A Memetic Algorithm for the Partition Graph Coloring Problem

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1 Introduction

The partition graph coloring problem (PGCP) belongs to the class generalized network design problems (GNDPs). This class is obtained by considering classical network design problems on a clustered graph where the original problem's feasibility constraints are expressed in terms of the clusters, i.e., node sets instead of individual nodes. Formally, the PGCP is defined on an undirected graph $G = (V, E)$ where vertex set V is partitioned into p mutually exclusive nonempty clusters $V_1, \ldots, V_p$ with $V = V_1 \cup \ldots \cup V_p$ and $V_i \cap V_j = \emptyset \; \forall i, j \in \{1, \ldots, p\}, i \neq j$. The goal is to find a subset $V^* \subseteq V$ that contains exactly one vertex for each cluster and a coloring for V^* so that in the graph induced by V^* two adjacent vertices have different colors and the total number of used colors is a minimum. An example for the PGCP is shown in Figure 1.

Li and Simha [6] introduced this $\mathcal{NP}$-hard problem which was motivated by the wavelength routing and assignment in an optical network. In this context several approaches have been proposed: construction heuristics [6], a tabu search algorithm [9], a branch-and-cut [3] and a branch-and-price [5]. Finally, Demange *et al.* [2] analyzes the complexity for some special graph classes.

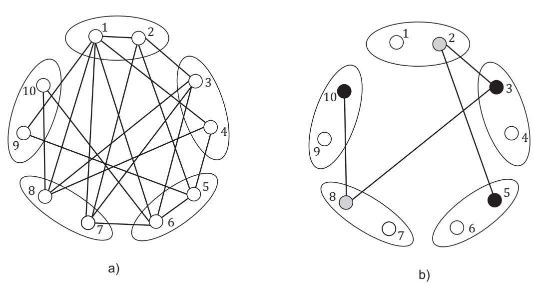

Fig. 1. a) An instance of PGCP and b) an optimal solution with two colors

2 A Memetic Algorithm for the PGCP

We propose a memetic algorithm (MA) that combines efficient genetic operators with a local search procedure. The population is initialized with a randomized version of the onestepLF algorithm [6]. For the genetic representation, a solution S is characterized by the color classes $U = \{U_1, U_2, \ldots, U_k\}$ where $U_i,\ i \in \{1, \ldots, k\}$ denotes the set of vertices assigned to color i and $U_1 \cup \ldots \cup U_k \subseteq V$. Furthermore, U contains exactly one node of each cluster.

The crossover operator is adopted from [7] which was originally designed for the classical graph coloring problem. For the PGCP, we choose two parents U^1 and U^2 from the population via a binary tournament selection and perform the following steps:

1. start from parent U^1, select the color partition with the most number of vertices and copy it to the offspring;
2. delete from both parents the selected vertices and all other vertices that belong to the same clusters of the selected ones;
3. repeat this procedure for $U^2, U^1, U^2, \ldots$ until all vertices are either assigned to the offspring or removed.

We mutate a solution by removing a randomly selected vertex from its color class and reinserting it in a different color class so that no conflicts occur. If necessary, the number of color classes is increased by one. Both crossover and mutation only generate feasible solutions.

We apply with a certain probability local improvement on an offspring after it has been created by the genetic operators. For this purpose we use an incomplete solution representation that only specifies the selection of vertices without specifying the coloring information, i.e., solution $S = \{s_1, \ldots, s_p\},\ s_i \in V_i,\ i \in \{1, \ldots, p\}$. This is a popular approach for GNDPs, but the complexity of decoding a solution for this problem is equal to solving the classical graph coloring problem which is $\mathcal{NP}$-hard. Therefore we apply the DSATUR heuristic [1] which does not guarantee an optimal result, but has a complexity of only $O(p^3)$. Using exact approaches such as mixed integer programming [8] or even metaheuristics [4] for this purpose presumably consume too much time for large instances. For local search we apply a standard vertex exchange neighborhood structure, i.e., a neighbor solution is derived by changing the selected vertex of a cluster.

Preliminary results on instances that are also used in [3, 5] show that our MA runs very fast and is able to find the optimal solutions on graphs with up to 70 vertices in less than one second. Its speed documents excellent scalability and encourages us to use more sophisticated genetic operators and/or local search procedures. Besides more powerful decoding algorithms such as the DANGER heuristic [4] we would also like to investigate the possibility of other incomplete representations that exploit other aspects of the problem.

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