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A Dynamic Analysis of Schelling's Binary Corruption Model: A Competitive Equilibrium Approach

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Abstract Schelling (in Micromotives and Macrobehavior, Norton, New York, 1978) suggested a simple binary choice model to explain the variation of corruption levels

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across societies. His basic idea was that the expected profitability of engaging in corruption depends on its prevalence. The key result of the so-called Schelling diagram is the existence of multiple equilibria and a tipping point. The present paper puts Schelling's essentially static approach into an intertemporal setting. We show how the existence of an unstable interior steady state leads to thresholds such that history alone or history in addition to expectations (or coordination) is necessary to determine the long-run outcome. In contrast to the related literature, which classifies these two cases according to whether the unstable equilibrium is a node or a focus, the actual differentiation is more subtle because even a node can lead to an overlap of solution paths such that the initial conditions alone are insufficient to uniquely determine the competitive equilibrium. Another insight is that a (transiently) cycling competitive equilibrium can dominate the direct and monotonic route to a steady state, even if the direct route is feasible.

Keywords Corruption · Schelling diagram · Intertemporal competitive equilibria · Thresholds · History versus expectations

1 Introduction

This paper embeds Schelling's static description of corruption within a dynamic framework along the lines suggested in Krugman [1] for labor markets. In this way, we clarify the crucial and important distinction between history and/or expectation-dependent outcomes and find an additional and interesting point concerning the potential Pareto ranking among the intertemporal equilibria. In brief, the shortcut to the steady state may be dominated by first spiraling around the steady state.

Corruption is an old phenomenon that has a substantial impact on economic growth (mostly negative) and that affects all societies, albeit to different degrees. The organization Transparency International compares corruption internationally and documents a substantial variation across states. Several papers try to explain why similar socio-economic structures can give rise to such different levels of corruption (see, e.g., Andvig and Moene [2]). Given the long history of the phenomenon, it is no surprise that corruption has already been analyzed by the ancients (e.g., in Plato) and in the political science literature. Friedrich [3] documents historical examples of corruption (in the Roman empire, monarchical England, Prussia, Russia, France, and the United States). These examples, as well as the recent revolutions in the Arab world, reveal a link between corruption and dictatorships, confirming the famous quotation from Lord Acton (taken from Popper [4, p. 137]) that all power corrupts and that absolute power corrupts absolutely. Although democracy itself seems not a sufficient guarantee against bribery, some political scientists, e.g., Friedrich [3], claim at least an inverse relation between corruption and popular support. However, corruption need not always harm economic development if it provides the grease to oil the bureaucracy. According to Friedrich [3], some even argue that "politics needs all these dubious practices; it cannot be managed without … corruption." These practices might even help to lessen the abuse of state authorities. Furthermore, Lui [5] argues that, in order to increase income from bribes, bureaucrats speed up the administrative process; social welfare might increase because of this.

Early economic investigations of corruption include Rose-Ackerman's [6, 7] (with a recent follow-up, [8]) and Schelling's [9, 10]. Shleifer and Vishny [11] differentiate between beneficial and harmful corruption. Concerning the latter, think for instance about the disastrous effect if different bureaucracies with overlapping jurisdictions must each be bribed to obtain permits and licenses (e.g., for operating a business), as happens, for example, in post-communist Russia. These papers, as well as the bulk of this literature, are static. Therefore, one objective here is to investigate corruption within a dynamic framework accounting for rational actors (e.g., bureaucrats) and (social) interdependence among them. The starting point is Schelling's model [9, 10], which illustrates how the *same* bureaucrat within the *same* political and economic system can be corrupt or not depending on the aggregate level of corruption.

More precisely, this investigation focuses on both dynamics and social interactions. Dynamics are a crucial characteristic of corruption that evolves, spreads, and turns out to be hard to eradicate. Examples of dynamic treatment of the evolution of corruption are investigations of (optimal) deterrence of corruption. Lui [5] observed that, in Communist China, episodes of pervasive corruption were followed by anti-corruption campaigns. Feichtinger and Wirl [12] attempt to endogenize such episodes of crusades against corruption alternating with periods of little or no deterrence. Recently, Dong and Torgler [13] presented an overlapping generations model in order to empirically assess the link between democracy, inequality, property rights, and corruption. Likewise, Caulkins et al. [14] consider the problem of a political leader whose corrupt actions influence the corruption level of a bureaucracy in a dynamic framework.

The incidence of corruption varies strongly across nations, regions, societies, and also time. For example, Singapore, which today occupies always a top position among the least corrupt countries in Transparency International rankings, was considered to be very corrupt in the 1950s. Russia and other ex-Soviet republics, in contrast, moved from corrupt (a characteristic of the Socialist economies that was overlooked in the West, see Friedrich [3]) to worse. Given these experiences, an important question is why more people are not corrupt? An important observation is that the profitability of a corrupt transaction compared to that of rejecting a bribe depends in part on the number of other people accepting bribes (compare Andvig [15, p. 69]). A number of papers recognize this importance of social pressure for understanding corruption. Social interactions are the subject of Glaeser et al. [16], and this (static) concept is applied to crime in Glaeser et al. [17]. More recently, Dong and Torgler [18] find social interaction significant for corruption in a cross-province panel in China 1998–2007. However, papers that account for both dynamics and social interdependence are rare. For example, Aidt's rather comprehensive survey [19] does not include dynamic approaches to corruption in spite of referring to multiple equilibria. Wirl [20] and Epstein [21] investigate dynamics and social interactions within the context of corruption via cellular automata; this approach adds spatial elements but ignores intertemporal optimization. Feichtinger et al. [22] try to explain corruption as a cyclical phenomenon within a dynamic model, but the model is only descriptive.

The objective of this paper is to account for rational agents who optimize intertemporally and have rational expectations about the other agents' behavior. A corresponding starting point is Krugman's analysis [1] of competitive agents who can choose between two activities: working either in agriculture, which yields a constant wage, or in manufacturing, where the individual wage depends positively on the aggregate size of the industry. (Krugman stresses the importance of these increasing returns, although this property is not necessary; see [23].) In our application, bureaucrats must choose to accept (or even solicit) a bribe or to reject that bribe. Facing costs for moving between these two activities, agents must solve an intertemporal optimization problem in which future payoffs depend on the other agents' actions. More precisely, payoffs depend on some *aggregate* level X, which is a given parameter that is not subject to *individual* influence although individual and aggregate corruption coincide along any symmetric competitive equilibrium.

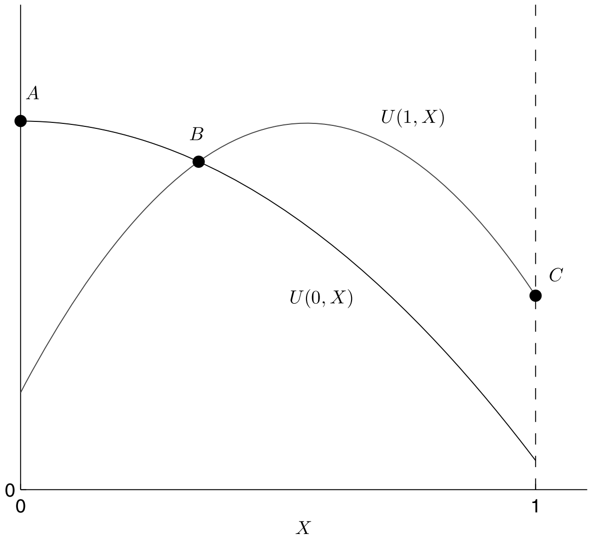
The contribution of this paper is twofold. Firstly, for the corruption literature, we provide a dynamic formulation that by and large corroborates Schelling's insight, produced 40 years ago, within a dynamic setting. Secondly, in the literature on rational expectation and intertemporal competitive equilibria (see, e.g., [1, 24, 25]) the following criterion is commonly applied: if the interior and unstable steady state is a node, then history—i.e., the initial state(s)—provides all the information needed to determine the future evolution. If the unstable steady state is a focus, then history alone is insufficient, and additional information about all the other agents' expectations is needed. However, this straightforward distinction is not entirely correct. The reason, as demonstrated within our application to corruption, is that even an unstable node can lead to an overlap, which requires, as in the case of a focus, additional information to determine future outcomes. As a consequence, the differentiation between the two cases, history versus history+expectations, is unfortunately more subtle and complicated than the literature suggests. Another and related point is that starting from the interior spiral of the saddle point path can Pareto-dominate the (usual) alternative of starting at the "envelope" and moving directly and monotonically to the steady state.

The paper is organized as follows. In Sect. 2, we sketch the basic idea based on the so-called Schelling diagram. The purpose of our analysis is to extend Schelling's model to an intertemporal setting. The model presented and analyzed in Sect. 3 shows how interior and boundary equilibria may occur: in particular, it illustrates how the existence of an unstable interior steady state leads to a tipping point. In Sect. 4, another (quadratic) utility function is considered to obtain additional insights. Finally, Sect. 5 concludes.

2 The Schelling Diagram

Thomas Schelling sketched out almost four decades ago that individual agents find it rational to accept or to reject bribes (compare [9, p. 388] and [10, Chap. 7]). Our exposition follows Andvig's notation [15, p. 70–75]. Denote by $X \in [0, 1]$ the level of corruption within a society; if $X = 0$, then everybody is "clean," while $X = 1$ refers to a totally corrupt society.

Fig. 1 The Schelling diagram. X represents level of corruption in a reference group; $U(0, X)$ denotes the individual utility of being noncorrupt, and $U(1, X)$ the individual utility of a corrupt action. A shows a stable boundary equilibrium with a "clean" society, B a stable boundary equilibrium with a "dirty" society, and C is an unstable interior equilibrium



Schelling assumes that the individual has only a binary choice $x \in \{0, 1\}$ to be either corrupt ($x = 1$) or not and that the individual is informed beforehand whether the others are corrupt. Schelling's core idea is that the profitability of individual corruption depends on the level of corruption in the society, X. Denoting the agent's payoff by $U(x, X)$, the decision to be corrupt or not depends on $U(0, X)$ and $U(1, X)$, see Fig. 1, and the analysis can be restricted to the following according to Andvig's survey [15]. Firstly, in a clean society the profitability of being corrupt is below that of being honest. (In other words, if (almost) everybody is noncorrupt, then breaking the rules may lead to higher feelings of guilt than in a "dirty" society. Furthermore, a black sheep in a herd of white ones can be caught more easily for a given capacity of enforcement interventions.) Thus, it is reasonable to assume that

$$U(0, 0) > U(1, 0). \tag{1}$$

Conversely, overall corruption renders individual corruption profitable (because there is no longer loss of individual reputation). Thus, we have

$$U(0, 1) < U(1, 1). \tag{2}$$

For our purposes, assumptions (1) and (2) are sufficient. As a consequence, there is at least one intersection of the two utility functions in the interior of the utility interval; the precise shapes of the two utility functions[1] are not crucial. Let us assume that there is a unique intersection of $U(0, X)$ and $U(1, X)$, denoted as B, as illustrated in Fig. 1. At B, the agent is *indifferent* between a noncorrupt and a corrupt action. However, this point B is an *unstable* equilibrium in the following sense: if only one

[1] In particular, we omit the discussion of the shape of the marginal utilities $U_X(i, X)$ for $i = 0, 1$; see, however, Andvig [15].

more member of the reference group is corrupt, it will pay to become corrupt. And since the higher profitability of corruption prevails right of B, the reference group will move to the high equilibrium C, i.e., the society will become "dirty." If one starts slightly below B, on the other hand, the agents have a private incentive to be honest, and the system converges to the stable low ("clean") boundary equilibrium denoted as A.

Thus, small changes in initial conditions around the unstable indifference point B will have a large impact on long-run behavior. This "history-dependence" has important economic and political consequences. A strong, but short-lived, anti-corruption campaign may move the society below the tipping point B. And since noncorrupt behavior is then more profitable, the whole system then will progress by its own movements to the clean stable equilibrium. The campaign may be lifted, provided that no exogenous shocks drive the society again above point B.

Schelling's approach provides a microfoundation for the observed widely different levels of corruption by linking macrolevel variables to individual (micro)profitability using the *same* set of economic assumptions. Although the Schelling diagram is vague concerning the dynamics of an equilibrium, it is a useful tool to explain some basic facts of economic corruption, such as multiple equilibria and tipping points.

We extend Schelling's analysis in two directions. First, we replace the binary corrupt/noncorrupt behavior with a continuous spectrum of various levels of individual corruption. Second, and more importantly, we generalize Schelling's static approach to rational agents with rational expectations solving a dynamic optimization model.

3 Dynamic Models of Corruption

Let $x \in [0,1]$ denote the individual decision maker's own degree of private corruption (i.e., the continuous share of corrupt acts of an agent), and $X \in [0,1]$ be the aggregate or average level of corruption. An agent's individual payoff is given by $U(x,X)$ since it depends on the average degree of corruption in the population X. The dynamics result from sluggish individual behavior, more precisely:

$$\dot{x}(t) = u(t), \qquad x(0) = x_0 \in]0,1[, \tag{3}$$

where u denotes the rate at which an individual decides to increase or decrease his or her degree of corruption. Any substantial change in individual behavior is costly, where the costs are given by $k(u,x,X)$. Costs of adjusting corruption may be due to moral (bad conscience) and economic reasons, because expanding corruption requires an expansion of business; a reduction in corruption, on the other hand, requires avoiding some otherwise fruitful contacts, alienation of former clients, etc. Marginal adjustment costs may increase or decrease in private or aggregate corruption; moral costs for becoming more corrupt decline as aggregate corruption increases, while the cost of expanding increases as everyone fights for the last niche to collect bribes. Therefore, the (competitive) agent's individual dynamic optimization problem is to

$$\max_{\{u(t)\in\mathbb{R}\}} \int_0^\infty e^{-rt}\big[U\big(x(t),X(t)\big) - k\big(u(t),x(t),X(t)\big)\big]\mathrm{d}t \tag{4}$$

subject to (3). Although the agents take the aggregate X as given (i.e., X cannot be influenced by individual actions), they know (rational expectations, i.e., perfect foresight) that

$$X(t) = x(t) \quad \forall t \geq 0 \tag{5}$$

for identical agents and a symmetric equilibrium. Summarizing, we have transformed Schelling's static approach to an infinite-time horizon dynamic optimization problem, where x is the state, u acts as the control variable, and X is the external factor affecting the decisions of individual agents. In spite of identity (5), X is not part of the optimization, i.e., it is only a parameter within the optimality conditions.

3.1 Reformulation as a Degenerate Game

To clarify the theoretical setting of the problem, we formally consider two decision makers. One is the (representative) individual agent considering the individual level of corruption $x(\cdot) \in [0, 1]$, and the other is the aggregate population with its corruption level $X(\cdot) \in [0, 1]$. Let $v(\cdot)$ denote the aggregate actions of the population; then the individual faces the following problem:

$$\max_{u(\cdot)} \left\{ \int_0^\infty \mathrm{e}^{-rt} \big[U\big(x(t), X(t)\big) - k(u\big(t, x(t), X(t)\big)\big] \mathrm{d}t \right\} \tag{6a}$$

$$\text{s.t.} \quad \dot{x}(t) = u(t) \tag{6b}$$

$$x(0) = X(0). \tag{6c}$$

Considering the dynamics of the aggregate population

$$\dot{X}(t) = v(t), \tag{6d}$$

we immediately find from assumption (5) and the initial condition (6c) that

$$v(t) = u(t), \quad t \geq 0, \tag{6e}$$

has to hold. Therefore, the problem of the aggregate population can also be formulated as an optimization problem yielding

$$\min_{v(\cdot)} \left\{ \int_0^\infty \mathrm{e}^{-rt} \big| u(t) - v(t) \big| \mathrm{d}t \right\} \tag{6f}$$

$$\text{s.t.} \quad \dot{X}(t) = v(t) \tag{6g}$$

$$X(0) = X_0. \tag{6h}$$

This reformulation shall help to clarify the mathematical foundation of the economically motivated problem. It also helps to explain the reason for the "deviating" behavior of the solutions. In reality, the problem is not an optimal control problem at all; it is rather a degenerate dynamic game. Moreover, the reformulation justifies the derivation of the necessary optimality conditions as it is done in the paper.

3.2 Linear Payoff U

The most simple case is that of a linear payoff: individual bribes increase linearly in aggregate corruption, while the payoff from honesty $(1-x)$ decreases with overall corruption. Therefore,

$$U(x,X) = x(a+bX) + (1-x)(c-dX) - \delta X^2,$$
$$a>0, b>0, c>0, d>0, \delta>0. \tag{7}$$

Figure 2 illustrates the above utility function. This chart ignores the last term, which is the public good externality of living in a corrupt society ($\delta > 0$) because this (additive) term is irrelevant for individual decisions.

A quadratic cost function k depending only on u and some parameter γ, which describes the adjustment costs, is assumed. For simplicity, the adjustment costs for becoming corrupt and noncorrupt are assumed to be symmetric. Note, however, an asymmetry that does not influence the second derivative of k with respect to u would only affect the location of steady states and not their stability properties. Therefore, asymmetries are not crucial for our analysis. The resulting model is very close to Krugman's labor market model (although Krugman [1] does not cite Schelling's earlier work [10]). The only difference is that the payoff of rejecting bribes declines with respect to X while the wage in agriculture is constant. Hence, the following analysis can be kept short. The advantage of (7)'s quadratic structure is that closed-form solutions of the steady states and precise characterizations of their stability properties are possible. The Hamiltonian of the associated agent's optimization problem (expressed below in current-values, see, e.g., Sethi and Thompson [26] and Grass et al. [27]) equals

$$\mathcal{H} = x(a+bX) + (1-x)(c-dX) - \frac{\gamma}{2}u^2 + \lambda u.$$

It is strictly jointly concave in the (u,x) such that all solution paths that satisfy the first-order conditions below are optimal since they also satisfy a limiting transversality condition.

By using Pontryagin's maximum principle we find that

$$u = \frac{\lambda}{\gamma}. \tag{8}$$

The costate equation is

$$\dot{\lambda} = r\lambda - (b+d)X + (c-a). \tag{9}$$

Differentiating (8) with respect to time and using $X=x$ and (8), we get

$$\dot{u} = ru + \frac{(c-a)-(b+d)x}{\gamma}. \tag{10}$$

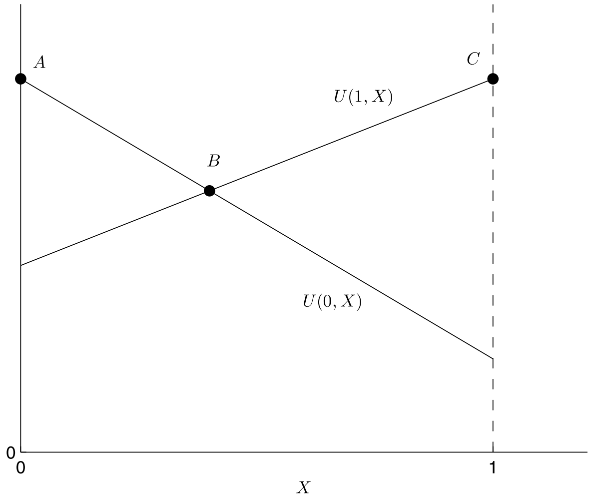


Fig. 2 Linear utility function $U(0, X)$ and $U(1, X)$ in the Schelling diagram

Therefore, the dynamic system (10) and (3) has one steady state[2] at the intersection shown in Fig. 2,

$$\hat{x}_B = \frac{c-a}{b+d}, \qquad \hat{u}_B = 0, \qquad \hat{\lambda}_B = 0. \tag{11}$$

This steady state is admissible if $c \geq a$ and $b + d \geq c - a$ and is unstable as the determinant of the Jacobian is $\det(J) = \frac{b+d}{\gamma} > 0$. Considering the eigenvalues of the Jacobian $\xi_{1,2} = \frac{1}{2}(r \pm \sqrt{r^2 - \frac{4(b+d)}{\gamma}})$, the point is a focus if $\frac{4(b+d)}{\gamma} > r^2$. Using the Lagrangian[3]

$$\mathcal{L} = \mathcal{H} + \nu_1 x + \nu_2(1-x),$$

we can find the following steady states at the boundary of the admissible region:

$$\begin{array}{lllll} \hat{x}_A = 0, & \hat{u}_A = 0, & \hat{\lambda}_A = 0, & \hat{\nu}_{A1} = c - a, & \hat{\nu}_{A2} = 0, \\ \hat{x}_C = 1, & \hat{u}_C = 0, & \hat{\lambda}_C = 0, & \hat{\nu}_{C1} = 0, & \hat{\nu}_{C2} = (b+d) - (c-a). \end{array}$$

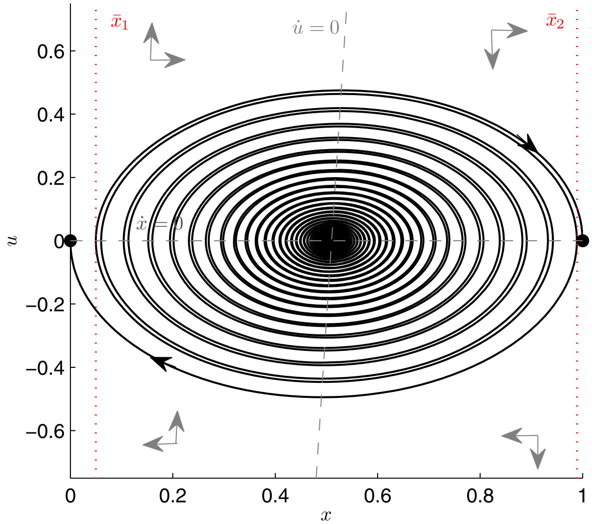
Thus, assuming $\gamma = 1$, the following classification is possible:

- $c - a < 0 < b + d$: one admissible steady state with maximal corruption,
- $0 < b + d < c - a$: one admissible steady state with no corruption,
- $0 < c - a < b + d$: three admissible steady states, where the interior steady state is

[2] Subscript B refers to the corresponding point in the Schelling diagram.

[3] For a detailed presentation of the Lagrangian technique taking into consideration inequality constraints, see Grass et al. [27, Sect. 3.6].

Fig. 3 Phase portrait for $0 < c - a < b + d$ with $r = 0.04$, $a = b = d = 0.5$, and $c = 1.01$



– an unstable focus iff $\frac{b+d}{\gamma} > \frac{r^2}{4}$,
– an unstable node iff $\frac{b+d}{\gamma} \leq \frac{r^2}{4}$.

The characterization of a steady state as a node or focus seems superficial given the instability of the steady state, but it has crucial economic implications. More precisely, the following distinction has been used in the literature since Krugman [1]:

1. Unstable node. Since the saddlepoint paths heading to the two boundary equilibria, $x \to 0$ or 1, do not overlap, history is sufficient to determine the future. More precisely, starting to the left of the unstable steady state implies $x \to 0$, while starting to the right implies an entirely corrupt society, $x \to 1$. Later we will show that this equivalence between node and history, i.e., starting to the left (right) of the node implies convergence to the steady state being situated on the left (right), does not hold in general, although it is assumed in all related papers. Flat linear functions (b and d are small), high adjustment costs (γ large), and high discounting favor this scenario.
2. Unstable focus. Here, the two saddlepoint paths overlap; see the discussion of the example in Fig. 3. This implies that history alone is insufficient to determine the future. Of course, the opposite properties, steep linear functions (b and d are large), small adjustment costs (γ small), and low discounting lead to fast adjustments that require additional information to determine the individual course of action.
3. Stable (focus or node) implies (local) indeterminacy, which is impossible in the above case of linear utilities and additive costs. However, indeterminacy cannot be excluded in general. Examples that allow for indeterminacy are Karp and Thierry [28], who consider an interaction between agriculture and polluting manufacturing, and Wirl [29], who assumes the adjustment costs to increase with respect to X (aggregate employment in manufacturing).

If the unstable steady state is a focus, then initial conditions (i.e., history) are insufficient to determine the competitive outcome transiently and in the long run. The reason is that both saddlepoint strategies can be reached for a set of initial conditions; in the example of Fig. 3 this domain where history is insufficient is extremely large because it contains the whole feasible region. While in the case of unilateral optimization the maximization objective provides an additional criterion to choose among different saddlepoint paths, a competitive equilibrium can rely only on the Nash property, i.e., individual behavior must be the best response given all others' actions.[4] Therefore, even if an individual knew that the way to $x = X = 0$ Pareto-dominates moving the other direction to the long run outcome $x = X = 1$, only a fool (in the context of the model) or a saint would choose to go to $x = 0$ when all others "agree" to move to the right. Krugman [1] calls this expectation dependence because agents must hold the same expectations within a symmetric competitive equilibrium. An alternative interpretation is that the agents must somehow coordinate their actions to choose either one of the two stable saddlepoint branches (e.g., via cheap talk). The particular feature of Fig. 3 is that, in contrast to Krugman's model, a very large overlap[5] results. More precisely, an initial overall corruption between $\bar{x}_1 = 5\ \%$ and $\bar{x}_2 = 99\ \%$ allows the system to reach both (extreme) steady states of all or no corruption. Expectations and/or coordination is crucial to choose between the two. Therefore, an improvement in coordination such as the one facilitated by modern media (e.g., Facebook, Twitter, mobile phones, SMS) may explain the drastic switch from one equilibrium to the other.

Kuran [30] suggests that revolutions often come as a surprise since people hide their true support for the opposition until it pays off to come forward; in fact, this static analysis bears some resemblance to the Schelling diagram in that individual payoffs from openly supporting the opposition depend on (expected) aggregate support of the opposition and unstable interior steady states. In our framework, fast changes occur only if the adjustment costs are very small, which they are presumably becoming as improved communications lowers uncertainties about individual exposures. This can explain why "liberal" dictators are usually removed fastest and those who play hardball much later if at all. More precisely, let us apply the insights from our corruption model to revolutions given the topical events in Arab countries. For this purpose, let the optimal control u describe the changing support of the government that depends on the adjustment costs γ. If a dictator leads a very strict regime and is able to block communications, both old (by prohibiting public meeting and demonstrations) and new (by limiting access to the Internet, Twitter, Facebook, mobile phones, etc.), then it is of course riskier and costlier for an individual to change toward supporting the opposition.

[4]For this reason, the payoff comparisons are irrelevant. To stress this point, note that the pure externality terms, here $-\delta X^2$, affect the payoffs but *not* the dynamics. Hence, by choosing a proper δ, we can choose the Pareto-dominant outcome at will.

[5]More precisely, Krugman [1] also shows a large overlap but only due to an error; the true overlap is small. In Fukao and Benabou [24] and Krugman [1], both long-run outcomes are attainable for almost all admissible initial conditions, $x_0 \in (0.05, 0.99)$.

4 Quadratic Payoff U

Given that the above simple model, with essentially linear payoff with respect to corruption, produces already interesting outcomes, the question arises how generalizations of payoffs and adjustment costs complicate the picture. It makes sense to keep the relation linear with respect to private corruption (since individual acts of corruption are negligible in the total). On the other hand, we impose that the payoff from each act of corruption depends on the aggregate in a nonmonotonic way: increasing at low levels, because few offers of corruption will be made even if the bureaucrat in question is corrupt, and decreasing at high levels, due to many other bureaucrats "selling" the same or a similar service. A simple version of this could be

$$U(x, X) = xX(1 - \varepsilon X) - \delta X^2 + \alpha(1 - x), \tag{12}$$

where the first term describes the financial benefit from bribes, the second describes the public good externality due to living in a corrupt society ($\delta > 0$), and the third term refers to the utility of being clean (no worries about getting caught, good conscience, etc., $\alpha > 0$). The parameter $0 \leq \varepsilon \leq 1$ determines the payoff of an individual corrupt act in a 100 % corrupt society because this payoff equals $1 - \varepsilon$. Hence, normalizing ε to 1 implies that individual payoffs from corruption are entirely eroded if everyone is willing to accept any bribe (perfect competition, thus no profits anymore). Figure 4 shows an example, ignoring again the external social costs of corruption since they are irrelevant for the individual optimization due to their public good characteristic.

The introduction of more complicated payoffs is only justified if it can provide additional insights. However, the complexity of indeterminacy—more precisely, both eigenvalues of the Jacobian being negative (or having negative real parts)—is impossible even if moving beyond the assumed pure adjustment cost. The reason is that (at least local) indeterminacy, i.e., the existence of a stable steady state, requires proper interaction between the individual control and the economy-wide externality (see Wirl [31]). Indeed, it is reasonable to assume that the costs of expanding individual corruption depend on the level of corruption within the society. Actually, the two conceivable costs, network effects and moral, point in the same direction: (i) as corruption spreads, the corresponding networks expand, and it becomes easier to join them, and (ii) the moral costs are also reduced if everybody else is already doing it. Therefore, it is reasonable to impose that

$$\frac{\partial^2 k}{\partial u \partial X} < 0.$$

This negative mixed derivative of the costs turns positive with respect to the total payoff so that indeterminacy can be ruled out (see Wirl [31, Proposition 5.7]) in the case of corruption of the Schelling type. Given this result, we continue using the simple adjustment cost framework. On the positive side, quadratic payoffs allow for interior and saddlepoint stable steady states. In addition, it allows us to make two formal points: (i) to question the usual classification in the literature that history is sufficient to determine the future in the case of an unstable node; (ii) to demonstrate nonmonotonicity of competitive outcomes in spite of the single-state framework, even if invoking additional criteria like Pareto-dominance in the case of multiple equilibria.

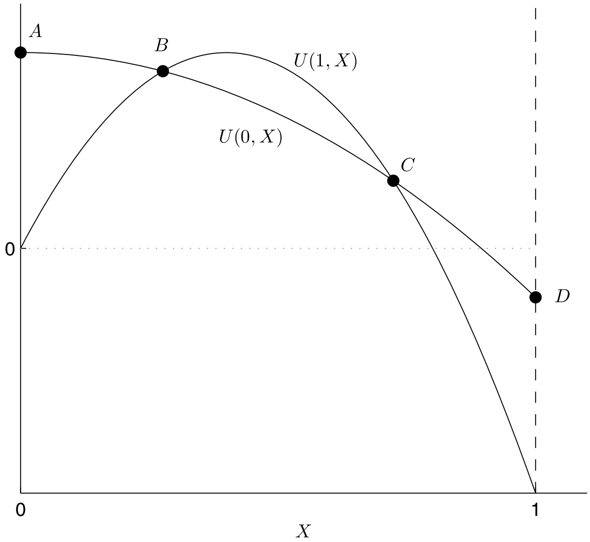


Fig. 4 Utility function $U(0, X)$ and $U(1, X)$ in the Schelling diagram

The corresponding current-value Hamiltonian

$$\mathcal{H} = xX(1 - \varepsilon X) - \delta X^2 + \alpha(1 - x) - \gamma \frac{u^2}{2} + \lambda u$$

implies the necessary conditions

$$u = \frac{\lambda}{\gamma}, \tag{13}$$

$$\dot{\lambda} = r\lambda - X(1 - \varepsilon X) + \alpha, \tag{14}$$

so that by differentiating (13) with respect to time and using $X = x$ we find

$$\dot{u} = ru - \frac{1}{\gamma}\big(x(1 - \varepsilon x) + \alpha\big),$$

and the following interior steady states (they correspond to the intersection of the static curves in Fig. 4 and are indexed accordingly):

$$\hat{x}_C = \frac{1 + \sqrt{1 - 4\varepsilon\alpha}}{2\varepsilon}, \qquad \hat{u}_C = 0, \qquad \hat{\lambda}_C = 0, \tag{15}$$

$$\hat{x}_B = \frac{1 - \sqrt{1 - 4\varepsilon\alpha}}{2\varepsilon}, \qquad \hat{u}_B = 0, \qquad \hat{\lambda}_B = 0. \tag{16}$$

The determinant of the Jacobian matrix evaluated at the steady states is $\det J(\hat{x}) = \frac{1}{\gamma}(1 - 2\varepsilon\hat{x})$, and the eigenvalues are $\xi_{1,2} = \frac{1}{2}(r \pm \sqrt{r^2 + \frac{8\hat{x}\varepsilon - 4}{\gamma}})$.

The first steady state is admissible if $0 \le \hat{x}_C \le 1$ and $1 - 4\varepsilon\alpha > 0$, or

$$1 - \varepsilon \le \alpha \le \frac{1}{4\varepsilon}.$$

Looking at the determinant of the Jacobian at the first steady state, which is $\det J(\hat{x}_C) = -\sqrt{\frac{1}{\gamma^2}(1-4\varepsilon\alpha)} < 0$, we see that this point is always a saddle point.

It can be easily shown that the second steady state is admissible if

$$0 \le \alpha \le \frac{1}{4\varepsilon}.$$

The determinant of the Jacobian at this steady state is $\det J(\hat{x}_B) = \sqrt{\frac{1}{\gamma^2}(1-4\varepsilon\alpha)} > 0$, and the trace of the Jacobian matrix is $\mathrm{tr} J(\hat{x}_B) = r > 0$. Thus, the steady state is always unstable, and it is a node if $r^2 + \frac{1}{\gamma}(8\hat{x}_B\varepsilon - 4) \ge 0$. Inserting $\hat{x}_B$, we find that the steady state is a node if and only if $\frac{1}{4\varepsilon} \ge \alpha \ge \frac{1}{4\varepsilon} - \frac{r^4\gamma^2}{64}$.

Note that while for certain parameters we can find an admissible interior saddle point $\hat{x}_C$ above the unstable steady state $\hat{x}_B$, it is not possible to find one below. The positivity of β implies that the marginal contribution to the objective of corruption is negative for x positive and sufficiently small, i.e.,

$$\mathcal{H}_x = X(1-\varepsilon X) - \beta < 0 \quad \text{for } X = x \text{ small enough.}$$

This implies that left of the smallest interior steady state it is optimal to converge to the origin, hence the instability of this steady state.

By looking at the Lagrangian function

$$\mathcal{L} = \mathcal{H} + \nu_1 x + \nu_2(1-x)$$

we find the following steady states at the boundary of the admissible region:

$$\hat{x}_A = 0, \qquad \hat{u}_A = 0, \qquad \hat{\lambda}_A = 0, \qquad \hat{\nu}_{A1} = \alpha, \qquad \hat{\nu}_{A2} = 0,$$
$$\hat{x}_D = 1, \qquad \hat{u}_D = 0, \qquad \hat{\lambda}_D = 0, \qquad \hat{\nu}_{D1} = 0, \qquad \hat{\nu}_{D2} = 1 - \varepsilon - \alpha,$$

where the one with $\hat{x}_D = 1$ is not feasible if $\varepsilon = 1$. Note that the boundary steady state $\hat{x}_D$ is only admissible if $\alpha < 1 - \varepsilon$, and thus the interior steady state with the higher level of corruption $\hat{x}_C$ and this boundary steady state cannot be admissible at the same time.

Thus, when assuming $\frac{1}{4\varepsilon} - \frac{r^4\gamma^2}{64} \ge 1 - \varepsilon$, we obtain the following scenarios:

- Region I: $\alpha > \frac{1}{4\varepsilon}$: one admissible steady state with no corruption.
- Region II: $1 - \varepsilon \le \alpha \le \frac{1}{4\varepsilon}$: two admissible interior steady states and one at the boundary with no corruption ($\hat{x}_A$). The interior steady state with the higher level of corruption is always a saddle point, while
  - $\frac{1}{4\varepsilon} - \frac{r^4\gamma^2}{64} \le \alpha \le \frac{1}{4\varepsilon}$: the interior steady state with lower level of corruption is an unstable node,
  - $1 - \varepsilon \le \alpha \le \frac{1}{4\varepsilon} - \frac{r^4\gamma^2}{64}$: the interior steady state with lower level of corruption is an unstable focus.
- Region III: $0 \le \alpha < 1 - \varepsilon$: three admissible steady states, where one is in the interior of the admissible region (unstable focus), and two are at the boundaries with no and maximal corruption.

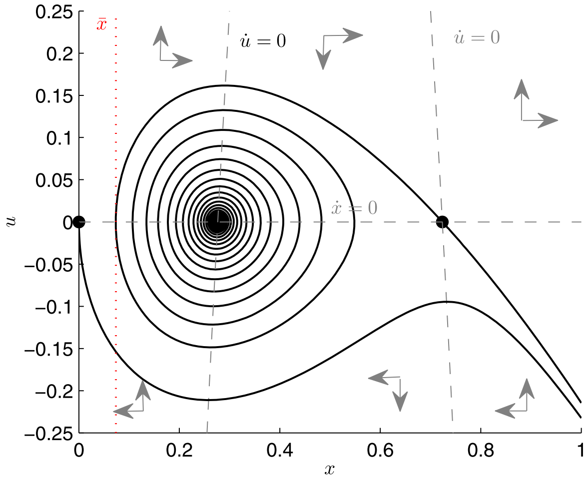


Fig. 5 Phase diagram for $r = 0.04$, $\gamma = 1$, $\delta = 0.25$, $\alpha = 0.2$, $\varepsilon = 1$

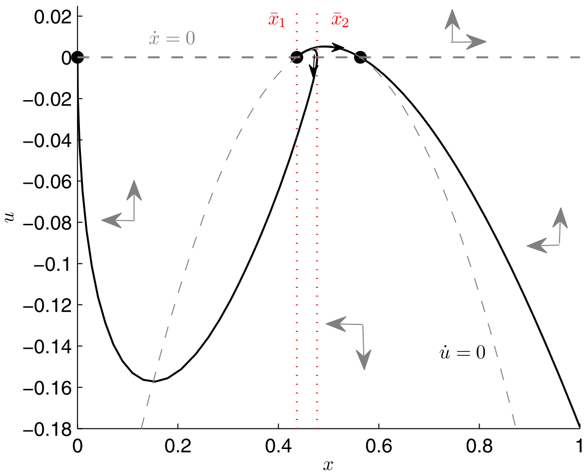


Fig. 6 Phase diagram for $r = 0.75$, $\gamma = 0.25$, $\alpha = 0.246$, $\varepsilon = 1$

Quadratic benefits allow for (saddlepoint) stable interior steady states and more complicated dynamics, such as the one shown in Fig. 5. In this example, the no-corruption outcome is globally attainable by a competitive equilibrium, while the high-corruption equilibrium (below 100 % now) requires sufficient initial corruption; again the overlap—i.e., the interval $[\bar{x}, 1]$—is large. However, the most important reason to study this extension to nonlinear (here just quadratic) payoffs is to correct common perceptions. The first is the claim that an overlap is equivalent to an unstable spiral, which is stated explicitly in Krugman [1, p. 664]: "An overlap exists if and only if $r^2 < 4\beta\gamma$" (which is equivalent to an unstable spiral). Figure 6 shows an example where the steady state is an unstable node that allows also for an overlap. Although the unstable steady state is a node, history is insufficient to determine the long-run outcome. The reason is that it is possible to pick either of the two saddlepoint branches for any $x_0 = X_0 \in (0.4368,\ 0.4771)$. Therefore, the common rule found

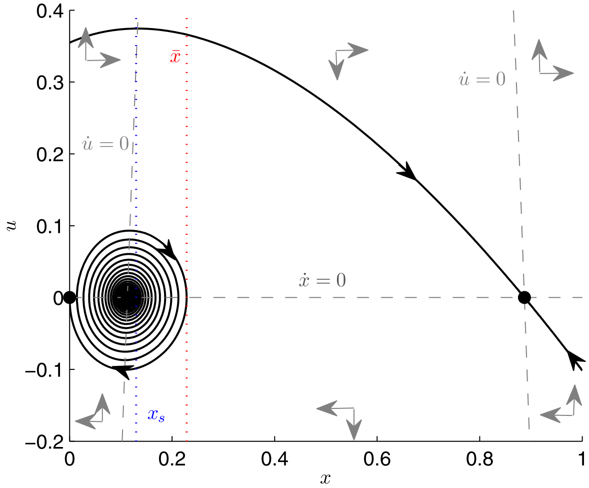


Fig. 7 Phase diagram for $r = 0.04$, $\gamma = 1$, $\delta = 0.25$, $\alpha = 0.1$, $\varepsilon = 1$

and applied in the related literature—if a node, then history is determinant, if a focus, then history is insufficient, and expectations or coordinations are needed to determine the long-run outcome—does not always hold. The unfortunate consequence of this finding is that tedious calculations are necessary to discriminate between these two different and highly policy-relevant cases instead of the relatively simple calculation of checking whether it has a node or focus.

The second point concerns the time paths of intertemporal competitive equilibria if the unstable steady state is a spiral. Krugman [1, footnote 5] already recognizes the complex pattern of multiple competitive equilibrium paths toward the two steady states; the purpose of the following example is to go deeper and to rank the many competitive equilibria in terms of their efficiency. Consider the example shown in Fig. 7. This allows the individuals to choose within the overlap (i.e., $x_0 = X_0 \in [0, \bar{x}]$) from a wide range of intertemporal competitive equilibria going either to the left or the right. (In this example, it is high corruption that is globally reachable, but only by a single path.) Given this multiplicity within the overlap (e.g., $x_0 = X_0 = 0.13$) the agents may find the "envelopes" as focal points (to use Schelling's famous term) because they converge monotonically to one of the two (stable) steady states. However, they may choose any point from inside along the spiral, which leads to indeterminacy (not to be confused with indeterminacy due to two eigenvalues having negative real parts), to non-monotonic solution paths, and to convergence to the left-hand—no-corruption—equilibrium. However, the choice of the monotonic paths is much less natural than it seems because starting at an interior part of the spiral can Pareto-dominate the monotonic (and thus not focal) alternative. Indeed, considering the example from Fig. 8 and $x_0 = X_0 = 0.15 > x_s$ and then choosing an initial condition inside the spiral that takes one turn around lead to a higher payoff than the direct route to the origin. This shows that (transient) cycling can be the most efficient competitive outcome in a one-state variable model. This means that a dynamic competitive equilibrium model can have a qualitatively totally different outcome compared with a one-state optimal control model because the latter always has a monotonic trajectory as its optimal solution (see Hartl [32]).

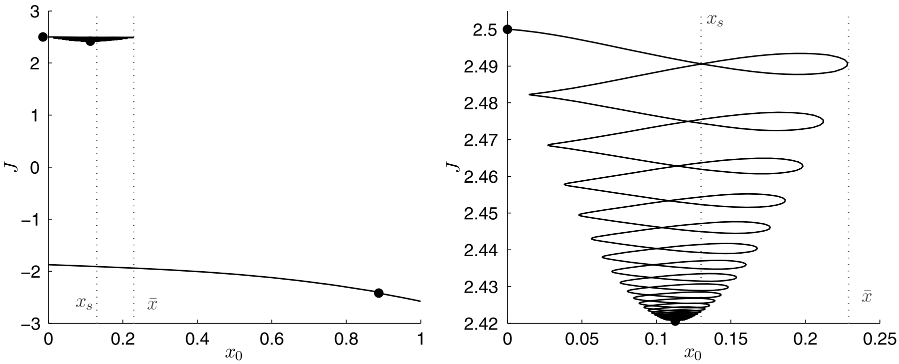


Fig. 8 Objective value evaluated for initial points lying on the stable path leading to the different steady states. The *right panel* is a zooming of the *left panel* depicting the objective value along the spiral seen in Fig. 7

5 Conclusions

This paper puts Schelling's model of corruption into a dynamic competitive equilibrium framework. As in Krugman [1], multiple equilibria are characteristic, and their determination can depend either on history or history plus a coordination of agents' expectations. Yet, while Krugman distinguishes these two cases along the local stability characteristics of the unstable steady state—node versus focus—it is shown that even a node can render the need for coordination. Furthermore, in the case of a focus, we show that transient cycling (and thus nonmonotonicity in the state) can Pareto-dominate the direct monotonic approach of the long-run equilibrium. This is striking because for the case of an optimal control model with one state, we know that optimal trajectories are always monotonic (see Hartl [32]). This stresses how crucial social interactions can be.

Natural extensions of the model are in the direction of more states, along the lines sketched above, or of including a dynamic externality about how corruption evolves or harms the economy. Another alternative is to apply the individual payoff as sketched in Schelling (and similar to Kuran [30]) in a political context to explain stability or instability of political institutions like dictatorships. A good starting point for this is Kuran's paper considering a similar payoff structure for the individual (to "falsify" his true political preference) in a static context. Putting this idea into a dynamic context might lead to interesting insights concerning sudden, radical switches toward support of the opposition.

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References

1. Krugman, P.: History versus expectations. Q. J. Econ. **106**(2), 651–667 (1991)

2. Andvig, J.C., Moene, K.O.: How corruption may corrupt. J. Econ. Behav. Organ. **13**(1), 63–76 (1990)
3. Friedrich, C.J.: The Pathology of Politics, Violence, Betrayal, Corruption, Secrecy, and Propaganda. Harper & Row, New York (1972)
4. Popper, K.R.: The Open Society and Its Enemies, vol. 1. The Spell of Plato. Routledge, London (1966)
5. Lui, F.T.: A dynamic model of corruption deterrence. J. Public Econ. **31**(2), 215–236 (1986)
6. Rose-Ackerman, S.: The economics of corruption. J. Public Econ. **4**, 187–203 (1975)
7. Rose-Ackerman, S.: Corruption: A Study in Political Economy. Academic Press, New York (1978)
8. Rose-Ackerman, S.: The law and economics of bribery and extortion. Ann. Rev. Law Soc. Sci. **6**, 217–238 (2010)
9. Schelling, T.: Hockey helmets, concealed weapons, and daylight saving: a study of binary choices with externalities. J. Confl. Resolut. **17**(3), 381–428 (1973)
10. Schelling, T.C.: Micromotives and Macrobehavior. Norton, New York (1978)
11. Shleifer, A., Vishny, R.W.: Corrupt. Q. J. Econ. **108**(3), 599–617 (1993)
12. Feichtinger, G., Wirl, F.: On the stability and potential cyclicity of corruption within governments subject to popularity constraints. Math. Soc. Sci. **28**(2), 113–131 (1994)
13. Dong, B., Torgler, B.: Democracy, property rights, income equality, and corruption. Fondazione Eni Enrico Mattei Working Papers 559 (2011)
14. Caulkins, J.P., Feichtinger, G., Grass, D., Hartl, R.F., Kort, P.M., Novak, A., Seidl, A.: Leading bureaucracies to the tipping point: an alternative model of multiple stable equilibrium levels of corruption. Eur. J. Oper. Res. **225**(3), 541–546 (2013)
15. Andvig, J.C.: The economics of corruption: a survey. Stud. Econ. **43**(1), 57–94 (1991)
16. Glaeser, E.L., Sacerdote, B.I., Scheinkman, J.A.: The social multiplier. J. Eur. Econ. Assoc. **1**(2–3), 345–353 (2003)
17. Glaeser, E.L., Sacerdote, B.I., Scheinkman, J.A.: Crime and social interactions. Q. J. Econ. **111**(2), 507–548 (1996)
18. Dong, B., Torgler, B.: Corruption and social interaction: evidence from China. J. Policy Model. **34**(6), 932–947 (2012)
19. Aidt, T.S.: Economic analysis of corruption: a survey. Econ. J. **113**(491), F632–F652 (2003)
20. Wirl, F.: Socio-economic typologies of bureaucratic corruption and implications. J. Evol. Econ. **8**(2), 199–220 (1998)
21. Epstein, J.M.: Modeling civil violence: an agent-based computational approach. Proc. Natl. Acad. Sci. USA **99**, 7243–7250 (2002)
22. Feichtinger, G., Rinaldi, S., Wirl, F.: Corruption dynamics in democratic systems. Complexity **5**(3), 53–64 (1998)
23. Wirl, F., Feichtinger, G.: History versus expectations: increasing returns or social influence? J. Socio-Econ. **35**(5), 877–888 (2006)
24. Fukao, K., Benabou, R.: History versus expectations: a comment. Q. J. Econ. **108**(2), 535–542 (1993)
25. Liski, M.: Thin versus thick CO2 market. J. Environ. Econ. Manag. **41**(3), 295–311 (2001)
26. Sethi, S.P., Thompson, G.L.: Optimal Control Theory Applications to Management Science and Economics, 2nd edn. Kluwer Academic, Boston (2000)
27. Grass, D., Caulkins, J.P., Feichtinger, G., Tragler, G., Behrens, D.A.: Optimal Control of Nonlinear Processes: With Applications in Drugs, Corruption and Terror. Springer, Heidelberg (2008)
28. Karp, L., Thierry, P.: Indeterminacy with environmental and labor dynamics. Struct. Chang. Econ. Dyn. **18**(1), 100–119 (2007)
29. Wirl, F.: Conditions for indeterminacy and thresholds in neoclassical growth models. J. Econ. **102**, 193–215 (2011)
30. Kuran, T.: Sparks and prairie fires: a theory of unanticipated political revolution. Public Choice **61**(1), 41–74 (1989)
31. Wirl, F.: Social interactions within a dynamic competitive economy. J. Optim. Theory Appl. **133**(3), 385–400 (2007)
32. Hartl, R.F.: A simple proof of the monotonicity of the state trajectories in autonomous control problems. J. Econ. Theory **40**(1), 211–215 (1987)