ELASTO-HYDRODYNAMIC LUBRICATION OF MACHINE ELEMENTS: THE BENEFITS OF THE HOMOGENISATION TECHNIQUE

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1 INTRODUCTION

Machine elements widely used in the automotive industry such as gears, roller bearings and cams, operate under elasto-hydrodynamic (EHL) lubrication conditions. Since the breakdown of a continuous lubricant film leads to adhesion and severe distress of the surfaces, a proper evaluation of the film thickness is crucial in any engineering design.

Several formulae for the central and minimum film thickness are available in literature, cf. that by Dowson and Higginson for line contacts or its counterpart for point contacts [1], [2]. These representations are based on a power law regression to the numerical solutions of the classical EHL problem for smooth contacts. However, in many practical engineering applications, the film thickness predicted by these formulae is comparable with the height of surfaces asperities. Therefore, the performance of the lubricated contact depends on both the macroscopic geometry of the contact itself and on the microscopic length scales of the roughness.

The first attempt to investigate the effect of surface roughness on the EHL film thickness was carried out by Patir and Cheng [3], [4] through a statistical average flow approach. In their formulation, the actual flow between rough surfaces is equated to an average flow between nominally smooth surfaces, while the roughness effects are included in a modified Reynolds' equation via the flow factors, which are obtained by numerical simulation on a small cell. This cell is supposed to be sufficiently large to include a representative portion of the roughness, but small enough compared to the characteristic length of the contact. The method is therefore intrinsically ambiguous, particularly in the specific choice of the dimensions of the cell problem and in its boundary conditions [5].

A second possible method consists in a deterministic approach. In this case, the Reynolds' equation is solved directly on a real, measured rough profile. However, Reynolds' equation is solved by using numerical methods, which means that the computational domain is divided into a certain amount of elements in which the

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solution is obtained. Considering a rough surface, a very large number of elements is required and the solution can be difficult to obtain and time consuming.

More recently, an effective approach to tackle the problem fully rationally was introduced, which adopts the technique of homogenisation [6], [7], [8]. Homogenisation is based on the asymptotic separation between the macro-scale of the contact and the micro-scale of the roughness. Interpreted as a rigorous (i.e. asymptotic) method, homogenisation provides *the* proper means for the description of the lubrication mechanism for largely arbitrary roughness patterns. Most important, it allows one to overcome the limits of other approaches notably the statistical one by Patir and Cheng [3], in the sense of a rational improvement of the underlying approximations and the inclusion of cross-flow effects generated by the roughness elements; see [5]. Furthermore, the solution of the homogenised problem can be obtained with a considerably reduced computational effort compared to that required by a fully deterministic approach. Homogenisation represents a mathematically rigorous method where the leading-order approximation of the “real-world” problem solution adopted here can, in principle, be refined by a higher-order approach.

2 HOMOGENISATION

Homogenisation is a mathematical technique that applies to physical phenomena which involve at least two separated (macro- and micro-) scales.

2.1 Fundamentals

Often, the mathematical description of effects occurring on the micro-scale but affecting the physics on the macro-scale is essentially difficult, or even impossible, to formulate. The idea behind homogenisation is then to formulate, if possible, a macroscopically equivalent “homogenised” phenomenon which behaves “on average” like the original one.

The possibility to homogenise the EHL problem has been already demonstrated, see [6]. The homogenisation process leads to a hierarchical but coupled system of equations which allows one to study the macroscopic behavior of a lubricated contact (i) subject to a given micro structure, (ii) without resorting to the explicit resolution of the micro-scale, i.e. the roughness, on the macro-scale. For the sake of clarity, we next briefly explain the homogenisation of the EHL problem.

2.2 Governing equations

A non conforming contact is represented locally by two elastic bodies, having paraboloidal shapes and touching in a single point *O*. We assume *O* as the origin of a Cartesian, orthogonal coordinate system $\boldsymbol{x} = (x_1, x_2, x_3)$ in which the x_1- and x_2- directions lay on the plane tangential to both paraboloids and x_3 in the direction of the common normal. We choose the x_1- and x_2- directions to be coincident with the principal axes of relative curvature, where R_{x1} and R_{x2} are, respectively, the corresponding curvature radii. This forms a frame of reference where the resulting EHL problem is considered as stationary. Under a sufficiently large load, the contact point *O* deforms to a Hertzian contact ellipse whose semi-axes lay in the x_1- and x_2- directions. We denote with *a* and *b* the semi-axes of the Hertzian contact ellipse and we assume $b \leq a$. Furthermore, we denote with p_{Hz} the maximum Hertzian pressure, with $v_m = (v_1+v_2)/2$ the mean velocity of the surfaces and with χ the angle between v_m and the x_1-direction. Due to the surfaces motion, the lubricant is forced to enter into the contact zone and generates a thin film that is able to separate the contacting surfaces. Let *h* denote the local thickness of this film, *p* the corresponding lubricant pressure, η_0 its viscosity at reference pressure, *α* the piezo-viscosity coefficient for isothermal conditions as assumed here and h_r the minimum approach of the contacting bodies when their elastic deformation is neglected. Furthermore, we introduce the following dimensionless quantities:

$$X_1 = \frac{x_1}{b},\ X_2 = \frac{x_2}{b},\ H = h\frac{R_{x_2}}{b^2},\ \Lambda = \frac{R_{x_2}}{R_{x_1}},\ e = \sqrt{1-\left(\frac{b}{a}\right)^2},\ \Gamma = \frac{12\eta_0 v_m R_{x_2}^2}{b^3 p_{Hz}},\ P = \frac{p}{p_{Hz}},$$
$$\Pi = \frac{1}{2\pi}\frac{e^2}{1-e^2}\left[\frac{E(e^2)}{1-e^2} - K(e^2)\right]^{-1},\ \bar{\alpha} = \alpha p_{Hz}$$

with *K(e)* and *E(e)* denoting the complete elliptic integrals of the first and second kind, respectively.

The film thickness *H* and the pressure distribution *P* depend explicitly upon both the macro- and micro-scale (roughness) of the problem. In order to take into consideration the influence of the micro-scale, we suppose that the roughness is *ε*-periodic in both the X_1- and X_2- directions, where the small fundamental wavelength ε will serve as a perturbation parameter. We therefore introduce a local coordinate system $\mathbf{Y}=(Y_1, Y_2)$ such that $Y_1 = X_1/\varepsilon$, $Y_2 = X_2/\varepsilon$ and $\mathbf{Y}$ = [0, 1]x[0, 1] on the thereby defined unit cell: see Figure 1. In physical terms, ε represents the parameter that controls the separation between the macroscopic and the microscopic scales of the problem. The distribution of the film thickness and the lubricant pressure can then be written in the form (1)-(2), with the subscript denoting the parametrisation in *ε* and satisfy the relationship (3). This represents the celebrated Reynolds' equation where piezo-viscosity is taken into account by virtue of the Barus law and the pressure-density dependence is neglected:

$$H_\epsilon(\mathbf{X}) = H\left(\mathbf{X}, \frac{\mathbf{X}}{\epsilon}\right) = H_0(\mathbf{X}) + R\left(\mathbf{X}, \frac{\mathbf{X}}{\epsilon}\right) \quad (1)$$

$$P_\epsilon(\mathbf{X}) = P\left(\mathbf{X}, \frac{\mathbf{X}}{\epsilon}\right) \quad (2)$$

$$\nabla \cdot \left(H_\epsilon^3 e^{-\bar{\alpha}P_\epsilon}\nabla P_\epsilon\right) = \Gamma\left(\frac{\partial H_\epsilon}{\partial X_1}\cos\chi + \frac{\partial H_\epsilon}{\partial X_2}\sin\chi\right) \quad (3)$$

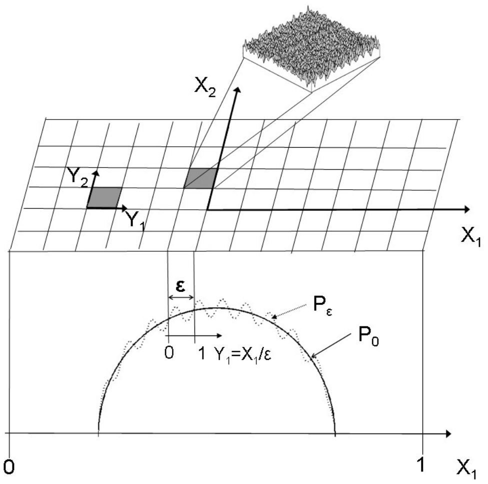

Figure 1: Central idea of the homogenisation method. The unit cell representing the roughness is supposed to be periodic in both the X_1- and X_2- directions

Homogenisation studies the limiting case $\varepsilon \to 0$, hence the case of infinite scale separation. According to [6], P_ε has an asymptotic representation of the form (4) in which the sole dependence of the leading-order term P_0 on the macro-scale variable $\boldsymbol{X}$ represents a central result of the homogenisation process:

$$P_\epsilon(\mathbf{X}) \sim P_0\left(\mathbf{X}\right) + \epsilon P_1\left(\mathbf{X}, \frac{\mathbf{X}}{\epsilon}\right) + \epsilon^2 P_2\left(\mathbf{X}, \frac{\mathbf{X}}{\epsilon}\right) + \cdots \tag{4}$$

By substitution of this expansion in (3), the homogenisation process leads to (5)-(9). The homogenised Reynolds' equation (5) is solved on the macro-scale, whereas the local unit cell problems (8), (9) on the micro-scale subject to periodic boundary conditions. The homogenised coefficients (6), (7) represent the link between the macro- and the micro-scale. Moreover, (5) is coupled with (10)-(12) describing, respectively, the film thickness, the elastic deformation of the bodies and the force-balance equation (δ_{ij} is the Kronecker delta):

$$\nabla \cdot \left[\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} e^{-\bar{\alpha} P_0} \nabla P_0 \right] = \sum_{i=1}^{2} \frac{\partial}{\partial X_i} \left(B_{i1} \cos\chi + B_{i2} \sin\chi \right) \tag{5}$$

$$A_{ij} = \int_0^1 \int_0^1 \left(H^3 \delta_{ij} + H^3 \frac{\partial W_j}{\partial Y_i} \right) \mathrm{d}Y_1 \mathrm{d}Y_2 \quad i,j = 1,2 \tag{6}$$

$$B_{ij} = \int_0^1 \int_0^1 \left(H \delta_{ij} - H^3 \frac{\partial \Omega_j}{\partial Y_i} \right) \mathrm{d}Y_1 \mathrm{d}Y_2 \quad i,j = 1,2 \tag{7}$$

$$\frac{\partial}{\partial Y_1}\left(H^3 \frac{\partial W_i}{\partial Y_1} \right) + \frac{\partial}{\partial Y_2}\left(H^3 \frac{\partial W_i}{\partial Y_2} \right) = -\frac{\partial H^3}{\partial Y_i} \quad i = 1,2 \tag{8}$$

$$\frac{\partial}{\partial Y_1}\left(H^3\frac{\partial \Omega_i}{\partial Y_1}\right)+\frac{\partial}{\partial Y_2}\left(H^3\frac{\partial \Omega_i}{\partial Y_2}\right)=\frac{\partial H}{\partial Y_i} \quad i=1,2 \tag{9}$$

$$H_0(\mathbf{X})=H_r+\Lambda\frac{{X_1}^2}{2}+\frac{{X_2}^2}{2}+U(\mathbf{X},P) \tag{10}$$

$$U\left(\mathbf{X},P\right)=\Pi\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{P(X_1',X_2')\mathrm{d}X_1'\mathrm{d}X_2'}{\sqrt{(X_1-X_1')+(X_2-X_2')}} \tag{11}$$

$$\frac{2\pi}{3\sqrt{1-e^2}}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X_1',X_2')\,\mathrm{d}X_1'\mathrm{d}X_2' \tag{12}$$

3 SIMULATIONS AND RESULTS

An attempt has been made to rigorously study the effect of surface roughness on non-conformal concentrated contacts under the action of elasto-hydrodynamic lubrication. Here the most intriguing phenomena of interest include the impact of transversal and oblique roughness on the EHL film thickness, i.e. due to grinding marks produced by machining process on engineering surfaces. This investigation may prove to be of interest from both a designer’s and a theoretician’s point of view.

First numerical results allow for assessing the deviations of the minimum and central homogenised film thickness from those referring to perfectly smooth surfaces. Three different base cells characteristic of typical engineering surfaces have been considered, see Figure 2: unit cell (a) refers to the aforementioned grinding marks, unit cell (b) to a double-periodic roughness (highest degree of isotropy) and unit cell (c) to a roughness resulting from a superfinishing process in which all the asperities are flattened. The differences of the micro-geometry are crucially reflected by the central results of the analysis of the micro-scale, namely the Poiseuille and Couette flow factors (PFF and CFF, respectively), which are shown below the sketches of the corresponding unit cells. The pressure (shear) flow factors represent the ratio between the mean Poiseuille (Couette) flow in the case of rough surfaces and in the case of perfectly smooth surfaces, if the fluid is locally considered isoviscous. The flow factors are shown as functions of the specific film thickness δ defined as follows:

$$\delta=h_{0\,min}/r_{max}=H_{0\,min}/R_{max} \text{ in case (a), (b)} \tag{13}$$

$$\delta=h_{0\,min}/|r_{min}|=H_{0\,min}/|R_{min}| \text{ in case (c)} \tag{14}$$

where h_{0min} and H_{0min} denote the minimum mean separations between the surfaces in dimensional and non dimensional representation and r_{max} (r_{min}) and R_{max} the corresponding maximum height of the roughness peaks.

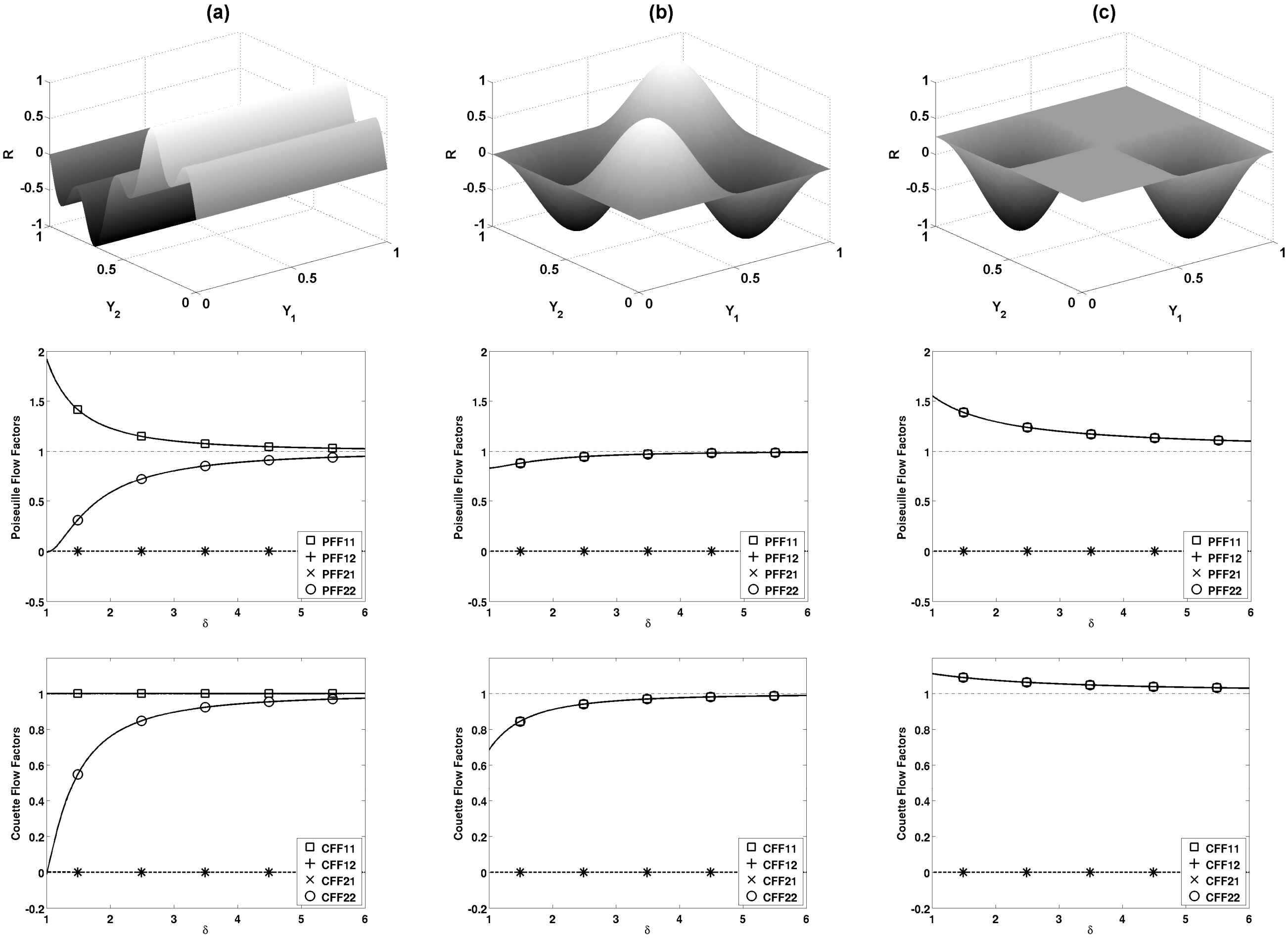

Figure 2: Analysis of the effects induced by the micro-scale on the Poiseuille and Couette flows. The velocity is in the Y_2 direction; (a) unit cell simulating grinding marks; (b) unit cell simulating double-periodic roughness; (c) unit cell simulating roughness resulting from a superfinishing process in which the peaks are removed and only the valleys remain.

We shall notice that in all the cases the cross flow factors vanish identically: $PFF_{12}=PFF_{21}=0$, $CFF_{12}=CFF_{21}=0$. This is due to the particular symmetry of the base cell as outlined in [5]. Moreover, it results in $PFF_{11,22} \to 1$, as $\delta \to \infty$. That means that the influence of the roughness decreases as the gap height increases. Specifically, in case (c) the decrease of the flow factors reflects the increasing depths of the individual pits. Correspondingly the lubrication problem assumes its classical form associated with perfectly smooth surfaces in the limit $\delta \to \infty$. On the contrary, the influence of the roughness increases as the gap height decreases. Particularly, for the case (a), the value $\delta = 1$ represents the extreme case in which the complete blockage of the flow in the Y_2 direction occurs (see PFF_{22}). Conversely, for the cases (b) and (c), even when $\delta = 1$ the flow can occur in the Y_2 direction and, due to the symmetry of the cell, $PFF_{11}=PFF_{22}$, $CFF_{11}=CFF_{22}$. Note that in case (c) the definition (14) of δ applies.

Figure 3 shows the pressure distribution and the film thickness for a circular concentrated contact and the cell (a) in the case $\delta = 1.2$. However, the impact of surface roughness on the EHL film thickness is hardly distinguished for the different cell cases (a)-(c) but can be clearly seen in Figure 4, where the ratio between the homogenised central (minimum) film thickness and that for perfectly smooth surfaces is plotted. For the cell (a) the effect of the roughness is to significantly increase this ratio, whereas for the cell (b) this increase is only marginal. On the contrary, for the cell (c), the effect of the roughness is to decrease the ratio between the homogenised central (minimum) film thicknesses referring to the rough and the smooth case.

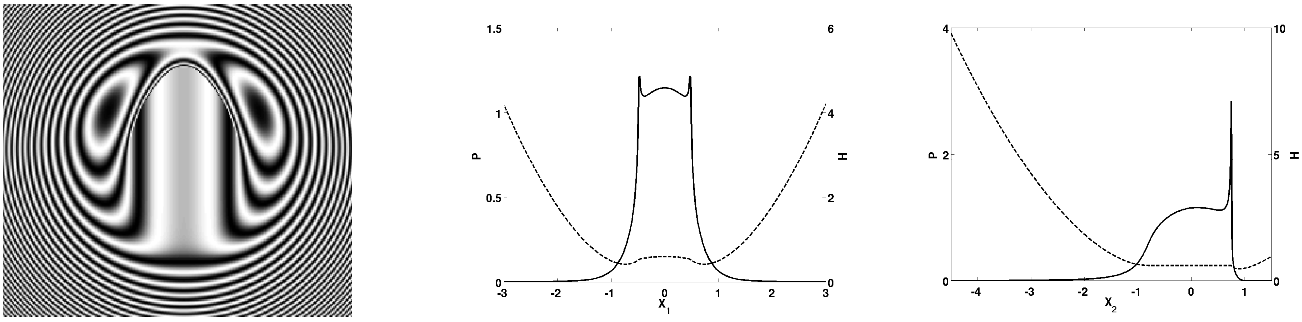

Figure 3: pseudo-interferometry of film thickness and cross sections of pressure and film thickness in both X_1- and X_2- directions for the cell (a) and $\delta = 1.2$. The velocity is in the X_2-direction.

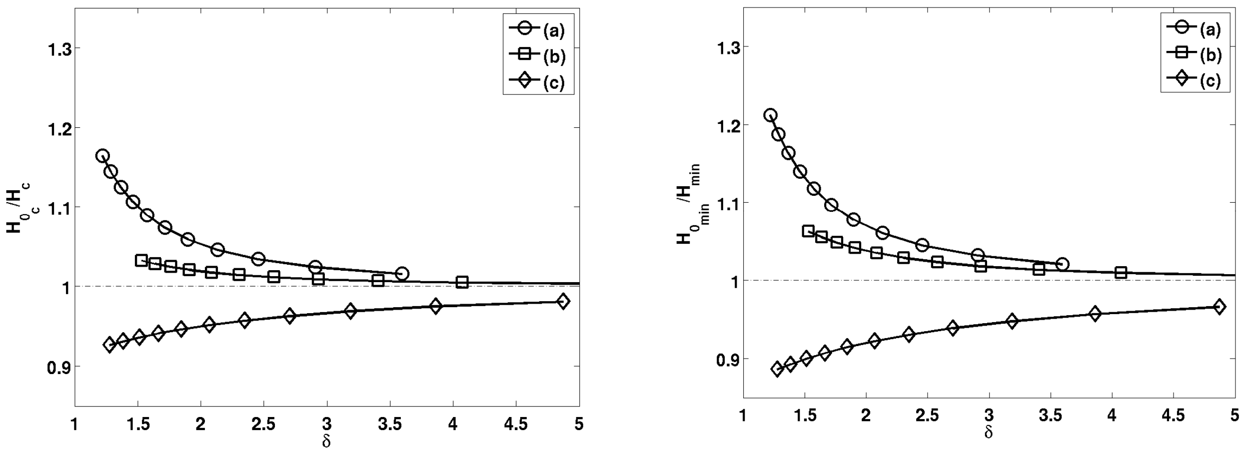

Figure 4: Effects of cells (a)-(c) on the film thickness. The curves show the ratio between the homogenised central and minimum film thicknesses (H_{0c}, H_{0min}) and that for perfectly smooth surfaces (H_c, H_{min}) as a function of the parameter δ

4 CONCLUSIONS

The performance of the EHL mechanism strongly depends on both the macroscopic geometry of the contact and on the microscopic length scales characteristic of the roughness. Homogenisation is a rigorous method which provides proper means for the description of the lubrication mechanism for largely arbitrary roughness patterns. This analysis is of interest in all the applications in which a proper evaluation of the film thickness is required. Most important, it allows one to surmount the limits of other (classical) approaches, notably the statistical one (no ambiguities in the definition of the dimensions of the cell problem and in its boundary conditions are avoided) and the deterministic one (considerably less computational effort).

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