Statistic Properties of a Sensorless Control Method for a Three Phase Permanent Magnet Biased Radial Active Magnetic Bearing

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Keywords
«active magnetic bearing», «three phase AMB» «permanent magnet biasing», «sensorless control», «INFORM method»

Abstract
In this paper a sensorless control method for position control of a radial active magnetic bearing is investigated. This novel bearing concept consists of a three phase design utilized by a three phase voltage source inverter. Additionally the stator coils themselves are used for position sensing by electrical current and voltage signals. Especially statistic properties of position estimation during static and sensorless position controlling are discussed. The control architecture of the complete system is presented. Measurement results of the magnetic bearing prototype are used for verification of the theoretical expectations.

Introduction
Self-sensing (sensorless) methods for Active Magnetic Bearings (AMBs) have been field of interest for many years [1],[2],[3]. This means replacing external position sensors by position estimation methods based on electrical parameters, so the actuator coils themselves are used as sensor. In this paper a voltage injection method based on the INFORM (Indirect Flux Detection by Online Reactance Measurement) method which is usually used for sensorless control of Permanent Magnet Synchronous Machines (PMSMs) is investigated. Due to the AMB design with only three stator coils and using a self-sensing method a cost reduction of the system is achieved. Additionally a reduction of losses during operation is achieved using a homopolar structure which reduces iron losses at high speed revolution and permanent magnet biasing reduces power consumption. Furthermore a standard three phase voltage inverter is used instead of two H-bridge inverters for classical four pole AMB designs. Finally a simplification of the complex mechatronic system is achieved and therefore the AMB results in a low cost realization which permits increased fields of applications.

Sensorless position detection method
The well known INFORM method for angular rotor position detection of PMSMs [4] is the basis of this sensorless control method for radial AMBs. The classical INFORM method is used for measuring differences of the inductances in d- and q-axis of a PMSM which are based on geometry or saturation effects. Due to the dependence of the coil inductances of the three phase AMB on the radial rotor position this method also can be used in a modified way [5]. In this case the rated inductance $L_{i1}$ depends on the displacement in the appropriate coil direction $x_1$ as
\[ I_{11} = \frac{1}{1 - \frac{x_1}{2\delta_0}} \]  

(1) where \( \delta_0 \) is the nominal airgap of the AMB. Combining three spatial displacements \( x_1, x_2 \) and \( x_3 \) a complex displacement phasor \( \chi = x + jy \) is built. The real part stands for the eccentricity in \( x \)-direction and the imaginary part represents the movement in \( y \)-direction. Finally, the rotor position in two orthogonal directions can be measured indirectly by evaluation of three coil inductances.

\[
\chi = -2\delta_0 \cdot \frac{2}{3} \left( \frac{1}{l_{11}} e^{i\theta} + \frac{1}{l_{22}} e^{\frac{2\pi}{3}} + \frac{1}{l_{33}} e^{\frac{4\pi}{3}} \right)
\]

(2) Using voltage pulses caused by the voltage inverter and current slope measurements of the three phase currents \( \Delta i_1, \Delta i_2 \) and \( \Delta i_3 \) the rotor displacement \( \chi \) determines as

\[
\chi = -2\delta_0 \cdot \frac{2}{u \cdot \Delta t} \left( \Delta i_1 + \Delta i_2 \cdot e^{\frac{2\pi}{3}} + \Delta i_3 \cdot e^{\frac{4\pi}{3}} \right)
\]

(3) From the implementation of the INFORM method at PMSMs different possible voltage pulse sequences are known and may also used for the AMB system. In figure 1 experimental results of current slopes using a classical voltage pulse sequence with symmetric measurement around the working point is presented. The influence of the rotor displacement in \( x \)-direction can be seen for different eccentricities.

![Graph](image)

**Fig. 1** Measurement of the current reaction of phase current \( i_1 \) for sensorless position detection at different displacements \( x \)

**Sensorless position control architecture**

As known from the classical magnetic bearing control as well as from the literature [6] for three phase bearings a PD control architecture is requested for a stable position control operation. Additional a cascaded current control loop is used. In the case of a self sensing bearing the differential behavior of the position controller is modified using the speed information of the mechanical observer. The sensorless control architecture of the three phase AMB system is presented in figure. 2.

The same current sensor signals are used for the current controller and for determination of the current slopes for position determination algorithm. The sensorless position detection algorithm uses three current slopes for the calculation of the sensorless rotor position in two directions \( (x_{\text{INF}}, y_{\text{INF}}) \). This information is fed to the mechanical observer. The mechanical observer is figured out in the next paragraph.
The current control loop of the three phase AMB is implemented in an two-axis reference frame \((x,y)\) and uses PID current controllers for both systems. The reference values of the currents are calculated from the position control loop including a stabilizing feedback using the rotor speed signals determined by the mechanical observer.

![Diagram](image)

**Fig. 2** Control structure of the realized sensorless controlled AMB

### The mechanical observer

Providing the sensorless detected rotor position to a dynamical model additional dynamic quantities as velocity and load force are observed. Therefore the following mechanical equations e.g. for the \(x\)-axis are used.

\[
\begin{align*}
\frac{d}{dt} x &= v_x \\
\frac{d}{dt} v_x &= \frac{1}{m} \left( f_{L,X} + f_{AMB,X} \right) \\
\frac{d}{dt} f_{L,X} &= 0
\end{align*}
\]

(4)

In this case no change of the load force \(f_{L,X}\) is assumed. The force \(f_{AMB,X}\) is the resulting force representing the Maxwell stress tensor generated by the electromagnetic system consisting of permanent magnets as well as the three coils. The classical linearization approach for the electromagnetic force behavior at an operation point is used. Considering the \(x\)-direction yields the following equation.

\[
\begin{equation}
\begin{split}
f_{AMB,X} &= f_{0,X}(i_x, \bar{z}) + K_{L,X}(i_x, \bar{z}) \cdot i_x + K_{X,X}(i_x, \bar{z}) \cdot x
\end{split}
\end{equation}
\]

(5)

This characteristic as well as other design parameters of the three phase AMB with permanent magnet biasing was investigated in [7],[8],[9]. Usually constant parameters are the linearization are used for the operating point of centered rotor position \(\bar{z}=0\). Based on the mechanical behavior of the rotor the continuous model of the observer is presented below. Also a notation in an time discrete system is feasible. In this observer model the measurement error feedback is introduced using three appropriate observer coefficients \(K_{L,X}, K_{L,X}, K_{L,X}\). These coefficients can be determined by pole placement or model error variation. Additionally the noise characteristic of the observer inputs can be used to optimize the observer coefficients.
\[ \frac{d}{d\tau} x_{EST} = v_{X,EST} + K_{1,\dot{X}} \cdot (x_{INF} - x_{EST}) \]
\[ \frac{d}{d\tau} v_{X,EST} = \frac{1}{m} (f_{l,x,EST} + f_{AMB,x}) + K_{2,X} \cdot (x_{INF} - x_{EST}) \]
\[ \frac{d}{d\tau} f_{l,x,EST} = K_{3,X} \cdot (x_{INF} - x_{EST}) \]  

(6)

**Statistic properties**

For sensorless control of PMSMs based on the INFORM method an investigation of statistical properties was already known from literature [4]. There, the statistical distribution of “measurement results” is described by a one dimensional normal (gaussian) distribution, because only the angular rotor position is taken into consideration. The probability density function is described by

\[ f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \cdot \exp \left( -\frac{1}{2} \left( \frac{x - \mu_x}{\sigma_x} \right)^2 \right) \]

(7)

where \( \sigma_x \) is the standard deviation and \( \mu_x \) the mean value of the measured angular rotor position signal. In the case of AMBs the measurement result becomes two dimensional, because the evaluation results in displacement phasor \( \vec{x} \) with a real and imaginary component. Therefore a two dimensional normal distribution is assumed for statistic distribution of the measured position \( \vec{x} = x + jy \). The behavior without any correlation term between both axes is written in the following equation. The approach neglecting the correlation terms is verified by the measurement results.

\[ f(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \cdot \exp \left( -\frac{1}{2} \left( \frac{x - \mu_x}{\sigma_x} \right)^2 + \left( \frac{y - \mu_y}{\sigma_y} \right)^2 \right) \]

(8)

**The prototype system**

For verification of the theory a complete prototype AMB system was built up.

![Fig. 3 Three phase homopolar AMB without rotor and coils](image1)

![Fig. 4 Three phase inverter prototype (200V/40A up to 50kHz PWM frequency)](image2)
This system consists of a three phase voltage inverter for high frequency switching (figure 4) as well as a three phase AMB with permanent magnet biasing (figure 3). The bearing is designed as modular concept for adaption of several parameters. The active iron length is split into three modules. Additionally the known permanent magnet ring often used in the literature is split up in single magnets for adaption of the bias flux value. Further NdFeB and Ferrite magnets are available for optimization. The bearing coils are designed as a concentrated winding structure and allow a simple adjustment of the coil inductances regarding the DC link voltage level by exchanging them. The nominal design parameters of the AMB are presented in table 1. A mechanical rotor position measurement system based on the eddy current measurement principle is available as reference system. Further a position control loop was implemented with this sensor feedback signal.

**Table I:** Characteristic data of the realized HMB prototype (with NdFeB-Magnets)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter</td>
<td>190mm</td>
</tr>
<tr>
<td>Total length</td>
<td>140mm</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>80mm</td>
</tr>
<tr>
<td>Nominal airgap</td>
<td>1.0mm</td>
</tr>
<tr>
<td>Pole area</td>
<td>2233 mm²</td>
</tr>
<tr>
<td>Turns of the winding</td>
<td>20</td>
</tr>
<tr>
<td>Nominal coil inductance</td>
<td>0.75 mH</td>
</tr>
<tr>
<td>Nominal radial force</td>
<td>750N</td>
</tr>
</tbody>
</table>

For design of the control behavior bearing parameter regarding equation (5) have to be known. From the Finite Element analysis at the centered rotor position and an arrangement with 24 Ferrite magnets bearing parameters of $K_{xX}(0,0)=17$ N/A and $K_{eX}(0,0)=700$N/mm were figured out.

**Stationary position measurement results**

For verification of the sensorless position detection method the stationary behavior was investigated. Therefore the rotor was fixed mechanically at certain rotor positions and several sensorless position measurements were evaluated statistically. The results for the centered rotor position as well as displacements of $|z|=0.4$mm are presented in figure 5. The appropriate statistical properties can be seen in table 2. For evaluation 1000 single position measurements were considered per position.

![Image](image_url)

**Table II:** Statistic properties of the stationary position measurements

<table>
<thead>
<tr>
<th></th>
<th>$x$-direction</th>
<th></th>
<th>$y$-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean value $\mu_X$ [mm]</td>
<td>Standard deviation $\sigma_X$ [mm]</td>
<td>Mean value $\mu_Y$ [mm]</td>
</tr>
<tr>
<td>A</td>
<td>0.418</td>
<td>0.025</td>
<td>-0.013</td>
</tr>
<tr>
<td>B</td>
<td>0.020</td>
<td>0.025</td>
<td>-0.405</td>
</tr>
<tr>
<td>C</td>
<td>0.035</td>
<td>0.024</td>
<td>0.402</td>
</tr>
<tr>
<td>D</td>
<td>-0.389</td>
<td>0.024</td>
<td>-0.004</td>
</tr>
<tr>
<td>E</td>
<td>0.004</td>
<td>0.025</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Fig. 5 Stationary evaluation of the sensorless rotor position measurements at real rotor positions of $|z|=0.4$mm

Exemplarily, the statistic properties of point C from figure 5 are evaluated with the approach of equation (8). A standard deviation of about $\sigma=25\mu$m is achieved in both directions and the mean
values fits well with the real position. The graphic in figure 6 shows the appropriate histogram of the measurement result of point C. Further, these statistical parameters are provided to the behavior of to the two dimensional normal distribution (8) and printed in figure 7. A comparison of both curves results in a good agreement. Further no correlation terms needs to be introduced in the probability density function for representing that behavior in a compact way.

**Fig. 6** Histogram of the position measurement of point C  
**Fig. 7** Normal probability density function of the evaluated parameters (table II)

**Dynamic position measurement results**

Additional investigations are done with respect to the dynamic behavior of the sensorless position detection method. Therefore based on the sensor signals a sensor controlled mode was implemented due to a parallel work of the sensorless position detection method. In figure 8 the AMB step response of a lift up procedure of the rotor from $x_{ref}=-0.2\text{mm}$ to $x_{ref}=+0.2\text{mm}$ is presented. The statistical evaluation of the sensor controlled mode is depicted in figure 9 and results in a standard deviation of $\sigma_x=19\text{\mu m}$.

**Fig. 8** Transient operation in a sensor controlled mode, sensorless method in parallel operation, rotor movement in x direction  
**Fig. 9** Evaluated probability density of the displacement error $\Delta x$ in a sensor controlled mode.

A full sensorless operation was implemented based on the control scheme shown in figure 2 and evaluation of the position quality is presented in figure 10. A comparison with the sensor controlled mode shows an increased position noise at the stationary reference values of $x_{ref}=-0.2\text{mm}$. Further the overshoot and oscillation behavior caused by the reference step to $x_{ref}=+0.2\text{mm}$ increases. Additionally
the movement in x-direction yields in a higher distortion within the behavior of the orthogonal y-axis. The statistical properties in the appropriate x-direction during sensorless levitation are printed in figure 11. The standard deviation gets nearly doubled to $\sigma_x = 37.9 \mu m$ and also the mean value of the position error has increased.

**Fig. 10** Step response during sensorless levitation, movement in x direction

![Graph showing step response during sensorless levitation](image)

**Fig. 11** Evaluated probability density of the displacement error $\Delta x$ at sensorless levitation

![Graph showing evaluated probability density](image)

**Conclusion**

In this paper a three phase radial active magnetic bearing was investigated with respect to sensorless control behavior. The three phase design of the magnetic bearing allows for an utilization of a standard three phase voltage inverter and furthermore the implementation of a self-sensing method. The so-called INFORM method was investigated and implemented in real hardware. To achieve a full sensorless operation a mechanical observer was introduced providing speed information to the control. The control structure with a PD position control behavior was adopted to be capable for sensorless operation. Measurement results of the manufactured prototype have shown the capability for the sensorless position detection method for active magnetic bearings. For evaluation of the achievable accuracy during sensorless levitation statistic methods were used. The resulting statistic properties were discussed during stationary operation as well as during levitation. Additionally a sensor controlled mode compared to the full sensorless operation has shown the influence of the generated noise from the INFORM method to the control behavior. Finally an accuracy of approximately $\pm 0.1 mm$, this is one tenth of the nominal airgap, was achieved during sensorless operation of the AMB prototype.

**References**


