bits with a code rate of $r_n$, and an outer repetition code with rate $1/N_n$ working on the output of the preceding channel code. Using the formulation described in [6, 7] and [8], [2] pointed out that the normalized accumulated mutual information of a jointly-processed HARQ data packet, denoted as $I^*$, can be approximated, for the case where all retransmissions experience the same Signal-to-Noise Ratio (SNR) $\gamma$, as

$$I^*\left(N_n,\gamma,r_n\right) \;=\; \frac{I_m\left(N_n,\gamma\right)}{r_n}, \tag{1}$$

where $I_m(.)$ is the Bit Interleaved Coded Modulation (BICM) capacity for the employed modulation, which encodes $m$ bits per symbol. From information theory, we know that the Block Error Rate (BLER) can be approximated by the probability

$$\varepsilon \;=\; \mathbb{P}\left[\frac{I_m\left(N_n,\gamma\right)}{r_n} < 1\right], \tag{2}$$

which can be also obtained by empirical BLER curves as

$$\varepsilon \;=\; \mathrm{BLER}_{M_q,r_n}\left(N_n,\gamma\right), \tag{3}$$

where $M_q$ is the modulation order associated with the $q$-th standard-defined combination of code rate and modulation scheme, termed CQI [9], with code rate $r_n$, and $\gamma$ is the SNR. Eqn. (3) states that, at a given $\gamma$ and modulation, the effect of HARQ on the BLER can be encapsulated in the two parameters $r_n$ and $N_n$. In other words, it is possible to model the error performance of an arbitrary CQI with any code rate by decoupling the effect of the parity-based inner code and repetition using parameters $r_n$ and $N_n$ respectively.

*Analytical derivation of $r_n$ and $N_n$:* As pointed out in [2], in order to be able to model the HARQ combining in this manner, the $r_n$ and $N_n$ code rates need to be obtained. In [2], they are numerically obtained by reverse-engineering the output of the LTE rate matcher.

In this section, simple closed-form expressions for these parameters are presented, which enables the respective values to be computed analytically.

Let $n$ be the transmission index, where $n=0$ corresponds to the initial transmission, and $n=1,2,3$ corresponds to the oissible retransmissions, up to three, as specified in the LTE standard [4]. Let $r'$ be the code rate which corresponds to CQI $q$ (not to be confused with the combined inner code rate $r_n$ or outer code rate $N_n$), the relationship between the code rate $r'$ and the number of systematic bits $X$ is

$$r' \;=\; \frac{X}{3X-Y} \;=\; \frac{1}{3-\eta}, \tag{4}$$

where $Y$ corresponds to the number of punctured bits from the mother code of rate $1/3$, and $\eta$ is the ratio of the number of punctured bit relative to the systematic bits. Here, in (4) the effect of the tail bits is omitted, as $X$ is assumed to be much larger than the number of tail bits. Also for cases where $r' < 1/3$, i.e., bits are repeated, $Y$ is negative.

*Theorem 1: If the redundancy versions of 0 to 3 are invoked sequentially, the level of repetition $N_n$ and the code rate $r_n$ associated with transmission index $n$ can be computed as*

$$N_n \;=\; \max\left(r_n n\left(3-\eta\right),\, 1\right), \qquad n=0,1,2,\ldots,3 \tag{5}$$

$$r_n \;=\; \max\left(\frac{4}{3n+4\left(3-\eta\right)},\, \frac{1}{3}\right), \qquad n=0,1,2,\ldots,3. \tag{6}$$

*Proof:* Let $T$ bits be taken from the circular buffer [4]:

$$T \;=\; 3X-Y = X\left(3-\eta\right), \tag{7}$$

as shown in Fig. 1, and starting at $n=0$, with $Y$ punctured bits. Let the code rate $r_n$, based only on the parity bits, be the ratio of the systematic bits to the total number of systematic + parity bits transmitted. Without loss of generality, we choose the starting position of the redundancy version index $n$ [10] within the circular buffer to be

$$S_n \;=\; k_0 + n\left(\frac{3X}{4}\right), \; n=0,1,\ldots,3, \tag{8}$$

where $k_0$ is the non-zero starting position given in [4]. The relationship between variable $X$, $Y$, $T$, $S_n$, and $k_0$ are shown in Fig. 1. For the sake of simplicity and without loss of generality, we can assume $k_0 = 0$.

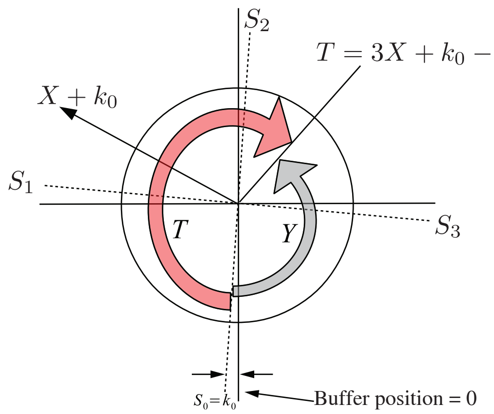


**Fig. 1** *Transmit data $T$ taken from the circular buffer starting at $rv=0$ with $Y$ punctured bits.*

From Fig. 1, it can be seen that the code rate for the first transmission is given by $r_0 = X/T$ as long as $T \leq 3X$. On the other hand, if $T > 3X$, the total number of parity bits are $2X$, and the code rate becomes $r_0 = 1/3$. Thus, the code rate is given by

$$r_0 \quad = \quad \max\left(\frac{X}{T}, \frac{1}{3}\right). \tag{9}$$

Assuming the redundancy version sequence is 0, 1, 2, 3, the code rate for the first retransmission is given by

$$r_1 \quad = \quad \max\left(\frac{X}{S_1+T}, \frac{1}{3}\right). \tag{10}$$

Using a similar reasoning, after $n$-th retransmission, and using (7) and (8), the code rate $r_n$ becomes

$$r_n \;=\; \max\left(\frac{X}{S_n+T}, \frac{1}{3}\right) \;=\; \max\left(\frac{1}{a_n+\left(3-\eta\right)}, \frac{1}{3}\right), \tag{11}$$

where $a_n = 3n/4$. For other retransmission orders, the same method applies with some minor modifications in the denominator of the first term in (11). The redundancy due to repetition can be expressed as

$$N_n = \frac{r_n}{X/(nT)}, \quad (12)$$

due to the fact that the the total number of transmitted bits includes both systematic bits, parity bits, as well as repeated bits. Using (7) and (8),

$$N_n = r_n n (3 - \eta). \quad (13)$$

Note that it is possible for (13) to be less than unity if no bits are repeated but some bits are punctured. In this case, the total number of transmitted bits should be the same as the total number of coded bits. In order to take the above-mentioned case into account, (13) is modified to be

$$N_n = \max(r_n n (3 - \eta), 1). \quad (14)$$

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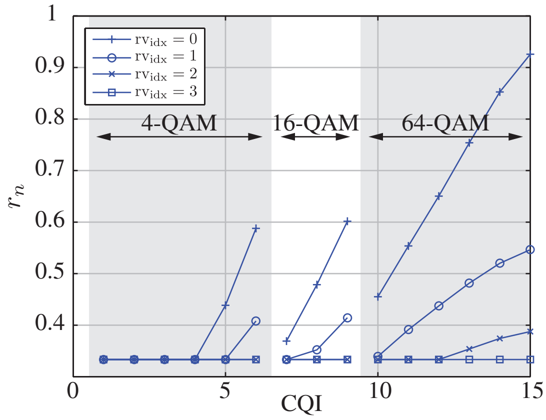


Fig. 2. Outer code rate $r_n$ as a function of CQI index.

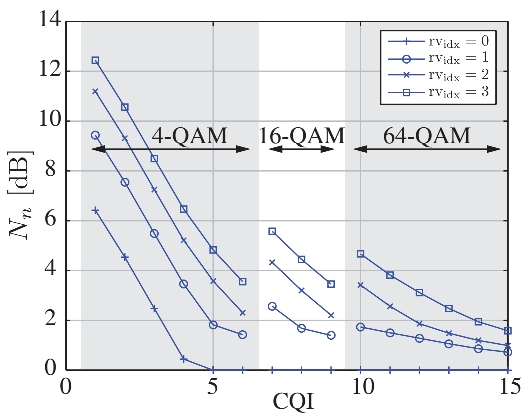


**Fig. 3** *Repetition gain $N_n$ for transmission $n$ in dB as a function of CQI index. Note that $N_n$ is expressed, as opposed to in (14), where it is in linear scale.*

Fig. 2 and Fig. 3 show HARQ model code rates for the inner code ($r_n$) and the outer repetition code ($N_n$) as a function of the 15 CQI indexes specified in LTE [9]. These results match those empirically extracted from an LTE rate matcher implementation in [2]. In this comparison, a fixed sequence of redundancy versions are assumed ($\mathrm{rv_{idx}} = 0, 1, 2, 3$), as that was the only sequence considered in [2]. However, the presented method can easily be adapted to derive expressions for $r_n$ and $N_n$ for other sequences.

*Conclusion:* In this paper, an analytical model for the effective code rate of the joint decoding of an HARQ-transmitted data packet employing the LTE standard has been derived. The model is designed after the LTE rate matcher and outputs the rates of the modeled inner channel code rate ($r_n$), as well as the outer repetition code, both of which are employed to model the HARQ gain. The presented analytical model matches prior simulation-obtained results, and offers the benefit of allowing arbitrary code rates.

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