Quantization-based Complexity Reduction for Range-dependent Modified Gilbert Model

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***Abstract*—In this contribution we propose an algorithm for dimension reduction of the previously introduced range-dependent modified Gilbert model. For reduction of the model dimension we suggest to jointly quantize model parameters estimated based on very narrow measurement intervals. We analyze the influence of the number of quantization levels on the model performance and based on this analysis find the optimal model dimension. Furthermore, we show that selected environmental and propagation effects, influencing the error pattern, can be identified by means of quantizing model parameters. Finally we demonstrate that these effects can be accurately reproduced by our model.**

I. Introduction

The design and optimization of all communication systems, including vehicular communication systems, require realistic models of the radio propagation channel. There exist different ways of modeling the propagation channel ranging from ray-tracing and replay models to stochastic channel models. For replay models, i.e., [1] the vehicular transmission is measured in realistic environment and the resulting trace is directly used as an input for simulator. However, the resulting models are constrained to reproduce the specific environment, namely the one where the measurements were taken. Ray-tracing models serving as an excellent approximation of the real-world measurements are more general [2]. However, computational complexity of the ray-tracing models is fairly high and the resulting precision is not always required.

In order to create accurate physical layer (PHY) model and yet keep the computational effort within manageable dimensions, stochastic models, describing the wireless channel characteristics from a macroscopic point of view, are often used. In this context, the authors of [3] suggest modeling the channel as a propagation graph with vertices representing transmitters, receivers, and scatterers, and edges representing propagation conditions between vertices. The authors of [4] propose to model time-variant radio channel such that individual multipath components are emerging and vanishing in a temporal birth-death alike manner.

Another approach for parsimonious PHY modeling is a so-called performance modeling. Performance models reproduce communication link in terms of packet error patterns and combine the effects of multiple factors influencing the physical transmission. These factors include transceiver architecture, distance between the transmitter and the receiver, vehicle velocity, traffic conditions and other environmental factors influencing wireless propagation media. In this context a range-dependent modified Gilbert model capable of reproducing realistic errors patterns for infrastructure-to-vehicle (V2I) communication was introduced in [5]. Building upon the previous work, we propose an algorithm for dimension reduction of this model. The advantages of the suggested dimension reduction algorithms are two-fold; on the one hand it allows to reduce the number of model parameters, and on the other hand it makes possible to localize and subsequently parametrize the influence of realistic performance limiting factors.

A. Experimental Setup

Parametrization of the range-dependent modified Gilbert model is based on the measured packet error traces. To collect these traces we have conducted an extensive series of measurements on the Austrian highway at a center frequency of 5.9 GHz. The average test vehicle speed was 80 km/h (22.2 m/s) with marginal deviations due to traffic.

As a receiver we have used the cooperative vehicle-infrastructure systems (CVIS) [6] platform, equipped with a radio module implementing the IEEE 802.11p protocol and a GPS receiver. The receiver was connected to the antenna mounted on the roof of the test vehicle. As receive antenna a vertically polarized broadband $(2.0-6.7\,\mathrm{GHz})$ double-fed printed monopole with a radiation pattern close to omnidirectional was used.

As transmitter we have used another unit of the CVIS platform. It was placed inside a weather protection cabinet close to a highway gantry. The radio front-end of the transmitter was connected via a 3 dB power splitter to a set of two identical directional antennas. The transmit antennas are right-hand circularly polarized with antenna gain of 10 dBi resulting in equivalent isotropic radiation power of 16.8 dBm. For the experiment presented here the antennas were mounted on the highway gantry, 7.1 m above the road and were pointing along both driving directions to ensure homogenous coverage.

The transmitter is constantly broadcasting packets of 200 Bytes with a data rate of 6 Mbit/s, corresponding to QPSK with coding rate $1/2$. The receiver was recording the detection events with corresponding time and location stamps only within the expected coverage range. All detection events underwent a cyclic redundancy check (CRC) used to determine whether the detected packet has been decoded correctly or not. Based on the result of the CRC a binary error pattern containing information about all detection events was created. This error pattern together with the GPS data are then used for model parameter estimation.

B. Preliminaries of Modeling Approach

Proposed in [5] range-dependent modified Gilbert model is a computationally effective method for generating realistic V2I error patterns. This model is an extension of a simple two-state hidden Markov model introduced by Gilbert [7]. As shown in Fig. 1, Gilbert's model is fully described by only three parameters: the transition probability from the bad state to the good state, P_{BG}, the transition probability from the good state to the bad state, P_{GB}, and the probability of an error P_E in the bad state. In this model, the good state is error-free.

Aiming at modeling real-world V2I measurements, we have concluded that a model with just two states cannot reproduce the link

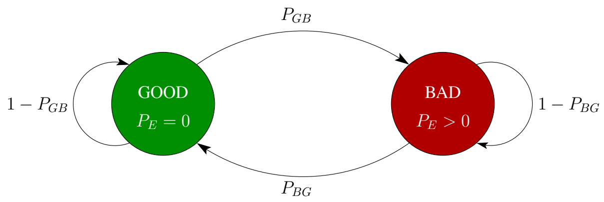


Fig. 1: Schematic illustration of the Gilbert model.

quality with sufficient accuracy. This is because the performance strongly depends on the (absolute) distance between transmitter and receiver. We therefore divide the measured error patterns into N parts, corresponding to N disjoint distance intervals of the same length (henceforth called "granularity"). The model parameters are then estimated for each interval using the Baum-Welch algorithm [8]. Once the model parameters for all N intervals are estimated, they are combined to form a range-dependent modified Gilbert model. This model retains all properties of the original Gilbert model, except for the fact that the model parameters change depending on the transmitter-receiver distance. We note that the initial state of the model in the $(n+1)^{\text{th}}$ interval is equal to the final state in the n^{th} interval.

II. Complexity Reduction Algorithm

Obviously, the granularity of the range-dependent modified Gilbert model constitutes a trade-off between the accuracy of the model and its complexity. We show in [5, Fig. 3] that an acceptable level of accuracy can only be achieved by estimating the model parameters with granularities $\leq$ 10 m. However, small granularities lead to a considerable increase of the number of intervals, thereby increasing the computational overhead of the model. In order to ensure high accuracy of the model while keeping the model dimension low, we suggest to use vector quantization (VQ). The main reason for using a VQ instead of, e.g., a scalar quantization (SQ) is that the model parameters P_{BG}, P_{GB} and P_E are not independent from one another, and hence joint quantization will improve the overall quality of the quantized representation. Furthermore, VQ frequently used for classification purposes in, e.g., speech or image recognition, can potentially be applied to localize and subsequently parametrize certain environmental and propagational aspects. Finally, jointly quantizing all three model parameters would reduce the number of possible parameter combinations by factor of K^2 as compared to SQ, where K is the number of quantization levels.

In the following subsections we will first introduce the VQ design and subsequently find an optimal number of quantization levels. In Sec. II-C arrangement of the quantized model parameters according to the absolute distance will be given. Based on this arrangement the whole communication range can be divided into non-overlapping quality areas, communication performance in which is modeled with a unique set of parameters. Finally, Sec. II-D reveals the insights of environmental effects captured by the quantized model parameters and suggests an accurate way to model them.

A. Quantizer Design

We first estimate the model parameters from a set of measured packet error traces. In order to achieve high accuracy level, the model parameters are estimated with the granularity of 1 m. Next we combine the model parameters for the nth interval into a vector $\underline{x}_n = [P_{BG,n}, P_{GB,n}, P_{E,n}]$, with $n = 1, \ldots, N$. In the following $\underline{X}_N = \{\underline{x}_n\}_{n=1}^N$ denotes the set of all model parameters. Using the LBG algorithm [9], we now aim to find a set of representative vectors

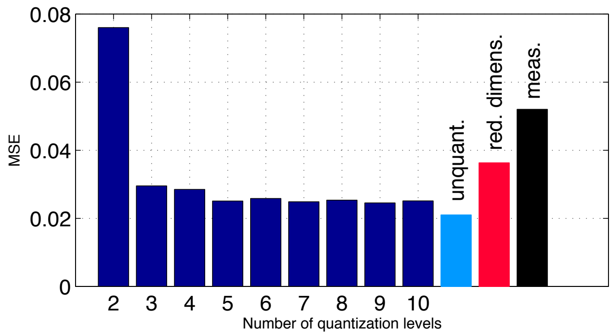


Fig. 2: Model performance in terms of averaged MSE for different quantization strategies.

$\underline{Q}_K = \{\underline{q}_k\}_{k=1}^K$ with $K < N$, such that the average distortion $\mathcal{D}$ (cf. (3)) is minimized. The vector quantizer design consists of the following steps:

1) **Initialization**: The iteration count i is set to 0, the initial set of representative vectors $\underline{Q}_K^{(0)} = \{\underline{q}_k^{(0)}\}_{k=1}^K$ is chosen randomly, and we set $\mathcal{D}^{(-1)} = \infty$.
2) **Assignment:** Each vector of model parameters $\underline{x}_n$ is allocated to its nearest-neighbor representative vector $\underline{q}_{k^\star}^{(i)}$ according to the smallest Euclidean distance, i.e.,

$$\underline{q}_{k^\star}^{(i)} = \underset{k\in\{1,\ldots,K\}}{\arg\min}\; d\Big(\underline{x}_n, \underline{q}_k^{(i)}\Big). \tag{1}$$

3) **Computation of average distortion:** All model parameter vectors $\underline{x}_n$, which have the representative vector $\underline{q}_{k^\star}^{(i)}$ as their nearest neighbor are grouped into the cluster

$$\mathcal{C}_{k^\star}^{(i)} = \left\{\underline{x}_n \in \underline{X}_N : \underline{q}_{k^\star}^{(i)} = \arg\min_k d\Big(\underline{x}_n, \underline{q}_k^{(i)}\Big)\right\}. \tag{2}$$

The average distortion $\mathcal{D}^{(i)}$ of the current set of representative vectors is then given by

$$\mathcal{D}^{(i)} = \frac{1}{N}\sum_{k^\star=0}^{K}\sum_{\underline{x}_n\in\mathcal{C}_{k^\star}^{(i)}} d(\underline{x}_n, \underline{q}_{k^\star}^{(i)}). \tag{3}$$

4) **Update:** If $\mathcal{D}^{(i)} < \mathcal{D}^{(i-1)}$, we update the set of representative vectors $\underline{Q}_K^{(i+1)} = \{\underline{q}_k^{(i+1)}\}_{k=1}^K$ by computing the component-wise arithmetic mean of all model parameter vectors in a cluster. Next, the iteration count i is incremented by one and the algorithm continues with the assignment step.

The above iterative vector quantizer design algorithm terminates if $\mathcal{D}^{(i)} \geq \mathcal{D}^{(i-1)}$.

B. Optimal Number of Quantization Levels

In order to evaluate the performance of our modeling approach, we first convert each error pattern into a packet delivery ratio (PDR) trace. The PDR is calculated as the number of error-free packets divided by the number of detection events in a time interval $t = \Delta d/v$, where $\Delta d = 10$ m and v is the velocity of the test vehicle. The sliding window is moved by 1 m, resulting in a single PDR component every 1 m of the measurement. Next we calculate a mean squared error (MSE) between two PDR traces, PDR_1 and PDR_2, as follows:

$$\text{MSE}_{1,2} = \frac{1}{T}\sum_{\tau=1}^{T}(\text{PDR}_1[\tau] - \text{PDR}_2[\tau])^2, \tag{4}$$

TABLE I: Quantized model parameter for $K = 3$.

	P_{BG}	P_{GB}	P_E
$\underline{q}_1$	0.92	0.01	0.04
$\underline{q}_2$	0.14	0.08	0.79
$\underline{q}_3$	0.05	0.91	0.98

here $\tau = 1, \ldots, T$ is the index of PDR component and T is the total number of PDR trace components. We use T as a normalization factor here, since the largest possible error in each PDR component is 1.

The dark blue bars in Fig. 2 show the performance of model, parameters of which are quantized with $K \in [2, \cdots, 10]$ levels. To obtain these MSE values, we use quantized model parameters to generate 1000 packet error traces and convert them into PDR. Thereafter, MSE is computed between every measured and model generated PDR trace and an average of all MSE values is taken. We note, that the data sets originating from the same measurement are divided into two subsets, one of which is used for model parameter estimation and the other for model performance evaluation. As expected the averaged MSE is reduced, when quantizing with larger number of levels. The performance improvements are particularly large, when using 3 quantization levels instead of 2. However, further increase in the number of quantization levels does not yield substantial MSE reduction.

MSE performance of the original (unquantized) range-dependent modified Gilbert model is shown with light blue bar in Fig. 2. Comparing the light blue bar to the dark blue bars, we clearly obtain some accuracy loss due to quantization. To verify that this loss is acceptable we compute a reference MSE level shown by the black bar in Fig. 2. Therefor, MSE between all combinations of measured PDR traces was calculated and an average over the resulting MSE values was taken.

Comparing the MSE resulting from the model generated traces with quantized parameters to the reference MSE, we conclude that quantizing the model parameters with 3 levels lead to acceptable model accuracy. The respective sets of quantized model parameters are given in Tab. I. Here the parameters are sorted according to the communication quality of the resulting error patterns. That is, the error patters generated by the Gilbert model with parameters $\underline{q}_1$ represent a high quality communication link. While model with parameters $\underline{q}_2$ and $\underline{q}_3$ should be used to reproduce intermediate and unreliable quality communication, respectively.

C. Complexity Reduction: Generic Case

The quantized model parameters P_{BG}, P_{GB} and P_E as a function of distance are shown in Fig. 3. The vehicular link quality captured by the model parameters is obviously distance dependent. Moreover, the communication range can be divided into three non-overlapping intervals, each of which is dominated by a single set of model parameters ($\underline{q}_1$, $\underline{q}_2$ or $\underline{q}_3$). By doing so, the number of intervals in the range-dependent modified Gilbert model is reduced to $N = 3$. Proposed boundaries between the three communication quality areas are shown with the dashed red lines in Fig. 3.

Thus, as generic modeling solution we suggest to classify the performance achieved in the radius of 330 m around the transmitter as a high quality communication range and use Gilbert model with parameters $\underline{q}_1$ to generate the respective packet error patterns. The link performance in the range from $(330 - 380]$ m is the best represented by parameters $\underline{q}_2$ and can be classified as an intermediate quality communication area. While the model with parameters $\underline{q}_3$ can be used to reflect the unreliable communication, obtained in the remaining coverage range, i.e., $(380 - 500]$ m.

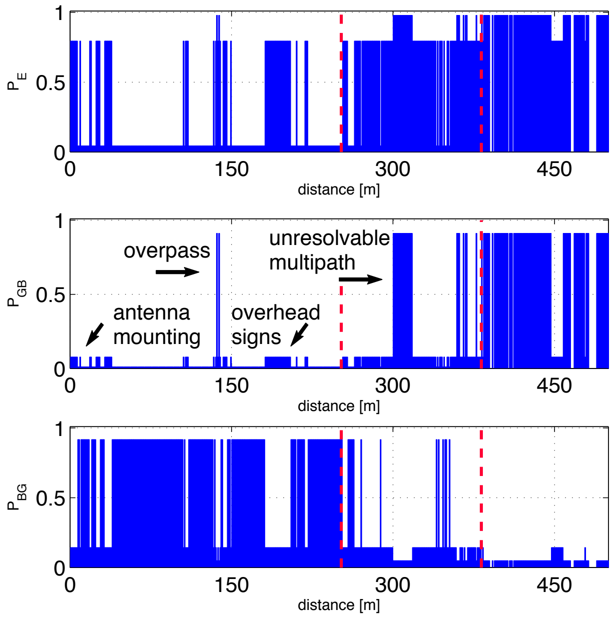
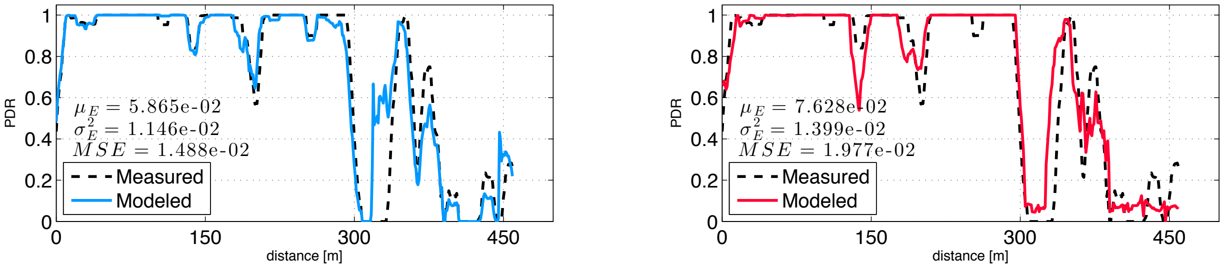


Fig. 3: Model parameters quantized with $K = 3$ levels.

D. Complexity Reduction: Realistic Case

The generic modeling approach is obviously very general and leads to disregard of realistic environmental and propagational aspects. Due to VQ, these performance limiting aspects can be easily localized and characterized in terms of model parameters (cf. P_{GB} plot in Fig. 3) and are as follows:

- **Antenna mounting:** In our previous work [10], we have shown that the performance drop in the close vicinity of the transmitter is caused by the specific antenna mounting. As mention in Sec. I-A, we used two directional antennas pointing along both driving directions. Therefore, the signal was not radiated directly under the gantry and in its close vicinity. This effect can be reproduced by randomly alternating between the model parameters of set $\underline{q}_1$ and $\underline{q}_2$ within the first 40 m of communication range. Parameters for each new interval are chosen randomly, according to empirically estimated probabilities $P(\underline{q}_1) = 0.85$ and $P(\underline{q}_2) = 0.15$.
- **Highway overpass:** Localized change of the model parameters in the high quality communication range at $(134 - 145]$ m to $\underline{q}_2$ and even $\underline{q}_3$ is caused by the highway overpass located 140 m away from the transmitter. To model a highway overpass effect, we suggest to use the Gilbert model with parameters $\underline{q}_3$ exactly at the position of the overpass and parameters $\underline{q}_2$ 5 m before and after the overpass location.
- **Highway overhead signs:** Large metallic objects, such as highway overhead signs, located within the high quality communication range impair the quality of communication link significantly by blocking the line-of-sight. In the measurements presented here, the overhead signs were located 190 m away from the transmitter. The communication link impairments introduced thereby, are reflected by using parameters $\underline{q}_2$ 10 m before and after the overhead signs' position.
- **Unresolvable multipath components:** The majority of our measurements, regardless of parameter settings and environ-



(a) Blue solid line shows a sample generated by the original model.

(b) Red solid line shows a sample generated by the model with reduced dimension.

Fig. 4: Measured sample (black dashed line) vs. model generated sample.

ment, exhibit sudden performance loss at the distance of approx. 300 m. Underlying reasons were investigated based on channel sounder measurements in [1]. The authors have concluded, that the number of multipath channel components in this range is significantly larger as at any other position. Since in our measurements, an of-the-shelf IEEE 802.11p receiver without any advanced signal processing was used, the large number of multipath components could not be resolved and a characteristic performance drop in this particular area was obtained for all measurement setups and for all measurement repetitions. This performance drop is modeled by using parameters $\underline{q}_3$ in the range $(300 - 330]$ m.

Therefore, to turn the generic model into realistic, we will exchange the generic parameters with realistic at the above specified positions. It is worth mentioning that by placing the realistic model parameters to these positions we reproduce the particular environment of our measurements. Other environments can be however easily reproduced as well, by appropriate placing of the realistic model parameters.

III. Results

To analyze the performance of the realistic model with reduced dimension, we calculate the average MSE between measured and model generated error patterns. The resulting MSE value is shown by the red bar in Fig. 2. As expected the performance is slightly worse than for the model, parameters of which were quantized with $K = 3$ quantization levels. However, MSE is still below the reference level (cf. black bar in Fig. 3) and thus, we conclude that the accuracy constraints are fulfilled by a model with reduced dimension.

For better visualization of the model accuracy, we compare a randomly chosen measured PDR curve shown with black dashed line in Fig. 4 to PDR resulting from original range-dependent modified Gilbert model, solid blue line in Fig. 4(a) and to PDR resulting from the model with reduced dimension, solid red line in Fig. 4(b).

Additionally we calculate the mean μ_E and the variance σ_E^2 of the absolute error between measured and modeled samples as follows:

$$\mu_E = \frac{1}{T}\sum_{\tau=1}^{T} |\mathrm{PDR}_{\mathrm{meas}}[\tau] - \mathrm{PDR}_{\mathrm{mod}}[\tau]|, \quad (5)$$

$$\sigma_E^2 = \frac{1}{T-1}\sum_{\tau=1}^{T} (|\mathrm{PDR}_{\mathrm{meas}}[\tau] - \mathrm{PDR}_{\mathrm{mod}}[\tau]| - \mu_E)^2, \quad (6)$$

The resulting values together with the MSE for the particular measurement and model realization samples presented in Fig. 4 are given beside the plots. The MSE and μ_E for the model with reduced dimension is just slightly larger than that of the original model. For comparison, μ_E and σ_E^2 calculated for measured PDR curves amount to $1.158 \cdot 10^{-1}$ and $3.79 \cdot 10^{-2}$, respectively.

IV. Conclusions

Quantization-based algorithm for dimension reduction of the range-dependent modified Gilbert model introduced in [5] was proposed. On the basis of the quantized model parameters, well-defined quality boundaries can be set. The entire communication range is divided into high, intermediate and unreliable quality communication ranges. Furthermore, realistic performance limiting factors are localized and characterized in terms of model parameters.

We show that the performance of the model with reduced dimension is only marginally worse than the performance of the original model. Moreover, none of the performance indicators for the model with reduced dimension exceed the reference level, defined by the difference between the repetitions of the same measurement. Therefore, we conclude that the proposed algorithm not only enables characterization of realistic environmental effects in terms of model parameters, but also allows to significantly reduce the model dimension at the cost of negligible accuracy reduction.

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