Signal-to-Noise Ratio Modeling for Vehicle-to-Infrastructure Communications

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Abstract—In this contribution we propose an extension to the range-dependent modified Gilbert model introduced in [1]. With the proposed extension, the model can be used to generate realistic vehicle-to-infrastructure signal-to-noise ratio (SNR) traces along with the corresponding error patterns. We model the SNR as a combination of correlated large scale fading and small scale fading. The model parameters are derived from real-world measurements at 5.9 GHz. The accuracy of our simple and yet effective modeling approach is confirmed by comparing the model generated SNR traces to the measured performance.

I. INTRODUCTION

The design and optimization of all communication systems, including vehicular communication systems, require realistic models of the radio propagation channel. There exist different ways of modeling the propagation channel ranging from ray-tracing and replay models to stochastic channel models. For replay models, i.e., [2] the vehicular transmission is measured in realistic environment and the resulting trace is directly used as an input for simulator. However, the resulting models are constrained to reproduce the specific environment where the measurements were taken. Ray-tracing models serving as an excellent approximation of the real-world measurements are more general [3]. However, computational complexity of the ray-tracing models is fairly high and the resulting precision is not always necessary.

In order to provide accurate representation of physical layer and yet keep computational effort within manageable dimensions, stochastic models describing the wireless channel characteristics from a macroscopic point of view are often used. In this context, the authors of [4] suggest modeling the channel as a propagation graph with vertices representing transmitters, receivers, and scatterers, and edges representing propagation conditions between vertices. The authors of [5] propose to model time-variant radio channel such that individual multipath components are emerging and vanishing in a temporal birth-death alike manner. The authors of [6] have developed a path-loss and fading model based on real-world non line-of-sight intersection measurements.

However, there rarely exist models that are capable not only to reproduce certain channel conditions, but also to generate the corresponding packet delivery statistics. Such models incorporate the realistic channel effects and the performance of a standard compliant transceiver. Following the idea of parsimonious packet-level performance modeling a computationally inexpensive approach to model packet errors for vehicle-to-infrastructure (V2I) communications was proposed in [1].

In this contribution we suggest an extension to the proposed modeling approach that allows generation of the realistic signal-to-noise ratios (SNR) corresponding to the model generated error patterns. We develop a V2I performance model based on the 5.9 GHz measurement data. The proposed SNR and error pattern generation model is distance dependent and is intended to be used in packet level load simulations.

II. EXPERIMENT AND MEASUREMENT SETUP

The parametrization of the extended range-dependent modified Gilbert model presented in this paper is based on measured packet error traces and SNR traces. The measurement data is obtained through an extensive series of real-world measurements on an Austrian highway at a center frequency of 5.9 GHz. The average test vehicle speed was 80 km/h (22.2 m/s) with marginal deviations due to traffic.

We have used the cooperative vehicle-infrastructure systems (CVIS) platform [7] as a receiver. The CVIS platform implements the IEEE 802.11p protocol stack and incorporates a GPS receiver for location and timing data. The receiver antenna (mounted on the roof of the test vehicle) is a vertically polarized broadband (2.0 – 6.7 GHz) double-fed printed monopole with an almost omnidirectional radiation pattern.

As a transmitter we have used another unit of the CVIS platform. The transmitter is placed inside a weather protection cabin close to a highway gantry, where it is connected to the mains and a local area network. The radio front-end of the transmitter is set to transmit with 10 dBm and is connected to two identical directional antennas via a 3 dB power splitter. The transmit antennas are right-hand circularly polarized with an antenna gain of 10 dBi. For the experiment presented here, the antennas are mounted on a highway gantry, 7.1 m above the road and were pointing in opposite directions the highway to ensure homogeneous coverage.

The transmitter is constantly broadcasting packets of 200 Bytes with a data rate of 6 Mbit/s. The receiver is recording detection events with their corresponding SNRs, as well as timing and location information within the expected coverage range. In the post-processing stage, all detection events underwent a cyclic redundancy check (CRC) used to determine whether the detected packet has been decoded correctly or not. Based on the result of the CRC, a binary error pattern for all detection events is created. The resulting error patterns together with the estimated SNR values and the GPS data are used in what follows.

III. PRELIMINARIES OF MODELING APPROACH

In our previous work [1], we have proposed a range-dependent modified Gilbert model as a computationally effective method for generating realistic V2I error patterns. This model basically is an extension of a simple two-state hidden Markov model introduced by Gilbert [8]. As shown in Fig. 1, Gilbert’s model is fully described by only three parameters: the transition probability from the bad state to the good state, $P_{BG}$, the transition probability from the good state to the bad state, $P_{GB}$, and the probability of an error $P_e$ in the
bad state. In this model, the good state is error-free and in the bad state errors occur with probability \( P_E \). The error pattern generated by the model is thus probabilistically linked to the state sequence and, hence, we cannot directly infer the state sequence by observing the error pattern.

Aiming at modeling real-world V2I measurements, we have concluded that a model with just two states cannot reproduce the link quality with sufficient accuracy. This is because the performance strongly depends on the (absolute) distance between transmitter and receiver. We therefore divide the measured error patterns into \( N \) parts, corresponding to \( N \) disjoint distance intervals of the same length (henceforth called “granularity”). The model parameters are then estimated for each interval using the Baum-Welch algorithm [9]. Once the model parameters for all \( N \) intervals are estimated, they are combined to form a range-dependent modified Gilbert model. This model retains all properties of the original Gilbert model, except for the fact that the model parameters change depending on the transmitter-receiver distance. We note that the initial state of the model in the \( (n + 1) \)th interval is equal to the final state in the \( n \)th interval.

Previously, we have shown that the granularity of the range-dependent modified Gilbert model constitutes a trade-off between the accuracy of the model and its complexity. We show in [1, Fig. 3] that an acceptable level of accuracy can only be achieved by estimating the model parameters with granularities \( \leq 10 \) m. However, small granularities lead to a considerable increase of the number of intervals, thereby increasing the computational overhead of the model. In order to ensure high accuracy while keeping the model complexity low, we jointly quantize the model parameters to obtain a small set of probabilities (cf. [10] for details). Prior to quantization, the model parameters are estimated using a granularity of 1 m aiming at higher model accuracy. Our results (cf. [10, Fig. 2]) have shown that quantizing (clustering) the model parameters with \( K = 3 \) levels (clusters) yields a negligible loss in model accuracy. The respective sets of quantized model parameters are given in Tab. I. Here, the parameters are sorted according to link quality levels. That is, the error patterns generated by the Gilbert model with the parameter sets \( Q_1 \) represent high quality link performance, and the parameter sets \( Q_2 \) and \( Q_3 \) reproduce intermediate and unreliable quality link performance, respectively.

**TABLE I: Parameters of the range-dependent modified Gilbert model, quantized with \( K = 3 \) quantization levels.**

<table>
<thead>
<tr>
<th>Quality level</th>
<th>( P_{BG} )</th>
<th>( P_{GB} )</th>
<th>( P_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1 )</td>
<td>0.92</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>0.14</td>
<td>0.08</td>
<td>0.79</td>
</tr>
<tr>
<td>( Q_3 )</td>
<td>0.05</td>
<td>0.91</td>
<td>0.98</td>
</tr>
</tbody>
</table>

The quantized model parameters \( P_{BG} \), \( P_{GB} \), and \( P_E \) are shown in Fig. 2 versus distance. Unsurprisingly, we observe that the vehicular link performance is obviously distance dependent. Thus, the model parameters in the radius of about 250 m around the transmitter are predominantly given by the quality level \( Q_1 \) (smallest error probability and largest probability of remaining in the good state). The V2I link performance in the range of about 250 m to 380 m is best reproduced by the quality level \( Q_2 \). Here, the probability to stay in the good state is still fairly high, but the probability of an error in the bad state is significantly larger as in quality level \( Q_1 \). The performance of the remaining coverage is modeled using the parameters of the poorest quality level, i.e., \( Q_3 \). Fluctuations in the quality level in Fig. 2 are mainly due to the specific propagation environment as described in detail in [10].

IV. SNR Modeling Approach

**A. Overview**

In this section, we introduce an extension of the range-dependent modified Gilbert model to additionally produces SNR traces corresponding to the error patterns. To this end, we need to know the probability of observing a given SNR value in each state for each quality level. Although this information is hidden initially, we can infer it from our data. The most likely state sequence can be determined recursively using the Viterbi algorithm (i.e., dynamic programming) once the model parameters are known. Subsequently, each measured SNR value can be associated to a state. The detailed SNR analysis and modeling can then be performed separately for each state and quality level. Fig. 3 shows the measured SNR values as a function of distance for the good state (upper plot) and the bad state (lower plot), respectively (different color represent different quality levels). As explained above, a granularity of 1 m is used to estimate the model parameters and therefore the median SNR trend (solid line in Fig. 3) is calculated separately for each 1 m interval. We note that with the chosen granularity and measurement parameters, each interval contains about 200 detection events and, hence, the median SNR is computed from approximately 200 data points. The median SNR curves in Fig. 3 are discontinuous since there are jumps between the individual quality levels (cf. Fig. 2) and in the good state each quality level exhibits different median SNRs.
By comparing the measured SNR values in the two states, the advantage of our state-dependent SNR modeling becomes obvious and allows us to conclude the following: On the one hand, the median SNR in the bad state is almost constant over the considered distances and the dependence on the quality level is negligible. On the other hand, in the good state the median SNR varies significantly over the distance and the variations depend on the considered quality level. As expected, the largest median SNR values are obtained in the highest quality level, $Q_1$. The SNR values obtained in the range of the poorest quality level ($Q_3$) are significantly smaller than the SNR of the other two quality levels and the median SNR in $Q_3$ is almost constant.

To model the variations of the receiver SNR versus the transmitter-receiver distance, a combination of path loss, shadowing, and small scale fading is frequently used. Path loss results from dissipation of the power radiated by the transmitter as well as effects of the propagation channel. Shadowing is caused by obstacles between the transmitter and receiver that attenuate signal power through absorption, reflection, scattering, and diffraction. The variations due to the combination of path loss and shadowing are also referred to as large scale fading, as they occur over large distances (relative to the wavelength). The median SNR in Fig. 3 can therefore be interpreted as large scale fading. The deviation of the measured SNR values from the median can be viewed as small scale fading which is due to constructive and destructive superposition of multipath signal components. Small scale fading occurs over relatively short distances which are in the order of the wavelength. We propose to use an approach based on combination of large scale and small scale fading to model the SNR values corresponding to the error patterns produced by our range-dependent modified Gilbert model with quantized parameters.

### B. Large Scale Fading

The off-the-shelf receiver used in our measurements, provides per packet SNR values with a resolution of 1 dB. The SNR values range from 0 dB to 34 dB and the estimated receiver noise power is $P_N = -110$ dB. The measured SNR is given by (all quantities in (1) are on a logarithmic scale)

$$SNR = P_{TX} - P_N - L_P - L_S,$$

where $P_{TX} = 10$ dBm is the transmit power, $L_P$ are the propagation losses, and $L_S$ is the system loss. The system loss incorporates the cable losses and the antenna gains, as well as any losses due to imperfections of the receiver hardware implementation. On the one hand, a sufficiently precise derivation of the system losses is infeasible due to the limited knowledge of the detailed receiver processing. On the other hand, we aim at developing a performance model that incorporates the effects of both, the channel and the standard compliant transceiver chain, and therefore modeling the system loss is not necessary. Instead of modeling the system loss and then deducing the propagation losses from the measured data, we focus on reproducing their joint effect which we refer to as transceiver chain loss. By subtracting the transceiver chain loss and the noise power from the transmit power, we obtain the SNR trend (without the effects of shadowing and small scale fading). The SNR trend is depicted by the blue line in Fig. 4.

To model the SNR trend, we fit the measured SNR values of the $k$th link quality level to an exponential function of the form

$$SNR_k = e^{d_k d}, \quad k = 1, 2, 3,$$

where $d$ is the distance from the transmitter. The fit is performed separately for both states and the results are given in Tab. II. From the resulting coefficients we conclude that the SNR trend in the bad state does not change significantly over the distance for all quality levels and amounts to approximately 5 dB. Furthermore, the SNR trend of the good state for the poorest quality level $Q_3$ is also constant and equals 6 dB.

To complete the large scale fading model, we next suggest a way of reproducing the shadowing effects. On the linear scale the shadowing is commonly modeled by a log-normal distribution of the received signal power. This corresponds to a normally distributed random variable on the logarithmic scale. In order to create a correlated Gaussian random process, we use a general autoregressive (AR) modeling approach. The correlation is essentially introduced by shaping the spectrum of a white input process using a linear filter.
TABLE II: SNR model parameters.

<table>
<thead>
<tr>
<th>State</th>
<th>Quality level ( Q_k )</th>
<th>SNR trend</th>
<th>Shadowing</th>
<th>Small scale fading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha_k )</td>
<td>( \beta_k )</td>
<td>( \gamma_1,k )</td>
<td>( \sigma^2_k )</td>
</tr>
<tr>
<td>Good</td>
<td>( k = 1 )</td>
<td>22</td>
<td>-0.0022</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>( k = 2 )</td>
<td>16</td>
<td>-0.0021</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>( k = 3 )</td>
<td>6</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Bad</td>
<td>( k = 1,2,3 )</td>
<td>5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The main advantage of using an AR model is the simplicity with which their parameters can be computed.

An AR process of order \( p \) can be generated via the time domain recursion

\[
y_k[n] = - \sum_{i=1}^{p} \gamma_{i,k} y[n-i] + w[n].
\]

Here, \( w[n] \sim N(0, \sigma^2_w) \) is the uncorrelated filter input signal and \( y[n] \) is the signal obtained at the output of the filter. The AR model for the \( k \)th quality level consists of the filter coefficients \( \{\gamma_{1,k}, \gamma_{2,k}, \ldots, \gamma_{p,k}\} \) and the variance \( \sigma^2_w \) of \( w[n] \).

In order to estimate the AR filter coefficients, we first subtract the general SNR trend from the median SNR for each of the \( N \) intervals. Given the autocorrelation function of the resulting deviations from the general SNR trend, the AR filter coefficients can be determined by solving the set of \( p \) Yule-Walker equations. These equations are guaranteed to have a unique solution and can be solved efficiently by the Burg algorithm [11]. Since estimation of the AR filter parameters for filter order \( p > 1 \) did not yield significant improvements, we decided to use the AR filter of order 1 (AR(1)).

The resulting filter coefficients \( \gamma_{1,k} \) and the variances \( \sigma^2_k \), \( k = 1,2,3 \), are given in Tab. II. Thus, to model the large scale fading component (LSF\(_{\text{STATE}}\)):

- normally distributed random variables with the specified variance are generated,
- the spectrum of these random variables is shaped by the AR(1) filter to introduce correlation,
- the resulting correlated process is added to the general SNR trend.

An example of the resulting large scale fading is depicted by the red curve in Fig. 4.

Since the deviations from the large scale fading in the bad state for all quality levels and in the good state for \( Q_3 \) are negligible, the shadowing effects are only modeled for the quality levels \( Q_1 \) and \( Q_2 \) in the good state. For the large scale fading value in the bad state we use LSF\(_{\text{BAD}} = \text{LSF}_{\text{STATE}}\) = \text{LSF}_{\text{BAD}} = 5 \text{ dB}.

C. Small Scale Fading

To model the small scale fading we compute the deviation of measured SNR from the median in each interval. The distribution of this deviation is closely approximated in the logarithmic scale by a clipped Laplace distribution, i.e., we have

\[
p(\eta) = \frac{1}{2\sigma_\delta} e^{-|\eta|/\delta}, \quad \eta \in \{-\eta_{\text{max}}, \eta_{\text{max}}\}, \tag{4}
\]

for the probability density function of the small scale fading value \( \eta \). The normalization factor \( c \) in (4) is given by

\[
c = \int_{-\eta_{\text{max}}}^{\eta_{\text{max}}} \frac{1}{2\delta} e^{-|\eta|/\delta} \, d\eta, \tag{5}
\]

and the values for \( \eta_{\text{max}} \) and \( \delta \) are given in Tab. II.

In order to obtain the modeled SNR value corresponding to a specific detection event we add a small scale fading realization according to (4) to the large scale fading LSF\(_{\text{STATE}}\) of the current interval. The details of the proposed modeling approach are summarized in Algorithm 1.

Algorithm 1 generation of error pattern and SNR traces

Input:
- \( N \) - number of non-overlapping intervals of length \( 1 \text{ m} \)
- \( K \) - number of quality levels
- \( P \) - number of error pattern digits per interval

Phase 1 - Large scale fading generation

if \( \text{STATE} = \text{BAD} \) then

\[
\text{LSF}_{\text{BAD}}(1:N) \leftarrow 5 \text{ dB}
\]

else

for \( k = 1 \) to \( K \) do

\[
\text{LSF}_{\text{GOOD}}(1:N) \leftarrow \text{SNR}_k(1:N) + y_k(1:N)
\]

end for

end if

Phase 2 - Error pattern and SNR generation

for \( n = 1 \) to \( N \) do

find quality level \( Q_k \) of interval \( n \)

generate length-\( P \) error pattern using parameters \( Q_k \)

\( \text{STATE} \leftarrow \) resulting sequence of states

for \( p = 1 \) to \( P \) do

\( \eta \leftarrow \) generate small scale fading realization

if \( \text{STATE}(p) = \text{GOOD} \) then

\( \text{SNR}(p) \leftarrow \text{LSF}_{\text{GOOD}}(n) + \eta \)

else

\( \text{SNR}(p) \leftarrow \text{LSF}_{\text{BAD}}(n) + \eta \)

end if

end for

end for

V. RESULTS

To demonstrate the proposed modeling approach, we compare the measured V2I performance with the performance originating from the extended range-dependent modified Gilbert model. The red curve in Fig. 5(a) shows an example of the measured SNR, while the blue curve shows an example of the modeled SNR (vs. absolute distance). Except for the very large SNR values in the close vicinity of the transmitter, it was possible to reproduce the measured performance very accurately. To validate this observation we have computed the normalized MSE between the modeled and the measured SNR sequences as follows:

\[
\text{MSE}_{1,2} = \frac{1}{N_{\text{SNR}}} \sum_{n=1}^{N} (\text{SNR}_1[n] - \text{SNR}_2[n])^2, \tag{6}
\]
where $\text{SNR}_{[n]}$ is the median SNR in the interval $n$ and $\text{SNR}_{\text{max}}$. $N$ is a normalization factor. As a reference for desired SNR model accuracy we computed the averaged MSE between 10 repetitions of the measurement. The reference MSE amounts to $\text{MSE}_{\text{ref}} = 0.35$. While the MSE between measured and model generated SNR traces was found to be $\text{MSE}_{\text{mod}} = 0.42$. The obtained $\text{MSE}_{\text{mod}}$ value is an average MSE computed between 1000 model generated and 10 measured SNR curves. This minor difference in the MSE underlines the accurate performance of the proposed SNR modeling approach.

Fig. 5(b) provides an example of the measured and modeled error pattern curves corresponding to the SNR in Fig. 5(a). For better visualization the error pattern is represented in form of a packet delivery ratio (PDR) curve plotted vs. distance. The PDR is defined as the number of error-free packets divided by the number of detection events in a time interval $T = \Delta d/v$. To calculate the PDR as a function of the distance, we compute a moving average of the corresponding error pattern where we set $\Delta d = 10$ m, and $v$ is the velocity of the test vehicle. Verifying the accuracy of the generated error pattern, we can state that the reference MSE computed between the measured PDR values amounts to 0.05 and the MSE between the measured and the modeled PDR is just 0.02.

Finally, Fig. 5(c) reveals the average dependence of the PDR on the SNR. The red line shows the averaged measured performance and the blue line shows the performance obtained as an average over 1000 realizations of the extended range-dependent modified Gilbert model. To compute this dependence, we averaged all PDR values obtained for the given SNR value. Both curves follow the same trend and strictly coincide for SNR values below 5 dB. In the higher SNR regimes, the measured PDR is always slightly larger than the modeled. However this difference in performance is negligible.

VI. CONCLUSIONS

In this contribution we propose an extension of the range-dependent modified Gilbert model with quantized parameters introduced in [10]. Based on quantized model parameters, the entire communication range is divided into high, intermediate and unreliable link quality (ranges) and the SNR performance is analyzed for each of them separately. Individual treatment of the SNR components obtained in different model states and quality ranges yields simple yet effective modeling approach.

We suggest to model SNR as a combination of large and small scale fading. The large scale fading consists of the exponentially decaying SNR trend and shadowing effect, modeled by the correlated Gaussian random process. The correlation is introduced by shaping the spectrum of a Gaussian random process with an autoregressive filter of order 1. The small scale fading is modeled as a Laplace distributed random variable, that depends neither on the distance, nor on the model state or quality range. The parameters required for SNR modeling were estimated from the realistic measurement data and are provided in this contribution. Finally, the accuracy of the proposed SNR modeling approach is shown by comparison of the model generated and measured performance in terms of SNR and PDR.

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