Performance Evaluation of Differential Modulation in LTE - Downlink

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***Abstract*—This paper evaluates the performance of differential modulation applied to the 3GPP UMTS Long Term Evolution downlink. With differential modulation, reference symbols for channel estimation are no longer necessary, thus increasing the potential maximum spectral efficiency. We present two basic modulation schemes and compare their performance to the coherent detection employed in Long Term Evolution downlink transmissions. The focus specifically lies on single input single output transmissions in single user scenarios.**

***Index Terms*—LTE, SISO, Differential Modulation**

I. Introduction

Current wireless communication systems like 3GPP UMTS Long Term Evolution (LTE) [1] require channel estimation for coherent detection. Channel estimation can become computationally demanding, especially for rapidly varying channels. Furthermore the reference symbols (RSs) required for channel estimation cannot be used for data transmission, which decreases the maximum spectral efficiency. A way to increase the spectral efficiency is to reduce the RS overhead for channel estimation. Noncoherent detection offers the possibility to receive information without knowledge of the channels, thus the RS for channel estimation are no longer required. Although this can provide a higher spectral efficiency, the disadvantage of noncoherent detection is a performance loss compared to coherent detection in terms of signal-to-noise ratio (SNR). In [2–4] the performance of differential modulation was investigated for different transmission systems. To our knowledge, however, the performance of differential modulation in LTE has not yet been evaluated. In this paper we consider differential modulation with noncoherent detection for the LTE downlink. The focus specifically lies on single input single output (SISO) transmissions in single user scenarios.

The remainder of this paper is organized as follows: Section II describes how differential modulation can be applied to the LTE downlink and presents the mathematical system model. Furthermore we discuss two noncoherent detection strategies which are denoted as conventional noncoherent detection (CND) and multiple symbol differential sphere decoding (MSDSD). In Section III we show how to choose the correct differential modulation scheme depending on the channel statistics and Section IV presents the simulation results. Finally, Section V concludes the presented work.

The following notation is used throughout this paper: $E\{x\}$ denotes the expected value of random variable x, diag$\{\mathbf{x}\}$ represents a diagonal matrix with components of the vector $\mathbf{x}$ on its main diagonal and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the conjugate transpose of a matrix, respectively. Furthermore, det$(\cdot)$ is the determinant of a square matrix and exp$(\cdot)$ is the natural exponential function.

II. System Model

The LTE downlink is based on Orthogonal Frequency Division Multiplexing (OFDM). OFDM systems convert a broadband frequency selective wireless channel into orthogonal narrowband frequency flat channels (subcarriers), by means of a Discrete Fourier Transform (DFT) and application of a cyclic prefix (CP).

The smallest physical resource in LTE is called resource element (RE) and represents one subcarrier during one OFDM symbol. The REs are grouped to resource blocks (RBs). Each RB consists of 12 consecutive subcarriers in the frequency domain and one 0.5 ms slot in the time domain. Two of these resource blocks are grouped to a resource block pair (RBP), which is the minimum scheduling unit in the LTE downlink. To explain how to apply differential modulation in the LTE downlink, we consider the RBP in the concept of a time-frequency grid as depicted in Fig. 1. The vertical axis f of the grid represents the frequency direction (i.e., subcarriers), the horizontal axis t the time direction (i.e., OFDM symbols). Each square of the grid is one RE which contains a modulated symbol. In the case of a normal CP length a RBP consists of 168 REs, i.e., 12 subcarriers in the frequency direction and 14 OFDM symbos in the time direction. In the following explanation, a RE of the n-th OFDM symbol and the k-th subcarrier is indexed as $x_{n,k}$ with $n \in \{1, 2, ..., 14\}$ and $k \in \{1, 2, ..., 12\}$.

For differential modulation in the LTE downlink we consider two modulation schemes:

- Frequency first modulation
- Time first modulation

The operation of the frequency first modulation scheme is depicted in Fig. 1. Both modulation schemes use the RE $x_{1,1}$ as reference symbol, marked light grey in Fig. 1. It does not convey information and is utilized as initial value of the data transmission. In the frequency first modulation scheme the differential modulation begins with modulating in the frequency direction

$$x_{n,k} = x_{n,k-1} d_{n,k} \ , \ k \in \{2, 3, ..., 12\} \ , \ n = 1. \qquad (1)$$

Here, $d_{n,k}$ represents an information symbol. The information is either transmitted by a phase change or by an amplitude and phase change between $x_{n,k}$ and $x_{n,k-1}$. This is referred to as Differential Phase Shift Keying (DPSK) [5] and Differential Amplitude and Phase Shift Keying (DAPSK) [3], respectively. After modulating in the frequency direction we obtain the data symbols $x_{1,1}$ to $x_{1,12}$. These symbols are subsequently used as reference symbols to modulate in the time direction

$$x_{n,k} = x_{n-1,k} d_{n,k} \ , \ n \in \{2, 3, ..., 14\} \ , \ k \in \{1, 2, ..., 12\} \,. \tag{2}$$

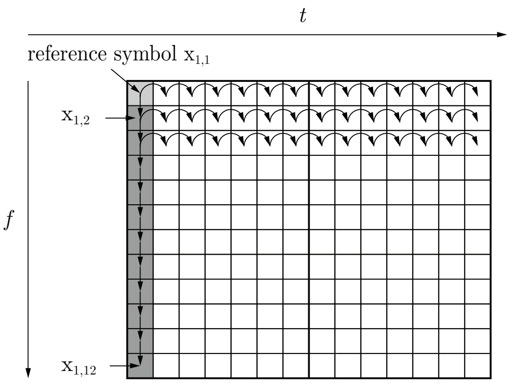

Fig. 1. Frequency first modulation scheme in the LTE downlink

The time first modulation scheme works similar to the frequency first modulation scheme, except that the differential modulation starts in the time direction and uses the obtained data symbols for modulating in the frequency direction.

After differential modulation the data symbols are transmitted to the receiver. For the channel we assume in general a finite impulse response linear time-variant (LTV) channel that fullfills the wide-sense stationary uncorrelated scattering (WSSUS) [6] assumption. Furthermore, the CP is assumed long enough so that no inter-symbol interference occurs. For an LTV channel with negligible inter-carrier interference the received data symbol in the frequency domain can be written as

$$y_{n,k} = H_{n,k} x_{n,k} + v_{n,k}, \tag{3}$$

where $H_{n,k}$ denotes the channel at subcarrier k and OFDM symbol index n. The noise $v_{n,k}$ is assumed to be independent zero mean complex Gaussian noise with variance σ_v^2.

The received data symbols $y_{n,k}$ are used to noncoherently detect the transmitted information symbols $d_{n,k}$. In the frequency first modulation scheme the detection begins in the frequency direction by estimating the information symbols of the first OFDM symbol. This is followed by estimating the remaining information symbols on each subcarrier in the time direction.

For the detection in frequency direction, in the most basic noncoherent detection scheme we utilize two data symbols to detect one information symbol $\hat{d}_{n,k}$ as

$$\hat{d}_{n,k} = y_{n,k} / y_{n,k-1}. \tag{4}$$

This is referred to as CND.

Alternatively, we can also apply N received data symbols to jointly detect $N-1$ information symbols. This is denoted as multiple symbol differential detection (MSDD) and was presented by Divsalar et. al. in [7]. In [8], Ho et. al. demonstrated that the performance of MSDD can further be improved if the correlation of the channel is exploited. Unfortunately, the complexity of MSDD increases exponentially with the number of observed data symbols N. In [9, 10] Lampe and Pauli showed how to reduce the complexity by the application of sphere decoding (SD). This is referred to as MSDSD.

In the following mathematical description MSDSD is considered in the frequency direction, following the derivations in [9]. For the sake of simplicity of notation the OFDM symbol index n is omitted.

Let $\mathbf{x} = (x_k, x_{k+1}, ..., x_{k+N-1})^T$ be the vector of N transmitted data symbols, $\mathbf{y} = (y_k, y_{k+1}, ..., y_{k+N-1})^T$ the vector of N received data symbols, $\mathbf{h} = (H_k, H_{k+1}, \cdots, H_{k+N-1})^T$ a vector of N channel coefficients and $\mathbf{v} = (v_k, v_{k+1}, ..., v_{k+N-1})^T$ the corresponding noise vector. The received data symbol vector $\mathbf{y}$ is input to a maximum likelihood sequence estimator (MLSE). The ML decision rule is obtained as [9]

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathcal{X}^{\mathbf{N}}}{\text{argmax}} \{p(\mathbf{y}|\mathbf{x})\} = \underset{\mathbf{x} \in \mathcal{X}^{\mathbf{N}}}{\text{argmax}} \left\{ \frac{\exp(-\mathbf{y}^H \mathbf{R}_{\mathbf{yy}}^{-1} \mathbf{y})}{\pi^N \det(\mathbf{R}_{\mathbf{yy}})} \right\}, \tag{5}$$

where $\mathbf{R}_{\mathbf{yy}}$ is the autocorrelation matrix of $\mathbf{y}$ conditioned on the transmitted data vector $\mathbf{x}$ and $\mathcal{X}$ is the utilized symbol alphabet of size M. The autocorrelation matrix $\mathbf{R}_{\mathbf{yy}}$ can be calculated as

$$\begin{aligned} \mathbf{R}_{\mathbf{yy}} &= E\{\mathbf{y}\mathbf{y}^H|\mathbf{x}\} = E\{(\mathbf{X}\mathbf{h} + \mathbf{v})(\mathbf{X}\mathbf{h} + \mathbf{v})^H|\mathbf{x}\} \\ &= \mathbf{X}\mathbf{\Sigma}_{\mathbf{hh}}\mathbf{X}^H + \sigma_v^2 \mathbf{I}_N, \end{aligned} \tag{6}$$

where $\mathbf{\Sigma}_{\mathbf{hh}} = E\{\mathbf{h}\mathbf{h}^H\}$ is the autocorrelation matrix of the channel, $\mathbf{I}_N$ is an $N \times N$ identity matrix and $\mathbf{X} = \text{diag}(\mathbf{x})$. In Equation (6) we used the fact that $\mathbf{X}$ becomes deterministic, if conditioned on $\mathbf{x}$ and that the noise $\mathbf{v}$ is assumed to be i.i.d. zero mean.

If $\mathcal{X}$ is an M-Phase Shift Keying (PSK) symbol alphabet, $\mathbf{X}$ is unitary and Equation (6) can be rewritten as

$$\mathbf{R}_{\mathbf{yy}} = \mathbf{X}(\mathbf{\Sigma}_{\mathbf{hh}} + \sigma_v^2 \mathbf{I}_N)\mathbf{X}^H. \tag{7}$$

In this case $\det(\mathbf{R}_{\mathbf{yy}})$ is constant and independent of $\mathbf{X}$ which simplifies the decision rule Equation (5) to

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathcal{X}^N}{\text{argmin}} \{\mathbf{y}^H \mathbf{R}_{\mathbf{yy}}^{-1} \mathbf{y}\}. \tag{8}$$

Defining $\mathbf{C} \triangleq \mathbf{\Sigma}_{\mathbf{hh}} + \sigma_v^2 \mathbf{I}_N$ and with the Cholesky factorization of the inverse matrix $\mathbf{C}^{-1} = \mathbf{L}\mathbf{L}^H$ to the the lower triangular matrix $\mathbf{L}$, we can rewrite Equation (8) as [9]

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathcal{X}^N}{\text{argmin}} \{||\mathbf{U}\mathbf{x}||^2\}. \tag{9}$$

Here we used $\mathbf{Y}_d = \text{diag}(\mathbf{y})$ and defined the upper triangular matrix $\mathbf{U} \triangleq \mathbf{L}^T \mathbf{Y}_d^H$. This is regarded as shortest vector problem which can be solved by application of the SD algorithm. A detailed description on how to implement MSDSD is found in [9, 10].

For M-Amplitude Phase Shift Keying (APSK) the SD algorithm can not be directly applied since $\mathbf{X}$ is not unitary. In this case let us define $\mathbf{X} \triangleq \mathbf{A}\mathbf{S}$, where $\mathbf{A} = \text{diag}(\mathbf{a})$ and $\mathbf{S} = \text{diag}(\mathbf{s})$ correspond to the transmitted amplitude and phase data symbols, respectively. For diagonal matrices $\mathbf{AS} = \mathbf{SA}$ and the autocorrelation matrix $\mathbf{R_{yy}}$ in Equation (6) can be written as [11]

$$\begin{aligned}\mathbf{R_{yy}} &= \mathbf{SA\Sigma_{hh}}(\mathbf{AS})^H + \sigma_v^2\mathbf{I}_N \\ &= \mathbf{S}(\tilde{\mathbf{\Sigma}}_{\mathbf{hh}} + \sigma_v^2\mathbf{I}_N)\mathbf{S}^H, \end{aligned} \tag{10}$$

with $\tilde{\mathbf{\Sigma}}_{\mathbf{hh}} = \mathbf{A\Sigma_{hh}A}^H$. The ML decision rule writes as

$$\hat{\mathbf{x}} = \hat{\mathbf{a}}^T \cdot \hat{\mathbf{s}} = \underset{\mathbf{a}\in\mathcal{A}^N,\, \mathbf{s}\in\mathcal{S}^N}{\operatorname{argmin}} \left\{ \frac{||\mathbf{Us}||^2}{\det(\mathbf{R_{yy}})} \right\}. \tag{11}$$

where $\mathcal{A}$ and $\mathcal{S}$ represent the amplitude and phase symbol alphabet, respectively, and $\mathcal{X} = \mathcal{A} \cup \mathcal{S}$. In the joint detection process of amplitude and phase, the amplitude vector $\mathbf{a}$ is varied inside the symbol alphabet $\mathcal{A}$ and the SD algorithm is applied to detect the phase vector $\hat{\mathbf{s}}$.

With the relation of Equation (1) and the corresponding symbol alphabet we obtain the transmitted information symbols d. A further detailed description can be found in [12].

The principle of MSDSD can be applied both in the frequency and in the time direction. For detection in the frequency direction, $\mathbf{\Sigma_{hh}}$ describes the correlation between subcarriers. For an linear time-invariant (LTI) channel it can be calculated as

$$\mathbf{\Sigma}^{(f)}_{\mathbf{hh}} = \mathbf{W}\mathbf{H}_{PDP}\mathbf{W}^H, \tag{12}$$

where $\mathbf{W}$ is a $K \times K$ DFT matrix and K stands for the size of the DFT. The superscript f indicates that the autocorrelation is considered in the frequency direction and the matrix $\mathbf{H}_{PDP}$ is of size $K \times K$ and has the coefficients of the power delay profile (PDP) on its diagonal. The PDP is of length L. In practical realizations neither the length L nor the shape of the PDP is known to the receiver. For the simulations, an exponential PDP with fixed length L is assumed at the receiver. The decay constant is set to one. For detection in the time direction the elements of the autocorrelation matrix $\mathbf{\Sigma}^{(t)}_{\mathbf{hh}}$ are calculated according to Clarke's model with

$$r^{(t)}_{hh}(\Delta n) = J_0(2\pi f_D T_S \Delta n). \tag{13}$$

Here, $J_0(\cdot)$ is the Bessel function of zeroth order, f_D is the Doppler shift and T_S is the OFDM symbol duration. The variable Δn denotes the discrete time difference between two OFDM symbols and the superscript t indicates that the autocorrelation is considered in the time direction.

III. Selection of the Modulation Scheme

The correct choice of the modulation scheme, i.e., to either apply the frequency first or the time first modulation scheme, depends on the correlation of the channel. For differential modulation a strongly varying channel is more likely to introduce errors during detection. Therefore it is recommendable to use the modulation scheme that modulates most of the data symbols in the direction with the highest correlation of the channel.

The correlation in the frequency direction depends on the delay spread of the channel, while the correlation in the time direction depends on the Doppler spread D_s. For Clarke's model D_s is determined by the relative velocity v between transmitter and receiver. In our investigations we discovered that for LTV channels we can find a velocity v at which it is recommendable to change from the frequency first modulation to the time modulation scheme. This velocity can be calculated by requiring equal correlation in the time and in the frequency direction

$$\begin{aligned}|r^{(t)}_{hh}(1)| &= |r^{(f)}_{hh}(1)| \\ |J_0(2\pi f_D T_S)| &= |\mathbf{w}_2\mathbf{H}_{\text{PDP}}\mathbf{w}_1^H|. \end{aligned} \tag{14}$$

The solution to this equation is

$$v = \frac{c_0}{2\pi T_S f_c} J_0^{-1}(|\mathbf{w}_2\mathbf{H}_{\text{PDP}}\mathbf{w}_1^H|). \tag{15}$$

The velocity v depends on the delay spread of the channel, which is represented by the PDP, the OFDM symbol duration T_S and the carrier frequency f_c of the system. A large delay spread, i.e., high frequency selectivity, leads to a low correlation in the frequency direction. In this case it is recommendable to apply the frequency first modulation scheme for velocities that are lower than the calculated v and to switch to the time first modulation scheme at higher velocities than v. On the other hand, a growing T_S leads to a lower correlation in the time direction. Thus, for an increasing OFDM symbol duration it is better to use the time first modulation schemes already at lower speeds. With a higher carrier frequency f_c the Doppler shift increases as well. This leads to a higher Doppler spread D_s which reduces the correlation in the time direction. Thus for a higher f_c it is also recommendable to switch to the time direction modulation at lower speeds.

IV. Simulation Results

This section presents simulation results for frequency selective LTI and LTV channels. The main simulation parameters are summarized in Table I. Soft information was not employed for the simulations. The confidence intervals are calculated for 95%. These are depicted as light grey bars in the simulation results.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
System bandwidth:	1.4 MHz
Subcarrier spacing:	15 kHz
Carrier frequency:	2.5 GHz
Number of subcarriers:	72 (6 RBP)
DFT size K:	128
Channel Model:	ITU PedB, ITU VehA [13]
Power delay profile:	exponential PDP with $L = 8$
Antenna configuration:	1 transmit, 1 receive
Receiver:	coherent: LS with ZF equalizer noncohrent: CND, MSDSD ($N = 4$, $N = 8$)
Modulation alphabet:	coherent: 4QAM, 16QAM, 64QAM noncoherent: 4DPSK, 16DAPSK, 64DAPSK
UE speed:	0 km/h for ITU PedB, 100 km/h for ITU VehA

Fig. 2 depicts the coded throughput over the SNR of both the coherent detection and the noncoherent detection for an LTI ITU PedB channel. According to Section III we utilized the frequency first modulation scheme. For the channel estimation in coherent detection we used least squares (LS) channel estimation with zero forcing (ZF) equalization. We applied

CND as well as MSDSD as noncoherent detection schemes. The number of observed symbols was set to $N = 4$ and $N = 8$ for 4-DPSK and to $N = 4$ for 16-DAPSK and 64-DAPSK. The simulation results show that the gain of MSDSD compared to CND detection is about 0.9 to 1.2 dB. Nevertheless, coherent detection outperforms noncoherent detection by about 2.9 dB. At very high SNR values around 33 dB noncoherent detection starts to offer a higher throughput than coherent detection. This can be explained by the fact that noncoherent detection needs only 1 RS in the whole RBP, while for channel estimation in LTE SISO 8 RS are required. This gives a theoretical possible throughput increase of 4.37%.

The second simulation depicted in Fig. 3 presents the coded throughput for an LTV ITU VehA channel with $v = 100\,\text{km/h}$. The velocity is assumed to be known to the receiver. With the simulation parameters summarized in Table I the speed at which to switch from the frequency first to the time first modulation scheme calculates to $v \approx 48\,\text{km/h}$. Thus we applied the time first modulation scheme. The simulation results show that the gain of MSDSD is only about 0.2 dB. This can be explained by the much lower correlation in the time direction. Furthermore, because of the high time selectivity of the channel the initially higher offered throughput of differential modulation cannot be achieved.

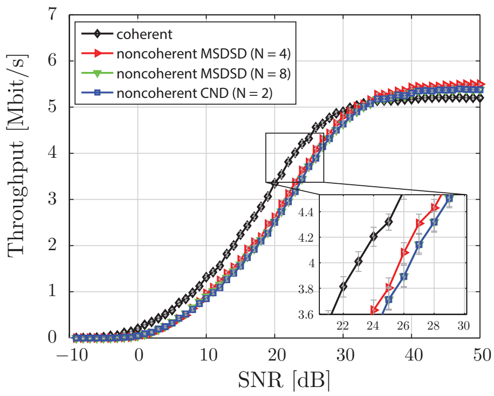

Fig. 2. Coded throughput for an ITU PedB channel with $v = 0$ km/h

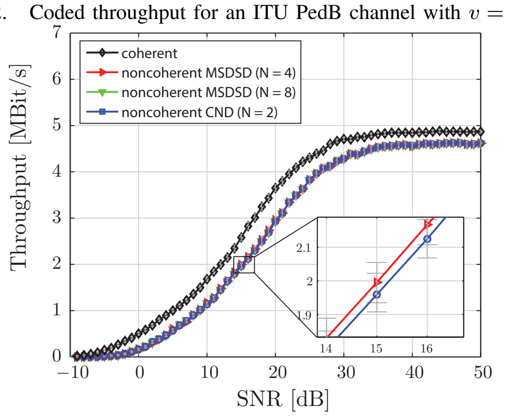

Fig. 3. Coded throughput for an ITU VehA channel with v =100 km/h

V. Conclusion

In this paper we presented the application of differential modulation and noncoherent detection to the LTE downlink. We proposed two modulation schemes, namely the frequency first and the time first modulation scheme and discussed two possible noncoherent detection methods denoted as CND and MSDSD. In Section III we argued that the correct choice of the modulation scheme depends on the channel statistics. We showed that for Clarke's model a certain speed can be calculated at which it is recommendable to switch from the frequency first to the time first modulation scheme. In the last section we presented simulation results for frequency selective LTI and LTV channels. We found that the employed coherent detection in the LTE downlink outperforms differential modulation with noncoherent detection, even if MSDSD is applied. The higher offered maximum throughput can only be achieved for very high SNR values of about 33 dB, which is not realistic for a wireless system. This suggests that although differential modulation provides a gain in spectral efficiency, with the techniques investigated, it is not a competitive alternative to the currently employed coherent detection in the LTE downlink.

Acknowledgments

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