Bottom-up thermalization from holography?

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Motivation

**quark gluon plasma**
- produced in heavy collisions at RHIC and LHC
- behaves as a strongly coupled liquid
- thermalization process not well understood

**goals**
- gain insight into the thermalization process
- modification of production rates of photons/dileptons
- which modes thermalize first: top-down or bottom-up?
- dependence on coupling strength

**strategy**
- SYM where strong and weak coupling regimes are accessible
Thermalization scenarios

**bottom up scenario**

- at weak coupling
- scattering processes
  - in the early stages many soft gluons are emitted which then thermalize the system (*Baier et al (2001)*)
  - driven by instabilities
    - instabilities isotropize the momentum distributions more rapidly than scattering processes (*Kurkela, Moore (2011)*)

**top down scenario**

- at strong coupling
- UV modes thermalize first
- in AdS calculations, follows naturally from causality
Photon emission in heavy ion collisions

Photons are emitted at all stages of the collision

- initial hard scattering processes: quark anti-quark annihilation:
  - on-shell photon or virtual photon → dilepton pair
- strongly coupled out of equilibrium phase: no quasiparticle picture
- additional (uninteresting) emissions from charged hadron decays
Probing the plasma

once produced photons/dileptons stream through the plasma almost unaltered

provide observational window in the thermalization process of the plasma

quantity of interest

spectral density: \( \chi_\mu = -2 \text{Im}(\Pi_{\mu}^{\text{ret}})(k_0) \)

number of photons emitted with given momentum

fluctuation dissipation theorem

\[ \eta^{\mu\nu} \Pi^{\leq \nu}_{\mu}(\omega) = -2n_B(\omega)\text{Im}(\Pi_{\mu}^{\text{ret}})(\omega) = n_B(\omega)\chi(\omega) \]

production rate

\[ k^0 \frac{d\Gamma_\gamma}{d^3 k} = \frac{\alpha}{4\pi^2} \eta^{\mu\nu} \Pi^{\leq \nu}_{\mu}(\omega = k^0) \]
Photon emission in equilibrium SYM plasma

perturbative result

- increasing the coupling: slope at \( k=0 \) decreases, hydro peak broadens and moves right

strong coupling result

- decreasing coupling from \( \lambda = \infty \): peak sharpens and moves left
Out of equilibrium

- equilibrium picture in SYM fairly complete
- how does photon/dilepton production get modified out of equilibrium
- can one access thermalization at finite coupling?
The falling shell setup

outside and inside spacetime

- metric:
  \[ ds^2 = \frac{(\pi TL)^2}{u} (f(u)dt^2 + dx^2 + dy^2 + dz^2) + \frac{L^2}{4u^2f(u)}du^2 \]
  \[ u = \frac{r_h^2}{r^2} \]
  \[ f(u) = \begin{cases} 
  f_+(u) = 1 - u^2, & \text{for } u > 1 \\
  f_-(u) = 1, & \text{for } u < 1 
\end{cases} \]

outside solution

\[ E_+ = c_+ E_{in} + c_- E_{out} \]
matching condition

Israel junction condition

- extrinsic curvatures match across the shell
  \[ [K_{ij}] - [K]g_{ij} = 0, \quad [K_{ij}] = K^+_{ij} - K^-_{ij} \]
- can be also adapted for other fields

Fourier transformation

- discontinuity in the time coordinate
  \[
  \frac{dt_-}{dt_+} = \sqrt{\frac{f_+}{f_-}} = \sqrt{f_m} \quad \Rightarrow \quad \int dt_+ e^{i\omega_+ t_+} = \frac{1}{\sqrt{f_m}} \int dt_- e^{i\omega_+ t_-},
  \]
- identification: \( \omega_- = \omega_+ / \sqrt{f_m} \)

matching condition:

\[
E_-(\omega_-)|_{u_s} = \sqrt{f_m}E_+(\omega_+)|_{u_s},
E'_-(\omega_-)|_{u_s} = f_mE'_+(\omega_+)|_{u_s}.
\]

quasistatic approximation:

- energy scale of interest >> characteristic time scale of shell’s motion
equation of motion

equation of motion for transverse electric field

\[ E'' + \frac{f'}{f}E' + \hat{\omega}^2 - \hat{q}^2 \frac{f}{uf^2} E = 0, \quad E'_{z} \equiv F'_{tz} \]

\[ \hat{\omega} \equiv \omega/(2\pi T), \quad \hat{q} \equiv q/(2\pi T) \]

\[ T = \frac{\tau_{h}}{\pi} \]

- this equation is solved numerically by the ansatz:

\[ E_{\text{in}} (u, \hat{\omega}, \hat{q}) = (1 - u)^{\mp \frac{i\hat{\omega}}{2}} y_{\text{in}} (u) \]

retarded correlator

\[ \Pi(\omega, q) = -\frac{N_{c}^{2}T^{2}}{8} \lim_{u \to 0} \frac{E'(u, Q)}{E(u, Q)} = -\frac{N_{c}^{2}T^{2}}{8} \Pi_{\text{therm}} \frac{1 + \frac{c_{-}}{c_{+}} \frac{E'_{\text{out}}}{E'_{\text{in}}}}{1 + \frac{c_{-}}{c_{+}} \frac{E_{\text{out}}}{E_{\text{in}}}} \]

- reproduce thermal case:

\[ \lim_{r_{s} \to r_{h}} \frac{c_{-}}{c_{+}} \to 0 \]

- behaviour of \( c_{-}/c_{+} \) crucial for out of equilibrium dynamics
Photon & dilepton spectral density

Photon spectral density for $r_s/r_h = 1.1, 1.01, 1.001$

Dilepton spectral density for $r_s/r_h = 1.01$ and $q=0,1,2$

- out of equilibrium effect: oscillations around thermal value
- as the shell approaches the horizon equilibrium is reached
Thermalization at infinite coupling: photons

- relative deviation from thermal equilibrium

\[
    R(\hat{\omega}) = \frac{\chi(\hat{\omega}) - \chi_{th}(\hat{\omega})}{\chi_{th}(\hat{\omega})}
\]

- thermalization: increase in frequency and decrease in amplitude

- top down thermalization: highly energetic modes are closer to equ. value

\[
    \chi(\hat{\omega}) \approx \hat{\omega}^{2/3} \left( 1 + \frac{f_1(u_s)}{\hat{\omega}} \right), \quad R \approx \frac{1}{\hat{\omega}}
\]
Thermalization depending on the virtuality

- virtuality
  \[ v = \frac{\hat{\omega}^2 - \hat{q}^2}{\hat{\omega}^2} \]
- parametrize
  \[ q = c \hat{\omega} \]
- thermalization depends on the virtuality
- photons are last to thermalize
- same conclusion was reached in other models of thermalization 
  \[(Arnold et al; Chesler and Teaney)\]
Photon production rate

Photon production rate for $r_s/r_h = 1.1, 1.01, 1.001$

- enhancement of production rate
- hydro peak broadens and moves right
Photon production rate for $r_s/r_h=1.1, 1.01, 1.001$

- enhancement of production rate
- hydro peak broadens and moves right
- combining the two allows to study thermalization at finite coupling!
Finite coupling corrections

action: \[ S_{IIB} = S_{IIB}^0 + S_{IIB}^{\alpha'}, \]

\[ S_{IIB}^0 = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{-g} \left( R_{10} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4.5!} (F_5^5)^2 \right) \]

\[ S_{IIB}^{\alpha'} = \frac{L^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( \gamma e^{-3\phi} (C + T)^4 \right), \quad \gamma \equiv \frac{1}{8} \zeta(3) \lambda^{-\frac{3}{2}} \]

\[ T_{abcde} = i \nabla_a F_{bcdef} + \frac{1}{16} \left( F_{abcd} F_{def} - 3 F_{ab} F_{dec} \right), \]

Paulos (2008)

solving Einsteins equations

\[ ds^2 = \frac{r_h^2}{u} \left( -f(u) K^2(u) dt^2 + d\vec{x}^2 \right) + \frac{1}{4u^2 f(u)} P^2(u) du^2 + L^2(u) d\Omega_5^2 \]

\[ K(u) = e^{\gamma [a(u) + 4b(u)]}, \quad P(u) = e^{\gamma b(u)}, \quad L(u) = e^{\gamma c(u)}, \]

\[ a(u) = -\frac{1625}{8} u^2 - 175 u^4 + \frac{10005}{16} u^6, \]

\[ b(u) = \frac{325}{8} u^2 + \frac{1075}{32} u^4 - \frac{4835}{32} u^6, \]

\[ c(u) = \frac{15}{32} (1 + u^2) u^4, \]

Gubser et al; Pawelczyk, Theisen (1998)
Finite coupling corrections

equation of motion

- after all the contractions are worked out the eom for a transverse electric field takes the simple form

\[ \Psi''(u) - V(u)\Psi(u) = 0 \]

Hassanain, Schvellinger

- making the ansatz

\[ \Psi_{\text{in}}^{\text{out}}(u, \gamma) = (1 - u)^{\pm \frac{i\omega}{2}} \left( \psi_{\text{in}}^{(0)} + \gamma \psi_{\text{in}}^{(1)} + \mathcal{O}(\gamma^2) \right), \]

- inside solution (pure AdS) stays the same (Banks, Green (1998)), but relation between frequencies gets corrected

\[ \omega_- = \frac{\omega_+}{\sqrt{f_m}}, \quad f_m \equiv f(u_s)K^2(u_s), \]

- all the corrections have to be taken into account, e.g

\[ \frac{c_-}{c_+} = C_0 + \gamma C_1 \]

- spectral density

\[ \chi(\omega) = \frac{N_c^2 T^2}{2} \left( 1 - \frac{265}{8} \gamma \right) \text{Im} \left( \frac{\Psi'_+}{\Psi_+} \right) \bigg|_{u=0} \]
Photon production rate at finite coupling

behaviour very similar to thermal limit

emission rate for $r_s/r_h = 1.01$ and $\lambda = \infty$, 120, 80, 40

rate for $r_s/r_h = 1.1, 1.01, 1.001$ and $\lambda = 100$

- behaviour very similar to thermal limit
Relative deviation from thermal limit

\[ R \text{ for } r_s/r_h = 1.01 \text{ and } \lambda = \infty, 500, 300 \]

- Behaviour of relative deviation changes at large frequency
Thermalization at finite coupling

relative deviation from thermal limit

![Graph showing the relative deviation from the thermal limit as a function of frequency.]

- behaviour of relative deviation changes at large frequency
- decreasing the coupling: change happens at lower frequency
- indicates a change of the thermalization pattern from top-down towards bottom-up?
Thermalization at finite coupling

Thermalization at finite coupling

$\Pi(\omega) \approx \Pi_{\text{therm}} \frac{1 + (C_0 + \gamma C_1) \frac{E'_{\text{out}}}{E'_{\text{in}}}}{1 + (C_0 + \gamma C_1) \frac{E'_{\text{out}}}{E'_{\text{in}}}}$

$C_0 \approx \frac{1}{\omega}, \quad C_1 \approx \omega$

- behaviour of the fields near the horizon is crucial
- originates from the Schroedinger potential

WKB approximation

$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{3}{2}} \left( 1 + \frac{3\zeta(3)}{8\lambda^{\frac{3}{2}}} + \frac{f_1(u_s)}{\hat{\omega}} + \frac{f_2(u_s)\hat{\omega}}{\lambda^{\frac{3}{2}}} \right)$
Thermalization at finite coupling

 behaviour of the fields near the horizon is crucial
 originates from the Schrödinger potential

WKB approximation

\[ \chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left( 1 + \frac{3\zeta(3)}{8\lambda^{\frac{2}{3}}} \hat{\omega} + \frac{f_1(u_s)}{\hat{\omega}} + \frac{f_2(u_s)\hat{\omega}}{\lambda^{\frac{2}{3}}} \right) \]

so far: only photons that get emitted from the plasma
what about plasma constituents themselves?
Future directions I: $\langle T_{\mu\nu}T_{\alpha\beta} \rangle$

Relative deviation of the shear channel: $\langle T_{xy}T_{xy} \rangle$

relative deviation $R$ for $r_s/r_h=1.1$ and $\lambda = \infty$, 100, 50

- finite coupling effects are weaker
- for large energies relative deviation becomes constant
- can be seen from the behaviour of $c_-/c_+$
Future directions II: QNM analysis

QNM for R current correlator at infinite coupling

- Flow of the imaginary part of the first QNM:

\[ \text{Im} \omega_1 = 2\pi T \left( -1 + \frac{c}{\lambda^{3/2}} \right) \]
Conclusion

**thermalization at infinite coupling**
- enhancement of production rate
- top down thermalization
- depends on virtuality: on-shell photons are last to thermalize

**thermalization at finite coupling**
- enhancement of production rate
- indication of thermalization pattern changing from top down towards bottom up

**open questions**
- why does the causality argument not apply
- go beyond quasistatic approximation
- can one include finite coupling corrections in more involved models of holographic thermalization