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# X. THROUGHPUT ANALYSIS OF S/R SHUTTLE SYSTEMS

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## Abstract

Shuttle systems are used in high performance automated storage/retrieval unit load systems. Each storage level is serviced by one transfer car travelling in dual command operation. One buffer slot is located at both ends of each level. This decouples horizontal travel from vertical input/output moves, which in this case requires two independent vertical reciprocating lifts at each end of the aisle. Other systems work with only one lift used in lower throughput applications. The content of this paper is treated in the following sections starting with a problem definition and a literature survey. This is followed by a detailed functional description of the system investigated here. A predictive model with analytical equations is derived for simplified calculations and a comparison with simulation results. A summary, conclusions and an outlook finalize the paper.

## Introduction and problem definition

During the past years a number of new automated storage and retrieval systems based on individual autonomous vehicles - also called shuttle systems - have been introduced to the European logistics market [1]. As compared to the conventional AS/R systems they offer greater operational flexibility in varying demand conditions. Several arrangements for the main components consisting of racks, lifts, buffers and vehicles are possible. A number of research papers has been published in the past to investigate standard configurations [2, 7, 8, 9, 10].

In this paper a special configuration of devices to be described later in more detail is investigated in its main performance measures throughput and cycle time. Analytical equations are presented to calculate throughput as the number of transactions per unit time and cycle time as the function of rack geometry (length, height) and kinematic data (velocities, accelerations, transfer times) from the beginning until the end of a transaction. The results of the analytical calculations verified with the simulation package ARENA. Furthermore a system comparison of the shuttle system with conventional SR systems is presented.

The following research questions are treated in detail:

- Calculation of cycle times and throughput depending on rack size
  - Are deadlock situations with blocking possible and by which operational rules can they be prevented
  - What is the effect on throughput and cycle times if shuttles perform single command (SC) operations instead of double command (DC) operations
  - Which improvement of throughput of a shuttle system can be expected compared to a conventional S/R system
- Whether research will be extended to the influence of the dwell point position, the I/O location and the number of buffers on the system performance.

## Literature survey

As mentioned a number of research papers [7, 8, 9, 10] have been published to investigate the performance measures of the shuttle technology in the last years. A major research has been published by Roy (2011) in his dissertation showing the basic system design in Fig. 1.

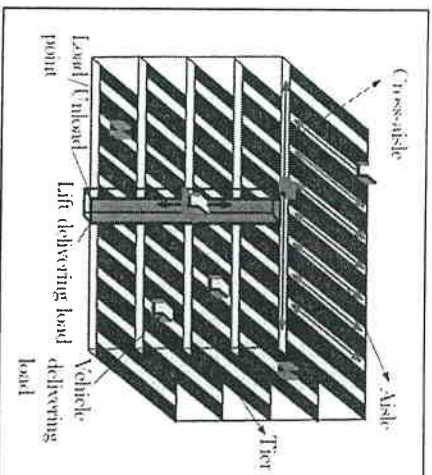


Figure 1: Autonomous vehicle based storage and retrieval (AVS/R) systems as described in [2]

A short functional description of the system investigated by Roy exhibits the following features:

- The shuttles operate on each tier in X- and Y- direction
  - Several shuttles operate on each tier simultaneously
  - Shuttles can change between tiers
  - Loads can enter or leave the system only by means of a shuttle at the I/O point
- As will be shown in the next section these operational specifications do not apply to the system investigated here.

Standard shuttle system configurations have also been treated in VDI 2692 (2013) [3]. The cycle time equations presented there allow only a separate calculation of shuttle and lift equations. VDI 2692 always assumes the lift to act as bottleneck and therefore considers only the lift cycle time to be relevant for the system performance.

Due to interference processes between shuttle, lift and waiting times transfer processes between the devices have to be considered

## Detailed description of the shuttle system

### System description (Figure 2, 3)

The shuttle system contains four devices:

#### Rack

The rack contains the storage locations for the unit loads and horizontal guidance rails for the shuttles. Average rack dimensions have lengths varying between 10 and 150 meters and heights between 2 and 30 meters. Storage locations can be single, double, triple and quadruple deep. Here only single deep locations are treated.

#### Shuttles

The shuttles contain transfer devices operating in orthogonal direction to the main guidance rails. After the load transfer on the shuttle it travels along the horizontal x-axis from the I/O point to the required storage location. Reverse operation takes place in case of a retrieval. Only one shuttle travels on each tier.

#### Lift

Vertical transportation along the z-axis is enabled by two separate lifts, one for the input and one for the output operation. The input lift moves the load from the I/O point at  $z = 0$  to the input buffer at the required tier. The output lift operates in the opposite direction.

#### Buffers

Each tier contains one input buffer and one output buffer arranged opposite of each other. When retrieving a load the shuttle transfers it from the vehicle platform to the buffer place, where from it will be removed by the output lift at a later point of time. Thus the buffer enables a decoupling of shuttle and lift operations. As the shuttle has no waiting time at the output lift it can service a different function and improve performance.



Figure 2: Shuttle system (OSR Shuttle by courtesy of KNAPP)



Figure 3 exhibits the geometric arrangement and the system layout of the shuttle system with buffer locations at the end of the main aisle. Also a double lift system is possible where lift locations are arranged at both ends of the aisle. But not investigated in this paper.

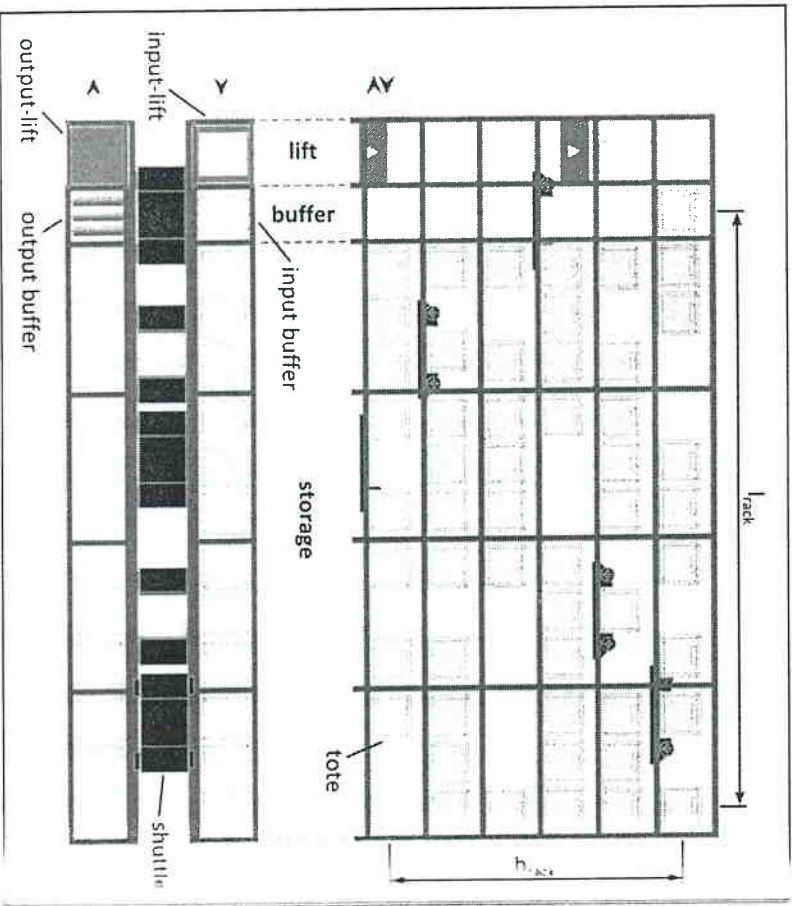


Figure 3: Shuttle system

## 1.1 Functional description

### 1.1.1 Input process

The input process describes the movement of loads from the I/O point to the storage position in the rack. A load to be stored waits in the queue at the I/O point until a vertical motion of the lift transports the load to the target level and transfers it to the input buffer position. Here it will wait until it can be transferred to the shuttle, which transports the load horizontally and transfers it to the final storage position (Figure 4).

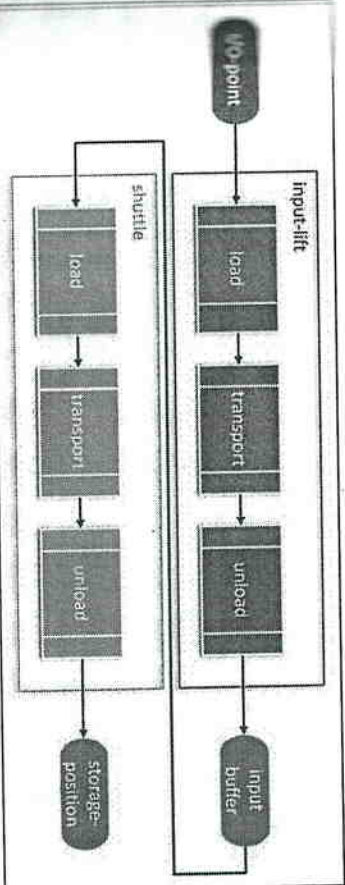


Figure 4: Input process

### 1.1.2 Output process

The output process describes the movement of loads from the retrieval location to the I/O position in front of the rack. A load to be retrieved waits in the order queue at the retrieval position until a horizontal motion of the shuttle transports the load to the output buffer position. After a possible waiting time to empty the buffer a transfer mechanism moves the load to the input buffer position. Here it will wait until it can be transferred to the vertical lift, which transports it to the final I/O position (Figure 5).

The control rules operate the lifts in single commands and the shuttles in dual commands. This means that a shuttle after transporting a load to the storage position moves in empty travel to the next retrieval position, wherefrom it transports the retrieval load to output buffer and transfers it by a lift operation.

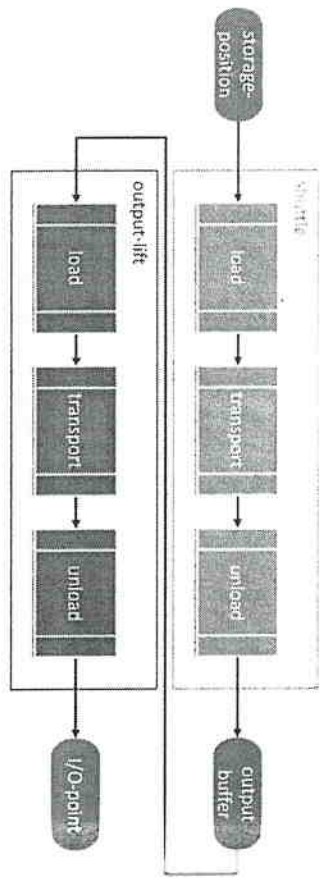


Figure 5: Output process

#### 4 Analytical calculation of cycle times and throughput

##### 4.1 Cycle times and throughput with lifts performing single cycles and shuttles performing double cycles

As stated before the input and the output lifts operate in single cycles and the shuttles in double cycles. These two operations are executed at the same time. For the calculation of the cycle time of the shuttle system the maximum of lift cycle time and shuttle cycle time is dominant.

Additionally to lift cycle times and shuttle cycle times there may occur waiting times. For example in the output process it is possible, that the output lift has to wait for totes brought by a shuttle. It is also possible that too many totes are brought to the output lift and have to wait to be served.

So the expected value of the cycle time of the complete shuttle system  $E(t_{doublecycle})$ :

$$E(t_{doublecycle}) := \max \left( t_{lift}, \frac{t_{shuttle}}{n} \right) + t_{wait}$$

The expected value of the cycle time of the lift is:

$$E(t_{lift}) := \frac{h_{rack}}{v_{lift}} + 2 \cdot \frac{v_{lift}}{a_{lift}} + t_{load\_lift} + t_{unload\_lift}$$

The expected value of the cycle time of the shuttle is:

$$E(t_{shuttle}) := \frac{4}{3} \cdot \frac{h_{rack}}{v_{shuttle}} + 3 \cdot \frac{v_{shuttle}}{a_{shuttle}} + 2 \cdot t_{load\_shuttle} + 2 \cdot t_{unload\_shuttle}$$

- $h_{rack}$  ... height of the rack
- $l_{rack}$  ... length of the rack
- $v_{lift}$  ... velocity of the lifts
- $v_{shuttle}$  ... velocity of the shuttles
- $a_{lift}$  ... acceleration of the lifts
- $a_{shuttle}$  ... acceleration of the shuttles
- $t_{load}$  ... loading time of the lifts
- $t_{unload}$  ... unloading time of the lifts
- $t_{load}$  ... loading time of the shuttles
- $t_{unload}$  ... unloading time of the shuttles

These equations were derived from the well-known formulas for the calculation of cycle times for AS/RS [4].

Additionally to the transportation times waiting times have to be considered:

- In the output process it is the waiting time of the shuttle until the output buffer becomes empty
- In the Input-Process it is the waiting time of the lift until the input buffer becomes empty

The waiting time can be calculated with the help of the queuing theory. Hereby different queuing models can be used. For example the G/G/1 model, M/G/1 model or the E<sub>k</sub>/E<sub>k</sub>/1

model. In section 5 it will be shown by simulation results that the M/G/1 model delivers the best approach.

If we consider the output process, the n shuttles in the rack describe the arrival process and the output lift describes the service process. If we look at the input process, the lift performs the arrival process and the n shuttles in the rack perform the service process.

So we can state for the output process:

$$\lambda_{\text{output}} := \frac{1}{E(t_{\text{shuttle}})} \cdot n$$

n ... number of levels in the rack

$$\mu_{\text{output}} := \frac{1}{E(t_{\text{lift}})}$$

$$\rho_{\text{output}} := \frac{\lambda_{\text{output}}}{\mu_{\text{output}}}$$

And for the input process

$$\lambda_{\text{input}} := \frac{1}{E(t_{\text{lift}})}$$

$$\mu_{\text{input}} := \frac{1}{E(t_{\text{shuttle}})} \cdot n$$

$$\rho_{\text{input}} := \frac{\lambda_{\text{input}}}{\mu_{\text{input}}}$$

the M/G/1 model the waiting times for the input and output process are [5]:

$$w_{\text{output}} := \frac{\rho_{\text{output}} \cdot t_{\text{lift}}}{2} \cdot \frac{1 + v_s^2}{1 - \rho_{\text{output}}}$$

$$w_{\text{input}} := \frac{t_{\text{shuttle}}}{2} \cdot \frac{\rho_{\text{input}}}{1 - \rho_{\text{input}}} \cdot \frac{1 + v_s^2}{2}$$

with the variational coefficient  $v_s$  for the service process.

$$v_s(\text{shuttle}) := \left( \frac{2 \cdot \frac{\text{rack}}{v_{\text{shuttle}}}}{18} \right)^2$$

$$v_s(\text{lift}) := \frac{\sqrt{\text{Var}(t_{\text{shuttle}})}}{n \cdot t_{\text{shuttle}}}$$

The calculation relies on the following assumptions, which have been verified by simulation

- 1. The output cycle time of a system with an utilization ratio of  $\rho_{\text{output}}$  equals the input cycle time of the same system with  $\rho_{\text{input}} = 1/\rho_{\text{output}}$ . A numerical example proves the output cycle time of a system with  $\rho_{\text{output}} = 2$  to equalize the calculation results with an input cycle time for  $\rho_{\text{input}} = 0.5$ .
- 2. Theoretically the waiting times would raise towards infinity in the region of  $\rho_{\text{output}} \approx 1$  or  $\rho_{\text{input}} \approx 1$ . By simulation however it was found that the average waiting time with the command operation never exceeded two seconds (Figures 6, 7):  $t_{\text{wait\_max}} = 2 \text{ sec}$

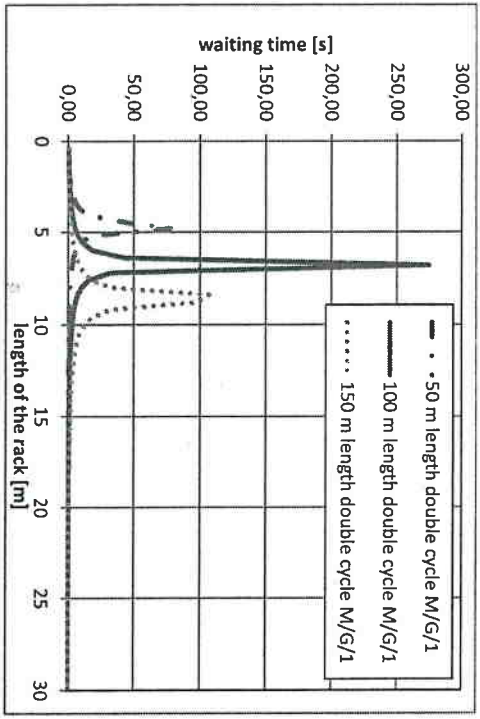


Figure 6: Waiting times

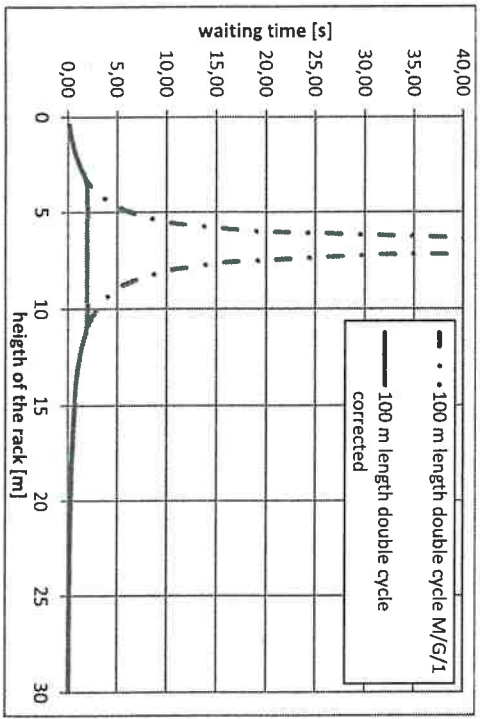


Figure 7: Waiting times for a 100 m rack with and without correction

the calculation the following lift and shuttle parameters for lifts and shuttles were defined:

$v_{max} = 2 \text{ m/s}$	$v_{min} = 5 \text{ m/s}$	velocities of shuttles and lifts
$a_{max} = 2 \text{ m/s}^2$	$a_{min} = 7 \text{ m/s}^2$	acceleration of shuttles and Lifts
$t_{load} = 5 \text{ s}$	$t_{unload} = 1,4 \text{ s}$	loading time of shuttles and Lifts
$t_{turnback} = 6,5 \text{ s}$	$t_{turnaround} = 1,4 \text{ s}$	unloading time of shuttles and Lifts
$\phi = 0,4 \text{ m}$		vertical pitch
$\theta = 0,5 \text{ m}$		horizontal pitch

Figure 8 shows the analytically calculated cycle times depending on the length and height of the rack.

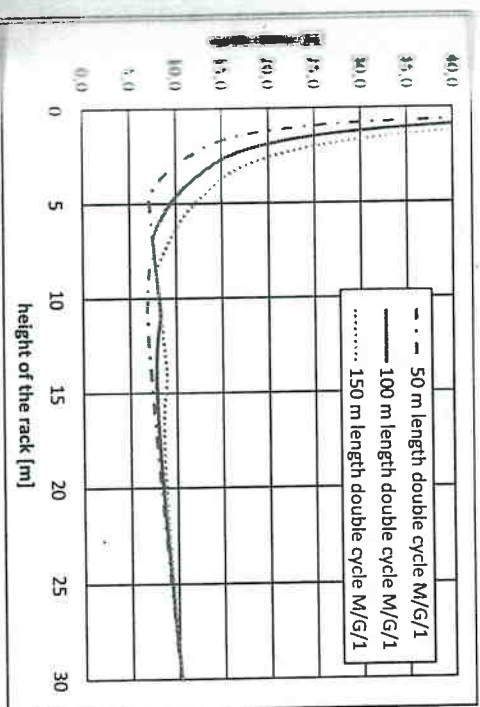


Figure 8: Cycle times of a shuttle system with lifts performing single cycles and shuttles performing double cycles

A few shuttles operate in the left region of figure 8 with rack heights below five meters. Therefore the long travel time of the shuttles is responsible for a high cycle time. Between heights of 5 and 20 meters we observe minimal cycle times for rack lengths of