Pushing IA to the (SNR) Limit:
Experimental Results from the Vienna MIMO Testbed

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Interference Alignment, Theory vs. Practice

Previous works on IA:

- “simulated” IA [El Ayach, Peters, Heath 2009]
- indoor, Ettus USRPs [Gollakota, Perli, Katabi, 2009]
- indoor, Ettus USRPs [Zetterberg, Moghadam, 2012]

Our objective: investigate the “fundamental” limits of IA

- over-the-air
- using lab-grade equipment
- in a standard-agnostic way
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K-user MIMO Interference Channel

\[ y_k = H_{kk} x_k + \sum_{j=1, j \neq k}^K H_{kj} x_j + n_k \quad \forall k = 1 \ldots K \]
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Based on linear precoding: \( x_i = V_i s_i \) with \( s_i \in \mathbb{C}^d \)

- Interference (in red) from multiple Tx aligns at the Rx
- Interference-free subspace used for communication
Mathematical formulation of IA\(^1\)

- interference $\sum_{j \neq i} H_{ij} V_j s_j$ does not occupy all receive dimensions

- IA solution equivalent to finding matrices $V_i$ and $U_i$

\[
\begin{align*}
\text{s.t.} \quad & \left\{ \begin{array}{l}
U_i^H H_{ij} V_j = 0, \quad \forall j \neq i \\
\text{rank} \left( U_i^H H_{ii} V_i \right) = d_i.
\end{array} \right.
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Why the hype?

Some attractive properties

- Low-complexity processing, just linear filters
- Achieves the full channel degrees of freedom at high SNR

\[
\text{DoF} = \lim_{\text{SNR} \to +\infty} \frac{C_i(SNR)}{\log \text{SNR}} = d
\]

BUT...

- Extensive CSI requirements: \( H_{ij} \ \forall j \neq i \)
- What good is the DoF result at finite SNR?
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The Vienna MIMO Testbed Set-Up

- Replicate an urban outdoor-to-indoor and indoor-to-indoor scenario, with 3 Tx / 1 Rx

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The Vienna MIMO Testbed Set-Up: Tx side

- 2 rooftop TX, Kathrein Scala Division XX-pol BTS antennas
- Indoor Tx, 2× Kathrein Scala Division X-pol directional ant.
The Vienna MIMO Testbed Set-Up: Rx side

- Rx using 4 custom-built $\frac{\lambda}{2}$ dipoles in a laptop shell
- On a positioning table

The Vienna MIMO Testbed Set-Up: Rx side
Testbed Characteristics

- 4 nodes (3 Tx + 1 Rx), 4 antennas per node
- Rubidium-synchronized clocks
- No duplexing, but high-speed feedback over dedicated fiber LAN
- Center frequency 2.503 GHz
- 200 MHz sampling rate
- OFDM with 15.02 kHz subcarrier spacing
- One instance of MATLAB at each node
IA-Specific Implementation Details

- Two additional “ghost” receivers to simulate the 3-user IC

- System dimensions allow IA with $d = 2$ streams per user

- IA precoder computed in closed-form based on eigendecomposition

- Consider a single subcarrier to keep complexity low
Closed-loop system with periodic training sequences

- Frame $(l)$
- Frame $(l + 1)$

- Calculate $\tilde{H}_{ij}^{(l)}$ and $V_j^{(l)}$
- Apply $V_j^{(l)}$

$(T_p \approx 70\text{ms})$
Covariance-Based Performance Metrics

- Mutual information
  - independent of the channel code
  - upper bound for the achievable rate

- Consider the covariance at the receiver

\[
E \{ y_1 y_1^H \} = H_{11} V_1 Q_{x_1} V_1^H H_{11}^H + H_{12} V_2 Q_{x_2} V_2^H H_{12}^H + H_{13} V_3 Q_{x_3} V_3^H H_{13}^H + Q_{n_1} + Q_{n_2} + Q_{n_3}
\]

- Assuming Gaussian signals:

\[
\mathcal{I}(x_1; y_1) = \log \det (Q_I + Q_N + Q_S) - \log \det (Q_I + Q_N)
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\]

desired signal \( Q_S \)

interference \( Q_I \)

noise \( Q_N \)

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Covariance-Based Performance Metrics

- Signal covariances *estimated over the air*

With perfect CSIT: \( Q_b = \mathbb{E}\{ y_b y_b^H \} = \mathbf{u}_i \mathbf{u}_i^H \text{diag}(\lambda_1, \ldots, \lambda_4) \begin{bmatrix} \mathbf{u}_i^H \\ \mathbf{u}_i^\perp \mathbf{u}_i^H \end{bmatrix} \)
Covariance-Based Performance Metrics

- Signal covariances *estimated over the air*

![Diagram showing signal covariances estimated over the air with perfect CSIT]

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![Diagram showing eigenvalues and interference rejection]

- \( \sigma_N^2 \) intereference rejection
- \( \lambda_4 \lambda_3 \lambda_2 \lambda_1 \) eigenvalues
Spatial stationarity: 5 Rx positions

Aligning interference from Tx2 and Tx3

\[ 10 \log_{10} \left( \frac{I}{I_{\text{supp}}} \right) [\text{dB}] \]

\[ \bar{\lambda}_2 \]

\[ \bar{\lambda}_3 \]

\[ \bar{\lambda}_4 \]

\[ \bar{\lambda}_5 \]

\[ \bar{\lambda}_1 \]

\[ I_{\text{supp}} = 49.2 \text{ dB} \]
Spatial stationarity: 5 Rx positions

Corresponding RSS for 5 positions

![Graph showing spatial stationarity for 5 Rx positions](image-url)
Spatial stationarity: 5 Rx positions

Aligning interference from Tx1 and Tx2

\[ \bar{I}_{\text{supp}} = 46.3 \, \text{dB} \]
Interference suppression

\[ E\left\{ \frac{\lambda_3}{\lambda_2} \right\} \text{ for signal of interest from Tx 1, 2 and 3} \]
Noise & Interference vs. Tx Power

Transmitter noise becomes significant at high Tx power!
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Conclusion

Experimental evidence (in our testbed) for

- Interference leakage (due to delayed CSI, synchronization, channel estimation noise)
  \[ I_{\text{supp}} \approx 45 - 50 \text{ dB at best} \]

- Transmitter noise (below \(-20\) dB SIR)
Thank you for your attention

Questions ?