DEVELOPMENT OF SUSTAINABLE METROPOLITAN AREAS
USING A MULTI-SCALE DECISION SUPPORT SYSTEM
FRACTALOPOLIS MODEL –
ACCESSIBILITY, EVALUATION & MORPHOLOGICAL RULES

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Why a Multi-Scale Approach to Planning?

Let us recall that sustainable planning deals with the development of strategies to reduce the use of resources, increase economic efficiency and improve integration of social aspects (e.g. pedestrian-friendly environments, well-balanced public and private transport modes, efficient street networks, land use, movement economy; access for all to jobs, retail, services; healthcare, culture and leisure). It is not only dispersed development (such as e.g. urban sprawl) that generates traffic overload and congestion; interestingly, the over-compact city also has this effect as it engenders longer journeys to green and leisure areas (cf. Schwanen et al. 2004). Minimising trip length to work places cannot be the only parameter. Accessibility to service and leisure facilities as well as green areas must be included in the overall assessment. On an urban scale, over-compactness causes ecological problems such as a lack of green wedges for supplying the city with fresh air (urban micro climate). Thus, we need to find a solution for managing dispersed development that marries the twin elements of green and built-up space in a highly efficient manner. This solution also needs to incorporate dynamic aspects of a city as well as minimizing traffic costs, emission and avoiding scouring of agricultural land.

Figure 1: Recap of some realities of urban sprawl
(from Frankhauser, Czerkauer-Yamu 2011).
As already pointed out, urban space is founded on the principle of *fractal geometry*. Morphological analyses of cities have shown that urban patterns after the industrial revolution, which are often perceived amorphous, mostly follow a fractal structural principle (Frankhauser 1994, 2008; Batty and Longley 1994; Batty 1996, 1999; Benguigui et al. 2000; Shen 2002; Salingaros 2003; Tannier and Pumain 2005; Franck 2005; Thomas et al. 2010). Urban growth appears to be governed by complex dynamic processes generating morphologically well-defined macrostructures. This is reminiscent of other evolutionary systems such as clouds, trees, leaves or the human vascular system.

Fractals are *multi-scale* and *self-similar*; their ordering principle is based on cascades with similar elements on different scales with different inherent levels of detail. In an urban context this is e.g.: house, block, quarter, city or: path, residential street, side street, main street, through road, freeway, and highway. According to Read (2000), different scales of hierarchy are distinguished by scales of mobility, and are designed to convey different scales of movement.

However, this hierarchical ordering principle is changing with increasing car traffic, with the effect that agglomerations are becoming more and more uniformly distributed due to the increased growth of remote suburbs (Frankhauser 2008).

Thus, the spatial-functional pattern of cities describes a relationship between their inhabitants’ usage of space and movement. This relationship has to be considered when developing sustainable and sustaining planning strategies and models (as the logic of the Haussmann’s intervention – fractal scaling).

Due to its inherent characteristics, the use of a fractal (multi-scale) logic for spatial planning supports sustainable and sustaining planning strategies.

Using fractal geometry for urban planning implicitly assumes that fractality corresponds to underlying optimization criteria, as is supposed to be the case in natural structures (see part one). Indeed, fractal surfaces seem to be optimal for spatial systems requiring a high articulation between subsystems. Then, hierarchical structures seem very efficient. This holds true for many natural networks such as lungs or vascular systems.

In urban planning, an example could be the urban road network. As described before, it has been shown that the street system of Paris including Haussmann’s street openings of the 19th century, indeed follows a fractal scaling logic (Frankhauser 1994).
Since every building must be accessible, transport networks generally play a crucial role in urban growth, consolidation and downsizing (shrinking). Therefore, during the trolley period, public transportation networks generated axial growth, as can still be seen in the case of Berlin, where the suburban railway network structured urban space. Railway networks are usually hierarchically organised and cover space less uniformly than road networks do nowadays. This explains why emerging patterns showed particularly fractal properties as long as public transport use predominated. In Berlin this type of growth (see Figure 9, p.47) became the basis for urban planning strategies by privileging development around the suburban railway axes. This holds even more explicitly for the Copenhagen’s Finger plan. Privileging transportation axes as development axes is an important aspect of the fractal (multi-scale) planning concept.

Starting from the underlying logic of fractals – the self-organising processes of cities and metropolitan areas – we can develop scenarios whose underlying concept takes advantage of these natural growth processes.

On an urban scale, the multi-scale planning concept (model) prevents interlaced peripheral roads from penetrating into green open space. Local recreation areas, which simultaneously function as ecological conversation areas and climate corridors, are brought into close proximity to residential areas. With their 1910 plan for Greater Berlin, Eberstadt, Möhring and Petersen developed 1910 the first archetype for such an organic link between the city and the open landscape.

Another well-known property of urban systems is the emergence of a central place hierarchy known as rank size distribution, which corresponds to a fractal hierarchy. The concept presented for the planning model refers to such a hierarchical organization of metropolitan areas. The hierarchical structure of an agglomeration, developed on the basis of social and economic interaction and interdependency between the locations (e.g. villages), has been investigated in urban geography for a long time. These observations served Christaller as the foundation for his Central Place theory, which is based on a reflection about the catchment areas of different levels of services depending on how often the services are used. That is why the services for everyday life (e.g. supermarkets) are close to housing, whereas weekly or monthly services require bigger catchment areas. The limitation of Christaller’s theory is that it is only concerned with the functional hierarchy, and does not reflect the spatial structure
(topography). This explains why in Christaller’s theory, locations are evenly distributed across the spatial surface plane. The accessibility of such a distribution is disadvantageous for several reasons. On one hand, it demands a pseudo-homogeneous traffic infrastructure; on the other hand, all of the remaining free spaces are approximately the same size. In our research, Christaller’s theory, which was already installed as a regional model in post-war Germany, undergoes a reconception that is clearly differentiated from Hillerbrecht’s ideal city structure of the Regionalstadt. Christaller’s conception leads further to the sustainable concept of a city of short distances supporting a functional, administratively sustainable urban planning concept.

The concept used modifies the Christaller scheme by introducing an uneven spatial distribution of settlements where urbanized areas are concentrated close to public transport axes (Frankhauser 2008). Nodes of a hierarchically structured transport network are the preferred locations for services and shopping areas. This calls to mind the concept of decentralised centralisation or, as Calthorpe (2001) formulates it, the regional town, which also enables an intraregional supply for in-between spaces of global axes. The first approach aiming at decentralised centralisation can be identified in Howard’s regional scheme (1902) and further in the New Towns (Les Villes Nouvelles).

These examples and the emergence of centre hierarchies in urban systems show that fractal geometry has attributes which can be used by planners to create sustainable, sustaining structures on all substantial scales – from a regional scale to an architectural scale. A further specific fractal attribute is the consistency through scale.

In particular, on an urban scale the interface between existing urban morphology and new potential development (option testing and scenario development) is an interesting challenge, as we have to deal with a non-linear urban fringe with underlying characteristics of self-organisation paired with former planning and building interventions. The strategy of merging simulated and existing networks is of major importance as the urban development and extension has to be continuous, without any noticeable phase transition either for the urban structure or the residents of the area. Standards have to be developed that correspond to a city’s population, ranking different levels of central places.
In general, fractal urban planning follows a naturalistic approach (*biophilic planning and design*; see further Salingaros 2010) being inspired by the emergence of urban patterns. Thus, we can describe the growth of cities and metropolitan areas as fractal entities (Batty and Longley 1994, Batty 1996, 1999, Benguigui 2000, Shen 2002, Thomas and Frankhauser 2008) – in line with fractal system descriptions of other evolutionary, biological systems like clouds, trees, leaves or the human vascular system.

Herein we explicitly present a multi-scale planning concept where fractal measures become norms for planning, starting from a metropolitan scale down to a local scale (urban context for growth, consolidation and downsizing scenarios). The metropolitan area is thus an organic entity in which different parts of the agglomerations are linked to each other.

Following features can be identified as being important for a sustainable and sustaining planning strategy:

- Hierarchical ordering principle of agglomerations (e.g. Christallerian logic);
- Interweaving of built-up and green open space;
- Interconnectedness of green areas for accessibility on all levels;
- Access for all to services and facilities as well as leisure;
- Hierarchy of the street and road network;
- Public transport;
- Strategic visibility for orientation and wayfinding;
- Density and city image.
METHODOLOGY AND FORMALISATION

Methodology and Formalisation of the Multi-Scale Decision Support System Fractalopolis

The herein presented multi-scale decision support system *Fractalopolis* refers for the regional model to a hierarchical organization of metropolitan areas (Christallerian logic) and on the local scale is based on an accessibility logic; in addition incorporating a population model.

Borsdorf stresses the fact that within Christaller’s system, surrounding villages near a central city could never gain a higher centrality, as Christaller’s theory took gravitation and transportation costs (= distance) as a basic principle (Borsdorf 2004, p.131). Borsdorf’s view on Christaller is right if we try to implement Christaller as a rigid, non-flexible system, that is not adapted to the surrounding built environment in which it is embedded.

Christaller’s conception leads further to the sustainable concept of a city of short distances supporting a functional, administratively sustainable urban planning concept. Christaller’s theory also follows a similar line of thought to the concept of decentralised centralisation which also enables an intraregional supply for in-between spaces of global axes. But, if we vary Christaller by viewing his scheme as a modular system and rescale it by adding new hierarchies and interfaces for agglomerations (working, living, leisure), we will find surprising new insights and possibilities for use in a differentiated spatial context, e.g. *Hillebrecht’s Regionalstadt* (regional town, 1962) incorporates central locations for commerce, services and workplaces.

Generally speaking, the core of idea of Christaller’s theory is the *application and mapping of a spatial hierarchy as a holistic system approach*. Further, we should not be trapped by the idea of mono-scale functional units when looking at Christaller. Borsdorf is right when he explains that Christaller’s theory deals first and foremost with spatial structures and not a strategy for the allocation of central function (Borsdorf 2004, p.132). This distinction is important to make when addressing new planning strategies based on the underlying idea of the Central Place theory.
With this in mind, within the framework of the PREDIT research programme a first draft of a planning scheme featuring Christaller’s centre hierarchy was developed at ThéMA, Université de Franche-Comté, France, in which spatial distributions are linked to the hierarchy of the traffic infrastructure, equating to a multifractal structure (Frankhauser et al. 2007, 2008).

![Diagram of Christaller's network of central places including traffic infrastructure and a multifractal hexagonal approach.](image)

Figure 2: Christaller’s network of central places including traffic infrastructure and a multifractal hexagonal approach. The hexagonal shape of this system is reminiscent of the Christaller scheme. Towns are concentrated in proximity to axes, which can be interpreted as public transport axes. Between the axes there is connected green open space (green wedges) which can be interpreted as areas of natural landscape and agricultural land. By its shape the system avoids fragmentation of these rural and natural zones (consistently across scales) (from Christaller 1933, reprint 1980; Frankhauser et al. 2006, 2008:31).

The agglomerations are thus pushed closer to the main traffic axes, decreasing distances and increasing accessibility from and to services. The structured services in Christaller’s centre hierarchy are localised at traffic nodes and have different sized catchment areas. The designed traffic system, using a radio-concentric principle, offers high accessibility with regard to its functional impact (Oberzentrum, Mittelzentrum, Unterzentrum, Kleinzentrum). This axes-oriented concept concentrates and lumps traffic flows and therefore allows public transport to be prioritised. In addition,
a hierarchically organised system of linked open spaces allows small green areas to be retained next to housing estates as well as nature reserves and vast woodlands. The green corridor principle is thus expanded as not only non-built-up surfaces and corridors are kept free, but the interweaving of urban space and open space on all scales becomes the predominant concept. The urban and agglomeration fringes are deliberately not chamfered, but linked to green spaces on all scales in order to reduce traffic flow and minimize travel distance to leisure areas.

Proceeding from Frankhauser’s multi-fractal model a further step was taken and the multiscale decision support system (and software) *Fractalopolis* was developed.

Of course, hierarchy is the underlying idea for the regional scale (and further the urban and architectural scale) in order to establish a principle that follows in the footsteps of Calthorpe and Fulton’s *regional city conception* (2001) as well as *Ebenezer Howard’s principle of city growth* (1898, 1902) and further developments such as the *Ville Nouvelles* (1965). The idea of hierarchy as a foundation for developing a regional growth model allows an efficient usage of space based on the *law of all living systems*, ergo a fractal logic.

Let us recall some features of fractal geometry according to a multifractal logic (for a sustainable and sustaining planning strategy):

- A fractal is based on a scaling law; the same structure appears on different scales
- Non-uniform distribution of mass; uniformity and concentration are limit cases
- A fractal is neither dense nor diluted; it is more or less contrasted
- Mass is distributed according to a precise law (Pareto-Zipf distribution)
- Strong hierarchical order
- Fractal structures may look “irregular/amorphous”, but may nevertheless be organised according to a fractal ordering principle. Such structures may be described by fractal scaling law.

The fractal law is further combined with social need with respect to accessibility, generating the distance and evaluation rules for services and leisure amenities for daily, weekly, monthly and occasional use.
However, let us emphasize that the proposed spatial system aims to take simultaneous account of different kinds of objectives:

- reducing travel distances required to access higher-order facilities;
- respecting the diversity of social demand i.e. taking into account the fact that certain types of households prefer living in a quiet, low-density environment with good access to green amenities;
- avoiding leapfrogging that lengthens the distances to acceded to centers and avoiding the fragmentation of natural or agricultural areas.

To fulfil these aims we introduce in the following a spatial model which uses iterative mapping procedures similar to those used for generating multifractal Sierpinski carpets. We assume that there exists a hierarchical system of central places structured according to the different levels of services and commercial amenities they provide. However, as we will see later, towns belonging to the same hierarchical level no longer have the same population: towns of a given level but which are close to a higher-ranked centre are assumed to concentrate more population than those lying close to lower-ranked centres.
Formalisation of Fractalopolis

To provide a convenient introduction to the quantitative modelling approach we consider a model version that is simpler than the multi-fractal hexagonal approach, but which follows the same logic.

Let us start by drawing a large-sized square of a certain base length which we will normalise to one. We assume that the most important centre of the system is localised in the centroid of the square. The surface of the square in some sense represents the catchment area (or area of attraction; sphere of influence) of our central place. Now we introduce a generator which consists of a square of base length \( r_1 < 1 \) centred on the first-order centred (framed with a dotted line). This square is surrounded by \( N = 4 \) smaller squares (sub-centres) with the base length \( r_0 < r_1 \). Let us emphasize that the generator lies just within the initial square so that the outer corners of peripheral squares are identical to that of the initial square. Moreover, no overlapping of squares is allowed (see also morphological rules).

We assume that the first step corresponds to the implementation of \( N = 4 \) second-order centres (or sub-centres) localised in the centroid of the smaller squares. The surface of the squares now corresponds directly to the catchment areas of these centres. Hence, the central square has a bigger second-order catchment area than the peripheral centres. In the next step we reiterate the procedure (Figure 43). Each of the existing squares is replaced by a smaller replication of the generator. In accordance with our logic we keep the already generated first-order and second-order central places and add third-order central places lying within the catchment areas of the second-order centres. Again, these centres are localised in the centroid of the generated smaller squares.
By the iteration process the reduction factors $r_1$ and $r_0$ are combined according to all possible combinations, which yields, e.g. for the second step:

$$r_1 \cdot r_1, r_1 \cdot r_0$$

[1]

Because of the commutativity property we have:

$$r_1 \cdot r_0 = r_0 \cdot r_1$$

[2]

This is the reason why the catchment areas of the second-order centres are as those of the third-order centres belonging to the highest-ranked centre. This corresponds to a peculiarity of multifractal structures and we will come back to this topic when considering the population distribution. Another consequence of this feature of multi-fractals is that the direct catchment areas belonging to the third-order centres no longer have the same size. Within the multifractal figure we find small squares of base length $r_0 \cdot r_0$ and large ones with base length $r_0 \cdot r_1$.

The next step adds another hierarchical level and we again discover that the size of the catchment areas of centres issuing from different iteration steps and thus corresponding to different hierarchical levels is the same;
and on the other hand that the catchment areas of centres belonging to the same level are different. Two logics can be distinguished:

- the first one generated the central place hierarchy by adding a lower level at each iteration step. Hence, the iteration step where the centroid is generated, determines its service level in the central place hierarchy;
- the second one is linked to the mentioned “degernation” effect. Since permutations are allowed we have direct catchment areas which have the same size but belong to different service levels.

However, we should emphasise that the logic of spatial configuration of the centres corresponds to the logic of the Central Place theory. The fact that the areas influenced by the centres are of different size depending on their localisation seems an interesting feature, since we can assume that cities lying close to important high-level centres are usually bigger than those lying close to low-level centres. This logic will be reconsidered when defining the theoretical population numbers.

By going on with iteration, it is of course possible to generate a more hierarchical spatial system. Let us recall that Christaller, for instance, distinguishes seven different service levels. However, in order to conserve a certain legibility, we shall restrict ourselves to the four service levels already introduced. These levels correspond to the following purchase rates or frequency levels:

- Level 1: *occasional* frequented services, shops or leisure amenities
- Level 2: *monthly* frequented services, shops or leisure amenities
- Level 3: *weekly* frequented services, shops or leisure amenities
- Level 4: *daily* frequented services, shops or leisure amenities
The hierarchical structure also determines the hierarchy of green open space.

Figure 4: Services and leisure amenities according to their level of frequentation in the context of their catchment areas.

Figure 5: Hierarchy of green open space.
The Coding System

A coding system is introduced in order to distinguish the different centres according to their service level. Hence for the first iteration step we distinguish the large central square which we denote by the digit 1 and the four smaller peripheral squares denoted by 0. In each following step we now add another digit to the right of the existing one, according to the same logic. This indicates that the hierarchy is created by combining just two factors. Hence in the next step the highest-order central square is now called 11, the four adjacent smaller ones 10. The four peripheral squares generated in the previous step are replaced, too, by the generator. The occurring central place are called 01 and the four peripheral ones 00. This procedure is reiterated in the third step (cf. figure 2c). We thus obtain a set of 8 different codes, each one consisting of three digits. The first-level center with the highest facility level \( m = 1 \) has the code 111. The four directly adjacent squares of level \( m = 2 \) have the codes 110. They correspond to suburban areas of the main center. The four centers 011 correspond to the four centers of level \( m = 2 \) generated at the first iteration step. The peripheral centers 101 and 001 are issued from the second iteration step and correspond to centers of the facility level \( m = 3 \). Of course the 101 centers belong to the catchment area of 111 for higher-level facilities, whereas the centers 001 belong to the catchment area of the second-level 011 centers. The small elements 100 and 000, adjacent to these third-level centres, are all low-level \( m = 4 \) centres.
The step-by-step generation of the elements can hence be represented as follows:

Figure 6: The coding system

Figure 7: The coding system; iteration step 0-2.
We can identify the following properties for the different kinds of elements:

<table>
<thead>
<tr>
<th>code</th>
<th>level</th>
<th>next superior center</th>
<th>number</th>
<th>surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$(S_1)^3$</td>
</tr>
<tr>
<td>110</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>$(S_1)^2 \cdot (S_0)$</td>
</tr>
<tr>
<td>101</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>$(S_1)^2 \cdot (S_0)$</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>3</td>
<td>16</td>
<td>$(S_1) \cdot (S_0)^2$</td>
</tr>
<tr>
<td>011</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>$(S_1)^2 \cdot (S_0)$</td>
</tr>
<tr>
<td>010</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>$(S_1) \cdot (S_0)^2$</td>
</tr>
<tr>
<td>001</td>
<td>3</td>
<td>2</td>
<td>16</td>
<td>$(S_1) \cdot (S_0)^2$</td>
</tr>
<tr>
<td>000</td>
<td>4</td>
<td>3</td>
<td>64</td>
<td>$(S_0)^3$</td>
</tr>
</tbody>
</table>

Table 1: Basic surfaces corresponding to the code and levels

Where we have set the basic surfaces as $(r_1)^2 = S_1$ and $(r_0)^2 = S_0$.

The codes inform us directly about the facility levels. Using a generalised code $ijk$, we obtain:

- $k = 0 \quad \rightarrow \quad m = 4$
- $jk = 10 \quad \rightarrow \quad m = 3$
- $ijk = 110 \quad \rightarrow \quad m = 2$
- $ijk = 111 \quad \rightarrow \quad m = 1$
By introducing these codes we have given up the previously discussed commutativity. Indeed, in the introduced system the codes 101 and 110 or 011 are not equivalent, even though the surface of their direct catchment area is the same. Hence, the code introduces a non-commutative operation. The consequence is that even multi-fractal, the system shows properties of unifractals. Thus, making an abstraction of their size, we verify that the total number of centres belonging to the different levels follows a geometrical series, except the transition from the highest to the next subordinate level.

<table>
<thead>
<tr>
<th>level</th>
<th>number</th>
<th>multiplicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2: Hierarchical logic of total numbers of centres

This corresponds to the usual hierarchical logic observed in fractals.
The Population Model

Pierre Frankhauser 2012
HAL: hal-00758864, version 1

http://hal.archives-ouvertes.fr/hal-00758864
oai:hal.archives-ouvertes.fr:hal-00758864
**Accessibility and Evaluation Rules**

The accessibility and evaluation rules for *Fractalopolis*, from a regional to a neighbourhood scale, allowing the three aspects of sustainability – economic, ecological and social – to be combined. On a local scale the rule set supports the creation of a pedestrian-friendly environment (daily and weekly facilities) at the same time balancing private and public space (morphological rules and multifractal IFS – iterated function systems). Further, the accessibility evaluation takes into account access to monthly and occasionally used facilities and open green space following the logic of TOD (Transport Oriented Development), prioritising public transport. (On a global scale the model employs a descriptive-normative approach.) The rules support interlacing of public and individual transport modes on all interwoven scales (metropolitan area to neighbourhood quarter) in order to support optimal land use and appropriation as well as economy of movement; access for all; and a crosslinking of work, trade, health care, culture, leisure, and green open space. Interwoven multi-fractal scenarios shown later on further enable the classic contraction of city and countryside to be overcome.

In summary, the Fractalopolis software delivers a “suitability map” coloured from red to green (traffic light principle) evaluating the distance to service clusters (daily and weekly used facilities), monthly and occasionally used facilities including green open space. It also shows the distance to the existing street and road network plus access to the public transport network. Depending on requirements, topographical conditions can be integrated as restriction zones restricted.
**Fractalopolis’ Accessibility Rules**

In Fractalopolis two main scales are considered:

*Macro* level: refers to amenities of type occasional and monthly  
*Micro* level: refers to amenities of type daily and weekly.

The fractal decomposition (as described above) starts with the macro level. As for the case study, the Vienna-Bratislava metropolitan region, it turns out that for Vienna the 3rd and 4th decomposition steps correspond to the scale of the municipal territories. Thus, it is possible to define centres according to the Christaller norms applied at this level, i.e. we distinguish:

- the *Oberzentrum*; includes all facilities up to level 1 (occasional)  
- The *Mittelzentrum*; includes all facilities up to level 2 (monthly)  
- the *Unterzentrum*; includes all facilities up to level 3 (weekly)  
- the *Kleinzentrum*; includes all facilities up to level 4 (daily)

Vienna combines levels 1 and 2. Thus, the whole metropolitan area is dealt with up to these steps.

Based on this, the classification of services and leisure amenities detailed below serves to generate the data base (GIS) for the accessibility evaluation of the *Fractalopolis* software. The classification applies to points and areas for the data base. The below-defined areas [ha] for open green space are based upon the *Accessible Natural Greenspace Standard* (ANGSt).

*N.b.* The following rule set is adjusted to the existing structure of the Vienna-Bratislava metropolitan region. It is flexible enough to be modified in the software for every spatial system.
Classification of Services and Leisure Amenities

Level 2 (monthly) = Level 1 (occasional)

Services

- university
- central public administration (e.g. ministry, court, embassy, etc.)
- cultural centre (opera, theatre, museum, etc.)
- specialised shops (cobbler, jewelers, tools shop, arms shop, etc.)
- shopping mall
- hospital and health centre
- DIY and garden centre
- casino

Distance Service, level 1 + 2:

0-20,000m $\mu(d) = 1$
20,000-40,000m $\mu(d) = 1-0$

Leisure Amenities

- skiing
- water sports (e.g. windsurfing, kitesurfing, sailing, etc.)
- golf
- recreation areas
- moors and heathlands
- forests
- mountains
- big natural areas (e.g. alluvial forests)
- UNESCO world heritage

Area Size, Leisure, level 1 + 2:

>150 ha

Distance Leisure, level 1 + 2:

0-60,000m; $\mu(d) = 1$
60,000-100,000m; $\mu(d) = 1-0$

Public Transport

- rail (station)
### Level 3 (weekly):

#### Services

<table>
<thead>
<tr>
<th>Service</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>post office</td>
<td>cinema</td>
</tr>
<tr>
<td>secondary school</td>
<td>household shop</td>
</tr>
<tr>
<td>bank</td>
<td>local cultural centre</td>
</tr>
<tr>
<td>hairdresser</td>
<td>drugstore</td>
</tr>
<tr>
<td>florist</td>
<td>place of worship</td>
</tr>
<tr>
<td>café, restaurant, bar</td>
<td>library</td>
</tr>
<tr>
<td>pharmacy</td>
<td>DIY and garden</td>
</tr>
<tr>
<td>car repair, bicycle shop</td>
<td>farmer’s market</td>
</tr>
<tr>
<td>supermarket</td>
<td>clothes shop</td>
</tr>
<tr>
<td>dentist</td>
<td>beauty salon</td>
</tr>
<tr>
<td>sports centre (competitive leisure)</td>
<td>spa centre (competitive leisure)</td>
</tr>
<tr>
<td>indoor climbing, gym, etc.</td>
<td>services; including beauty salon</td>
</tr>
<tr>
<td>local public administration</td>
<td></td>
</tr>
<tr>
<td>(including social facilities; e.g.</td>
<td></td>
</tr>
<tr>
<td>municipal office, etc.)</td>
<td></td>
</tr>
</tbody>
</table>

#### Distance Service, level 3:

- 0-3,000m; $\mu(d) = 1$
- 3,000-10,000m; $\mu(d) = 1-0$

#### Leisure Amenities

<table>
<thead>
<tr>
<th>Amenity</th>
</tr>
</thead>
<tbody>
<tr>
<td>small weekly recreation areas</td>
</tr>
<tr>
<td>sports areas (tennis, soccer, basketball, public swimming pool, etc.)</td>
</tr>
</tbody>
</table>

#### Area Size, level 3:

- 2-150 ha

(Reason for range: combines sports grounds with recreation areas)

#### Distance Leisure, level 3:

- 0-2,000m; $\mu(d) = 1$
- 2,000-15,000m; $\mu(d) = 1-0$

#### Public Transport

<table>
<thead>
<tr>
<th>Mode of Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>bus (stop)</td>
</tr>
<tr>
<td>rail (station)</td>
</tr>
</tbody>
</table>
**Level 4 (daily):**

<table>
<thead>
<tr>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>corner shop, organic store</td>
</tr>
<tr>
<td>primary school</td>
</tr>
<tr>
<td>kindergarten and crèche</td>
</tr>
<tr>
<td>newsagents and tobacconist</td>
</tr>
<tr>
<td>bakery</td>
</tr>
<tr>
<td>butcher</td>
</tr>
<tr>
<td>general doctor</td>
</tr>
<tr>
<td>cash machine</td>
</tr>
</tbody>
</table>

**Distance services, level 4:**

<table>
<thead>
<tr>
<th>Distance</th>
<th>µ(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-600m</td>
<td>1</td>
</tr>
<tr>
<td>600-1,200m</td>
<td>1-0</td>
</tr>
</tbody>
</table>

**Leisure Amenities**

<table>
<thead>
<tr>
<th>Leisure Amenities</th>
</tr>
</thead>
<tbody>
<tr>
<td>playground</td>
</tr>
<tr>
<td>dog exercise area</td>
</tr>
<tr>
<td>small park (Beserlpark)</td>
</tr>
</tbody>
</table>

**Area Size, level 4:**

<table>
<thead>
<tr>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2 ha</td>
</tr>
</tbody>
</table>

**Distance Leisure, level 4:**

<table>
<thead>
<tr>
<th>Distance</th>
<th>µ(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-400m</td>
<td>1</td>
</tr>
<tr>
<td>400-800m</td>
<td>1-0</td>
</tr>
</tbody>
</table>

**Public Transportation**

<table>
<thead>
<tr>
<th>Public Transportation</th>
</tr>
</thead>
<tbody>
<tr>
<td>bus (stop)</td>
</tr>
</tbody>
</table>
Accessibility Rules - MACRO Level

Level 1 (Occasional Level)

Services
- Distances are not taken into account (identification of potential).
- The logic follows a quality evaluation and checks whether amenities are present or not; multiple presence of same amenities is not important (competition between services is not taken into account on the regional scale – iteration step 1-3).
- Combination of amenities: no preference is assigned to specific types of amenities (all types have the same weight).

The presence of several amenities within a cell (square) is taken into account by means of a linear increase:

\[
\mu(S_i) = \frac{1}{4} \delta \quad \text{for} \quad \delta > 3
\]

where:
- \( S_i \) = service amenities level 1
- \( \delta \) = diversity of services

N.b. the services correspond to the same attribute (variable) and the different services are just different characteristics (values).

Figure 9: Linear increase function for services on level 1: 1 service amenity 0.25; 2 service amenities 0.5; 3 service amenities 0.75; and > 3 services 1.
Green and Leisure Amenities

- Distances are not taken into account (identification of potential).
- The logic follows a quality evaluation and checks whether amenities are present or not; multiple presence of same amenities is not important (competition between services is not taken into account on the regional scale – iteration step 1-3).
- Combination of amenities: no preference is assigned to specific types of amenities (all types have the same weight).

The presence of green amenities within a cell (square) is taken into account by means of a linear increase:

\[ \mu(L_1) = \frac{1}{4} \delta \]

where:

\( L_1 = \) green and leisure amenities level 1
\( \delta = \) diversity of green and leisure amenities

N.b. the green and leisure amenities correspond to the same attribute (variable) and the different services are just different characteristics (values).

\[\delta\]
\[\mu\]

Figure 10: Evaluation of green and leisure amenities on level 1.

No morphological rule.
Level 2 (Monthly Level)

Services

- Distances are not taken into account (identification of potential).
- The logic follows a quality evaluation and checks whether amenities are present or not; multiple presence of same amenities is not important (competition between services is not taken into account on the regional scale – iteration step 1-3).
- Combination of amenities: no preference is assigned to specific types of amenities (all types have the same weight).

The presence of several amenities within a cell (square) is taken into account by means of a linear increase:

\[
\mu(S_2) = \frac{1}{4} \delta
\]

\[
\mu(S_2) = 1 \quad \text{for} \quad \delta > 3
\]

where:

\( S_2 = \text{service amenities level 2} \)

\( \delta = \text{diversity of services} \)

N.b. the services correspond to the same attribute (variable) and the different services are just different characteristics (values).

Figure 11: Evaluation of services on level 2.
Green and Leisure Amenities

- Distances are not taken into account (identification of potential).
- The logic follows a quality evaluation and checks whether amenities are present or not; multiple presence of same amenities is not important (competition between services is not taken into account on the regional scale – iteration step 1-3).
- Combination of amenities: no preference is assigned to specific types of amenities (all types have the same weight).

The presence of green amenities within a cell (square) is taken into account by means of a linear increase:

\[
\mu(L_2) = \frac{1}{4} \delta \quad \text{for } \delta \geq 3
\]

where:

\[L_2 = \text{green and leisure amenities level 2}\]
\[\delta = \text{diversity of green and leisure amenities}\]

N.b. the green and leisure amenities correspond to the same attribute (variable) and the different services are just different characteristics (values).

![Figure 12: Evaluation of green and leisure amenities on level 2.](image)

No morphological rule.
**Level 3 (Weekly Level)**

**Services**

- The presence of several amenities is taken into account (diversity) with evaluation of number of the same category.
- Diversity is measured by means of the number of different types of facilities (e.g. for services: supermarket, lower-level administration, post office, etc.) present in the cell (or possibly in a cluster in accordance with the rules outlined below).
- Distances are not taken into account (identification of potential).
- Clusters are not taken into account (aggregation level is too coarse for a 800m cluster)

Combining the number and diversity yields:

\[
\mu(S_3) = \mu(n)\mu(\delta)
\]

where:

- \(S_3\) = service amenities level 3
- \(n\) = number of amenities
- \(\delta\) = diversity of amenities

Both criteria, i.e. diversity and number evaluation, are presumed to be “equivalent”. The product corresponds to a rather “pessimistic” evaluation: this seems realistic since the individual seem to be interested in both the criteria in equal terms an number of services (e.g. the square root or a potential weighting of one characteristic with respect to the other one to be too ,,optimistic“).

\[
\mu(n) = 0.25 \cdot n \quad \mu = 0 - 1 \quad \text{for} \quad n \leq 4 \quad ; \quad \mu = 1 \quad \text{for} \quad n > 4
\]

\[
\mu(\delta) = 0.20 \cdot \delta \quad \mu = 0 - 1 \quad \text{for} \quad n \leq 5 \quad ; \quad \mu = 1 \quad \text{for} \quad n > 5
\]

Figure 13: Evaluation of number and diversity on level 3.
Green and Leisure Amenities

- The presence of several amenities is taken into account (diversity) without evaluation of number of the same category (no competition, since usually publicly managed facilities).
- Distances are not taken into account (identification of potential).

The presence of several green amenities within a cell is taken into account in the following way:

\[
\mu(L_3) = \frac{1}{4} \delta 
\]

where:

- \( L_3 \) = green and leisure amenities on level 3
- \( \delta \) = diversity of amenities

N.b. the green and leisure amenities correspond to the same attribute (variable) and the different services are just different characteristics (values).

Morphological Rule

Distance between neighbouring cells decreases in a linear fashion (according to von Neumann logic); for morphological rule see p.154f.
Level 4 (Daily Level)

Services

- The presence of several amenities is taken into account (diversity) with evaluation of number of the same category (no competition, since usually publicly managed facilities).
- Distances are not taken into account (identification or potential)
- Clusters are not taken into account (aggregation level is too coarse for a 800m cluster).

Combining the number and diversity yields (same logic as above): [10]

\[
\mu(S_4) = \mu(n)\mu(\delta)
\]

where:

\[ S_4 \text{ = service amenities on level 4} \]
\[ n \text{ = number of amenities} \]
\[ \delta \text{ = diversity of amenities} \]

\[ \mu(n) = 0.25 \cdot n \quad \mu = 0 - 1 \quad \text{for} \quad n \leq 4 \quad ; \quad \mu = 1 \quad \text{for} \quad n > 4 \quad [11] \]
\[ \mu(\delta) = 0.20 \cdot \delta \quad \mu = 0 - 1 \quad \text{for} \quad n \leq 5 \quad ; \quad \mu = 1 \quad \text{for} \quad n > 5 \quad [11a] \]

Figure 15: Evaluation of number and diversity on level 4.
Green and Leisure Amenities

- The presence of several amenities is taken into account (diversity) without evaluation of number of the same category (no competition, since usually publicly managed facilities).
- Distances are not taken into account (identification of potential).

The presence of several green amenities within a cell is taken into account in the following way:

\[
\mu(L_4) = \frac{1}{3} \delta \quad \text{for} \quad \delta \geq 3
\]

where:

\(L_4\) = green and leisure amenities on level 4
\(\delta\) = diversity of amenities

N.b. the green and leisure amenities correspond to the same attribute (variable) and the different services are just different characteristics (values).

![Figure 16: Evaluation of green and leisure amenities on level 4.](image)

Morphological Rule

Distance between neighbouring cells decreases in a linear fashion (according to von Neumann logic); for morphological rule see p. 154f.
Combining the Criteria for Each MACRO Level

Amenities (services and facilities):
- $F_4$: Daily frequentation, services $S_4$ and leisure amenities $L_4$; morphological rules $M_4$
- $F_3$: Weekly frequentation, services $S_3$ and leisure amenities $L_3$; morphological rules $M_3$
- $F_2$: Monthly frequentation, services $S_2$ and leisure amenities $L_2$
- $F_1$: Occasionally frequentation, services $S_1$ and leisure amenities $L_1$

Centrality levels:
- $P_4$: Important central place (e.g. Vienna; prime city) “Oberzentrum”
- $P_3$: Intermediate central place (town) “Mittelzentrum”
- $P_2$: Small central place “Unterzentrum”
- $P_1$: Petit central place (village, hamlet) “Kleinzentrum”

At each level the three or respectively two types of criteria (service, leisure and morphology evaluation) for $M_4$ and $M_3$ are weighted and the arithmetic mean is computed, i.e.

Level $P_1$

\[
A(P_1)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) + 0.33\mu(L_4) + 0.33\mu(M_4)
\]

\[
A(P_1)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) + 0.2\mu(M_3)
\]

\[
A(P_1)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.4\mu(S_2) + 0.6\mu(L_2)
\]

\[
A(P_1)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.4\mu(S_1) + 0.6\mu(L_1)
\]

Level $P_2$

\[
A(P_2)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) + 0.33\mu(L_4) + 0.33\mu(M_4)
\]

\[
A(P_2)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) + 0.4\mu(L_3) + 0.2\mu(M_3)
\]

\[
A(P_2)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.4\mu(S_2) + 0.6\mu(L_2)
\]

\[
A(P_2)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.4\mu(S_1) + 0.6\mu(L_1)
\]

Level $P_3$

\[
A(P_3)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) + 0.33\mu(L_4) + 0.33\mu(M_4)
\]

\[
A(P_3)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) + 0.4\mu(L_3) + 0.2\mu(M_3)
\]

\[
A(P_3)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.45\mu(S_2) + 0.55\mu(L_2)
\]

\[
A(P_3)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.45\mu(S_1) + 0.55\mu(L_1)
\]
Combining the Levels for MACRO Level (Accessibility Rules)

The rules depend on the areas considered (area of level 1, 2, 3, 4 – occasionally, monthly, weekly and daily). The logic strictly follows the means defined by Tannier (2012; working paper) modified by Czerkauer-Yamu, Frankhauser:

The accessibility $A$ is hierarchically structured. From a functional point of view the explicit hierarchical approach allows a relational link to be made between frequency of different amenities and the corresponding distances.

Oberzentrum $P_1$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_1) = 0.25A(P_1)[F_4] + 0.25A(P_1)[F_3] + 0.25A(P_1)[F_2] + 0.25A(P_1)[F_1] \quad [17]$ 

Mittelzentrum $P_2$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_2) = 0.25A(P_2)[F_4] + 0.25A(P_2)[F_3] + 0.25A(P_2)[F_2] + 0.25A(P_2)[F_1] \quad [18]$ 

Unterzentrum $P_3$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_3) = 0.3A(P_3)[F_4] + 0.3A(P_3)[F_3] + 0.2A(P_3)[F_2] + 0.2A(P_3)[F_1] \quad [19]$ 

Kleinzentrum $P_4$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_4) = 0.5A(P_4)[F_4] + 0.25A(P_4)[F_3] + 0.1875A(P_4)[F_2] + 0.0625A(P_4)[F_1] \quad [20]$
Accessibility Rules – MICRO Level

Within the “fine” scale accessibilities via networks are included.

Level 1 (Occasional Level)

Services

- Distances $d_i$ are computed on the network for each cell $i$ (centroid) to the gravity centre of the set of the level 1 facilities.

![Diagram showing processing of distance for service amenities on level 1.](image)

Figure 17: Processing distance $d_i$ for service amenities on level 1.

- Neither diversity nor number are important since they are the same for all sites in the highest-level catchment area. Hence, only distance is taken into account. This is a linearly declining $\mu$-function.

$$
\mu_i(S_i) =
\begin{cases} 
1 & \text{for } d_i \leq 20\text{km} \\
2 - \frac{1}{20} d_i & \text{for } 20\text{km} < d_i \leq 40\text{km} \\
0 & \text{for } d_i > 40\text{km}
\end{cases}
$$

[21]

where:

- $S_i$ = service amenities on level 1
- $d_i$ = distance [km]
- $i$ = cell

![Graph showing evaluation of distance for services on level 1.](image)

Figure 18: Evaluation of distance for services on level 1.
Green and Leisure Amenities

- The presence of several categories of amenities is taken into account (diversity) without evaluation of numbers of the same category (no competition since usually publicly managed facilities).
- No clusters
- Distances are taken into account: nearest distance to amenity for each category and mean distance for all categories.
- The green and leisure amenities are identified by their accessibility points. These points correspond to the centroid for sports areas etc.
- For area objects such as e.g. forests we take the intersection point between the shortest route between the gravity centre of the cell and the area object’s boundary.

![Figure 19: Processing distance $d_i$ for green amenities on level 1.](image)

- For each leisure category we take the nearest leisure amenity; further all categories are then combined. The evaluation of distance again follows a linearly declining function.

$$
\mu_i(L_i) = \begin{cases} 
1 & \text{for } d_i \leq 60 \text{km} \\
\frac{5}{2} - \frac{1}{40} d_i & \text{for } 60 \text{km} < d_i \leq 100 \text{km} \\
0 & \text{for } d_i > 100 \text{km}
\end{cases}
$$

[22]

where:

$L_i = \text{green amenities on level 1}$

$d_i = \text{distance [km]}$

$i = \text{cell}$

![Figure 20: Linear distance decrease for services on level 1.](image)
**Level 2 (Monthly Level)**

**Services**

- Distances \(d_i\) are computed on the network for each cell \(i\) (centroid) to the gravity centre of the set of the level 2 facilities.

\[
\delta \text{ and } d_i \text{ are important, not the amount (number) of services. Diversity is considered as the different central places of level 2 may have different types of services. Hence, we combine diversity and distance.}
\]

\[
\mu_i(S_2) = \mu(d_i)^{(\delta)}
\]

where:

- \(S_2 = \text{services on level 1}\)
- \(d_i = \text{distance [km]}\)
- \(\delta = \text{diversity}\)
- \(i = \text{cell}\)

*Diversity is of importance for distinguishing the attractiveness of the different central places of the same level. This formalisation has been chosen since diversity is more important than distance.*

This is a linearly declining \(\mu\)-function:

\[
\mu_i(d_i) = \begin{cases} 1 & \text{for } d_i \leq 20\text{km} \\ 2 - \frac{1}{20}d_i & \text{for } 20\text{km} < d_i \leq 40\text{km} \\ 0 & \text{for } d_i > 40\text{km} \end{cases}
\]

where:

- \(d_i = \text{distance [km]}\)
- \(i = \text{cell}\)
Green and Leisure Amenities

- The presence of several categories of amenities is taken into account (diversity) without evaluation of numbers of the same category (no competition since usually publicly managed facilities).
- No clusters
- Distances are taken into account: nearest distance to amenity for each category and mean distance for all categories.
- The green and leisure amenities are identified by their accessibility points. These points correspond to the centroid for sports areas etc..
- For area objects such as e.g. forests we take the intersection point between the shortest route between the gravity centre of the cell and the area object’s boundary.

- For each leisure category we take the nearest leisure amenity; further all categories are then combined. The evaluation of distance again follows a linearly declining function.
No Morphological Rule.

Level 3 (Weekly Level)

Services

- The presence of several amenities is taken into account (diversity $\delta$) with evaluation of numbers $n_j$ of the same category
- Different centres lying within a distance range are taken into account (3 km), usually by means of a linearly declining function ($\mu(d) = 1$ up to 3km, linear decrease up to 10km, then $\mu(d) = 0$
- Clusters are introduced (range: 800m)
- Distance $d_{ij}$ from cell $i$ (centroid) to cluster $j$ is taken into account
- The formalization strictly follows that proposed by Tannier according to the MUP-city logic (Tannier, Vuidel, Houot, Frankhauser 2012).

(Zimmermann-Zysno operator combining the different effects as in MUP-city\textsuperscript{40} with other distance standards).

\[\mu_i(L_2) = \begin{cases} 
1 & \text{for } d_i \leq 60\text{km} \\
\frac{5}{2} - \frac{1}{40}d_i & \text{for } 60\text{km} < d_i \leq 100\text{km} \\
0 & \text{for } d_i > 100\text{km}
\end{cases} \]  

where:

\[d_i = \text{distance [km]}\]

\[i = \text{cell}\]

Figure 24: Linear distance decrease for services on level 2.
\[ Y_j = [\mu(n_j)\mu(\delta_j)]^{1-\mu(d_{ij})} \cdot [1 - \mu(n_j)\mu(\delta_j)]^{1-\mu(d_{ij})} \]  \[26\]

where:
- cell = \( i \)
- services = \( j \)
- number of services = \( n_j \)
- diversity (number of different types) for aggregation \( j = \delta_j \)
- distance for every cell \( i \) and aggregation \( j = d_{ij} \)
- accessibility for a cell \( i \) and aggregation \( j = Y_{ij} \)

The operator \( \mu(S_j) \) evaluates the accessibility of the cell \( i \) to the set of service clusters with weekly frequentation:

\[ \mu(S_j) = 1 - \prod_j (1 - Y_{ij}) \]  \[27\]

**Green and Leisure Amenities**

- The presence of several amenities is taken into account (diversity \( \delta_j \)) \textit{without} evaluation of numbers of the same category.
- No clusters.
- Distances \( d_i \) are taken into account: nearest distance to amenity for each category and further mean distance for all categories.
- Green and leisure amenities are identified by their accessibility points. These points correspond to the centroid for \textit{sports areas} etc.
- For area objects such as e.g. forests we take the intersection point between the shortest route between the gravity centre of the cell and the area object’s boundary.

![Figure 25: Processing distance \( d_i \) for green amenities on level 3.](image)
- For leisure then we take the nearest leisure amenity; further all categories are then combined. The evaluation of distance again follows a linearly declining function.

\[
\mu_i(L_i) = 1 \quad \text{for } d_i \leq 2km \\
\mu_i(L_3) = \frac{15}{13} - \frac{1}{13} d_i \quad \text{for } 2km < d_i \leq 15km \\
\mu_i(L_3) = 0 \quad \text{for } d_i > 15km
\]

where:

\(L_3 = \text{green amenities on level 3}\)

\(d_i = \text{distance [km]}\)

\(i = \text{cell}\)

Figure 26: Distance evaluation for leisure and green amenities on level 3.

**Morphological Rule**

Distance between neighbouring cells decreases in a linear fashion (according to Von Neumann logic); for morphological rule see below.
**Level 4 (Daily Level)**

**Services**

- The presence of several amenities is taken into account (diversity $\delta_j$) with evaluation of numbers $n_j$ of the same category
- Different centres lying within a distance range are taken into account (3 km), usually by means of a linearly declining function $\mu(d) = 1$ up to 3 km, linear decrease up to 10 km, then $\mu(d) = 0$
- Clusters are introduced (range: 800 m)
- Distance $d_{ij}$ from cell $i$ (centroid) to cluster $j$ is taken into account
- The formalization strictly follows that proposed by Tannier according to the MUP-city logic (Tannier, Vuidel, Houot, Frankhauser 2012).

(Zimmermann-Zysno operator combining the different effects as in MUP-city with other distance standards).

\[
Y_{ij} = \left[ \mu(n_j)^{\mu(\delta_j)} \mu(d_{ij}) \right]^{1-\mu(\delta_j)} \cdot [1 - (1 - \mu(n_j)^{\mu(\delta_j)})(1 - \mu(d_{ij}))]^{\mu(\delta_j)}
\]  \[29\]

where:
- cell = $i$
- services = $j$
- number of services = $n_j$
- diversity (number of different types) for aggregation $j = \delta_j$
- distance for every cell $i$ and aggregation $j = d_{ij}$
- accessibility for a cell $i$ and aggregation $j = Y_{ij}$

The operator $\mu(S_4)$ evaluates the accessibility of the cell $i$ to the set of the service clusters with daily frequentation:

\[
\mu(S_4) = 1 - \prod_j (1 - Y_{ij})
\]  \[30\]
Green and Leisure Amenities

- The presence of several amenities is taken into account (diversity $\delta_j$) without evaluation of numbers of the same category.
- No clusters.
- Distances $d_i$ are taken into account: nearest distance to amenity for each category and mean distance for all categories.
- Green and leisure amenities are identified by their accessibility points. These points correspond to the centroid for sports areas etc..
- For area objects such as e.g. forests we take the intersection point between the shortest route between the gravity centre of the cell and the area object's boundary.

Figure 27: Processing distance $d_i$ for green amenities on level 4.

- For leisure then we take the nearest leisure amenity; further all categories are then combined. The evaluation of distance again follows a linearly declining function.

$$
\begin{align*}
\mu_i(L_4) &= 1 \quad \text{for } d_i \leq 0.8 km \\
\mu_i(L_4) &= 2 - \frac{5}{4} d_i \quad \text{for } 0.8 km < d_i \leq 1.6 km \\
\mu_i(L_4) &= 0 \quad \text{for } d_i > 1.6 km
\end{align*}
$$

where:

$L_4 =$ green amenities on level 4

$d_i =$ distance [km]

$i =$ cell

Figure 28: Distance evaluation for services on level 4.
Morphological Rule
Distance between neighbouring cells decreases in a linear fashion (according to Von Neumann logic); for morphological rule see below.
Combining the Criteria for Each MICRO Level

On the microscale global evaluation is of course only of interest for the cells which lie within the selected mesh. Thus, the only relevant rules are those which refer to the type of mesh selected. decreases in a linear fashion rules in awareness of the fact that only some of them will be used in a given context.

Amenities (services and facilities):
- $F_4$: Daily frequentation, services $S_4$ and leisure amenities $L_4$, morphological rules $M_4$
- $F_3$: Weekly frequentation, services $S_3$ and leisure amenities $L_3$, morphological rules $M_3$
- $F_2$: Monthly frequentation, services $S_2$ and leisure amenities $L_2$
- $F_1$: Occasionally frequentation, services $S_1$ and leisure amenities $L_1$

Centrality levels:
- $P_1$: Important central place (e.g. Vienna; prime city) “Oberzentrum”
- $P_2$: Intermediate central place (town) “Mittelzentrum”
- $P_3$: Small central place “Unterzentrum”
- $P_4$: Petit central place (village, hamlet) “Kleinzentrum”

At each level the three or respectively two types of criteria (service, leisure and morphology evaluation) for $M_4$ and $M_3$ are weighted and the arithmetic mean is computed, i.e.

**Level $P_1$**

\[ A(P_1)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) + 0.33\mu(L_4) + 0.33\mu(M_4) \]
\[ A(P_1)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) + 0.4\mu(L_3) + 0.2\mu(M_3) \]
\[ A(P_1)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.4\mu(S_2) + 0.6\mu(L_2) \]
\[ A(P_1)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.4\mu(S_1) + 0.6\mu(L_1) \]

**Level $P_2$**

\[ A(P_2)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) + 0.33\mu(L_4) + 0.33\mu(M_4) \]
\[ A(P_2)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) + 0.4\mu(L_3) + 0.2\mu(M_3) \]
\[ A(P_2)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.4\mu(S_2) + 0.6\mu(L_2) \]
\[ A(P_2)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.4\mu(S_1) + 0.6\mu(L_1) \]
Combining the Levels for MICRO Level (Accessibility Rules)

The rules depend on the areas considered (area of level 1, 2, 3, 4 – occasionally, monthly, weekly and daily). The logic strictly follows the means defined by Tannier (2012; working paper) modified by Frankhauser, Czerkauer-Yamu (2012):

The accessibility $A$ is hierarchically structured. From a functional point of view the explicit hierarchical approach allows a relational link to be made between frequentation of different amenities and the corresponding distances.

Oberzentrum $P_1$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_1) = 0.25A(P_1)[F_4] + 0.25A(P_1)[F_3] + 0.25A(P_1)[F_2] + 0.25A(P_1)[F_1]$  \[36\]

Mittelzentrum $P_2$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_2) = 0.25A(P_2)[F_4] + 0.25A(P_2)[F_3] + 0.25A(P_2)[F_2] + 0.25A(P_2)[F_1]$  \[37\]

Unterzentrum $P_3$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_3) = 0.3A(P_3)[F_4] + 0.3A(P_3)[F_3] + 0.2A(P_3)[F_2] + 0.2A(P_3)[F_1]$  \[38\]

Kleinzentrum $P_4$ to amenities $F_4$, $F_3$, $F_2$, $F_1$:

$A(P_4) = 0.5A(P_4)[F_4] + 0.25A(P_4)[F_3] + 0.1875A(P_4)[F_2] + 0.0625A(P_4)[F_1]$  \[39\]
Morphological Rule –
Lacunarity Rule including Landscape View Rule

- We measure the minimum distances $d_{\text{min}}$ of separate cells in proximity of same order (independent of their size). Thus, we measure distances between two cells of order 1 and distances of two cells of order 2.
- If one or more buildings are located between the two assessed cells, the minimum distance $d_{\text{min}}$ is taken to the building located in closest proximity to the cell's border under scrutiny. The minimum distance is always taken in all directions.

a) With this theoretical configuration open and green spaces are the biggest as possible in the context of well connected spaces consistent through scale.

b) Evaluation for two equal sized meshes of $d_{\text{min}}(2.1;2.2), d_{\text{min}}(2.1;2.4) = d_{\text{min}}(2.1;2.4)$

Figure 29 (a, b): Morphological rule set for lacunarity including landscape view.

- We measure the minimum distances $d_{\text{min}}$ of separate cells in proximity of same order (independent of their size). Thus, we measure distances between two cells of order 1 and distances of two cells of order 2.
- For the evaluation the logic of a Manhatten metric is used (x and y are equal).
- Within a mesh the distances are only evaluated with respect to the same order elements (at least for this simple version of the model with only two reduction factors). The neighbourhood of elements of different order are allowed to be adjacent.
- For adjacent meshes the same rules apply, regardless of the cell’s size (order takes priority over).
- The size of buffer corresponds to $l$. The buffer is defined by the base length of a “sub-cell” (order 2) within a mesh. The buffer $l$ is potentially different for each assessed mesh.

_N.b._ The evaluation should take into consideration the base length of every architectural object (e.g. house) and the corresponding distance from the cell’s border to the object. As the programming for this is not yet possible a simplified evaluation rule. In the absence of detailed information we take the mean value of the minimum distance $d_{\text{min}}$ of all present distances to buildings and the best evaluation ($l=1$).

This takes account of the type of housing, e.g. a small single family house versus a linear housing block blocking the view. Thus, using the arithmetic mean we assume that there are still (remaining) open views to the surrounding landscape.

By applying the rule at different decomposition steps for different levels of analyses levels we take into account open and green space of decreasing size. Let us remind here that the size of open and green spaces is directly related to their frequency of use. Small open spaces are daily used, whereas medium sized ones are weekly and very big ones are monthly or rarely used.

_Example:_ Element 2.1.

- Evaluation $\min (d(2.1;2.2), d(2.1;2.4)) = d(2.1;2.4)$ evaluated according to linearly declining function:

$$\mu(d_{\text{min}}) = \begin{cases} \frac{d_{\text{min}}}{l} & \text{for } d_{\text{min}} \leq l \\ 1 & \text{for } d_{\text{min}} > l \end{cases}$$

[40]

In the event that neighbouring cells (belonging to different meshes) are of the same order but different size, the smallest cell size corresponds to $l$. 
Definition of Distances and Information of the Fractalopolis Software

In the following we define criteria for determining the distances used in the previous evaluations. The best evaluated metric distances are chosen and used. The path time is computed and users can also retrieve this information. We now define the “metric“ according to the facility levels.

**Level 1 (Occasional Level) and Level 2 (Monthly Level)**

- Car accessibility via road network (including speed limits)
- Public transport network (PTN) (railway network)

![Figure 30: Depiction of the rule set for accessibility; green lines correspond to pedestrian access by using the street network.](image)

Three options for evaluation:
- Car access
- PTN access
- Car and PTN access: best evaluation ist taken for evaluation function

*(behaviour as usual)*
**Level 3 (Weekly Level)**

The weekly level incorporates different alternatives for evaluation:

- Car accessibility via road network
- PTN (regional bus network)

![Figure 31](image1.png)

Figure 31: Depiction of the rule set for accessibility; green lines correspond to pedestrian access by using the street network.

**Three options for evaluation:**
- Car access
- PTN access
- Car and PTN access: best evaluation is taken for evaluation function *(behaviour as usual)*

**Level 4 (Daily Level)**

**Evaluation:**
- Pedestrian accessibility by using the street network (according to Tannier, Vuidel, Frankhauser 2010)

![Figure 32](image2.png)

Figure 32: Depiction of the rule set for accessibility to clusters (modified; original Tannier 2012).
The Fractalopolis Software – A Simple Guide.

Based on the previous formalisation of Fractalopolis we will explain in the following how the software Fractalopolis 0.6 works and how the user can create multi-scale spatial scenarios on a macro and micro scale including accessibility and morphological evaluations. Please note, that the Fractalopolis software is an ongoing research.

Before we begin
Prepare shape files (ESRI format for geodata) containing areas and points including max. distances and definition of levels according to the formalisation for the area under scrutiny. For a minimum set you need to prepare the following layers:

For MACRO Level:
- Built-up area
- Population (e.g. at municipal level)
- Highways and motorways
- Railway network
- Green areas
- Restricted zones
  (these are zones where building is subject to special requirements, including landscape conservation zones and slope restrictions)
- Water
  (-Hillshade and agriculture can be helpful)

For MICRO Level:
- Detailed built-up area
  (this will also serve as a basis for the 3D model)
- Population (e.g. at municipal level)
- Road network (detailed)
- Railway network and stations
- Bus network and stops
- Green areas
- Services
- Leisure
- Restricted zones
- Water

The software was programmed at ThéMA, Université de Franche-Comte, France by Gilles Vuidel.
To create a new project from scratch the built-up area layer and the population layer have to be loaded. A folder will be created containing all loaded shape files including a project file. This helps to transfer projects in general. Additional layers can be added using File – Set layer.

Once the layers are loaded you have the possibility to change the colour for each individual layer and add labels by ticking the box Draw labels.

Once all layers are set the fractal generator can be defined (Iterated Function System – IFS Editor \(^{42}\)). At the moment two different ranks can be calculated; rank 1 for centres and rank 0 for sub-centres and/or periphery. This has an impact on the population model. IFS can be set individually for macro and microscale.

N.b. the fractal generator links to the Christallerian idea; the generated cells (iteration steps) will define the different sizes of potential development areas on an iteration level (multi-fractal logic).

\(^{42}\) The IFSs are a method of constructing fractals.
From the very beginning you will have the initial figure (blue square; iteration step 0) defining the catchment area for the area under scrutiny. The initiator can be varied in size and position, which will influence the scenarios. Below we show iteration steps 1 and 2 (for a theoretical multifractal see left images). (For an easy colouring of the layers you can move the initiator to the right within the window.) For developing planning scenarios, each cell can be moved within the mesh in order to indicate a potential strategic development area – either for an urban infill, consolidation, downsizing (by identifying the worst measures for accessibility measures to facilities and leisure), or extension of an area (by identifying the best accessibility measures). Further, any necessary economic, ecological and social enhancement (e.g. more shops for daily use; public transport stops; schools and kindergartens) can be discussed and analysed.
The iteration steps are obtained by opening the *macro scale monitor* for macro scale and the *micro scale monitor* for micro scale using menu – *macro scale – monitor* and the same for micro scale. For the iteration steps simply click on *Add*. With *Next* and *Previous* you can browse through the iteration steps and make changes to your scenarios at any time. After going backwards and forwards and making changes you also need to update the statistics by clicking *Update stats*. By clicking on *Init* (initiator) you can remove all fractal steps. *Limits* helps you to stay within the catchment area when moving cells within a mesh, whereas *Overlap* prevents overlapping of cells.

This monitor provides a number of features. The most important features are always visible.

*Step*: Current iteration step; below this, information on the current step in relation to the previous step and the initiator (step 0) is displayed.

*Build*: Built-up area according to GIS files; percentage calculated between previous and current iteration step; percentage calculated from initiator

*Pop*: Real population of current iteration step; percentage – same logic as before

*Urban Pop*: Urban population; from this measure we know how much population is in the countryside (see population model); percentage – same logic as before

*Model pop*: is the population determined by the population model with coefficients estimated by regression.

*User pop*: is the population determined by the population model with coefficients given by the user.
View allows us to display a coloured image of the cells for a chosen feature, e.g. density. Transparent view is the default setting (blue cells). The monitor further offers a Pop model, which allows you to define a model population and redistribution of existing population for the different ranks.

To change between macro and micro scale, choose a cell of interest at any iteration step on the macro scale and go to menu – micro scale – create. The micro scale monitor appears with the chosen cell as the new initiator for micro scale. Hence, a new IFS can be defined. If you do not define a new IFS for micro scale the previously defined IFS for macro scale will be used. On the vertical layer bar micro scale will be added. (Experience shows that it makes sense to change the scale, macro to micro, at iteration step 3 or 4 by a given basic length of the initiator on macro scale of appr. 200km.)

On macro scale Fractalopolis is a a normative model, whereas on micro scale it follows the logic of accessibility. Of course, accessibility can be calculated on macro and micro scale, though it is preferable and more useful to do it on the micro scale. (Note: We need to map the whole catchment area = initiator on macro scale for services and facilities as well as leisure and public transport)

Add all layers as mentioned above (see list). By clicking update statistics and next the accessibility evaluation will be colour coded from green (= 1) to red (= 0) in the scale monitor. In the scale monitor you can view different accessibility evaluations including the morphological evaluation and a global accessibility measure. The accessibility distance measures, the combination of facilities as well as their preponderation (aggregation levels) and can be changed to reflect any metropolitan area under scrutiny. When browsing through the iteration steps the colour code corresponding to the evaluation on each level will be kept.
The simulation is useful down to an architectural scale (plot and block size). The scenarios can be exported as shapefiles, svg and TIFF files using menu – file – export. The export as shapefiles supports further handling and processing in GIS and also the creation of a 3D model.

Accessibility parameters

Example of evaluation for a theoretical multifractal.
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