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Controlling pollution and environmental absorption capacity

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Abstract This pollution accumulation model shows that the environmental absorption capacity is impacted by economic activity. The resulting optimal control problem has two inter-related state variables: the stock of pollution and the absorption capacity of the environment. The stock of pollution decreases with environmental absorption capacity and increases with the rate of current emissions, which is controlled by a production level as well as an emissions reduction effort. However, the environmental absorption capacity is positively affected by an absorption development effort, and negatively impacted by the stock of pollution. Under specific conditions, it is shown that an optimal path, which can be either monotonic or following transient oscillations, leads to a (nontrivial) saddle-point characterized by a positive environmental absorption capacity.

Keywords Pollution · Environmental absorption capacity · Production rate · Emissions reduction effort · Absorption development effort

1 Introduction

For quite some time, a considerable stream of economic literature has pointed out the bounded absorption capacity of pollution by the environment and the necessity for global economic activity to comply with environmental limits (e.g., Georgescu-Roegen 1971;

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Table 1 Economic consequences of possible emissions reduction strategies

Production rate	Technological effort for emissions reduction	
	High	Low
High	*High production revenues* *Moderate pollution costs* *High emissions reduction costs*	*High production revenues* *High pollution costs* *Low emissions reduction costs*
Low	*Low production revenues* *Low pollution costs* *High emissions reduction costs*	*Low production revenues* *Moderate pollution costs* *Low emissions reduction costs*

Costanza and Daly 1992). Despite these recurrent warnings, the carbon emissions trend has been constantly rising over the years, and increased from a yearly rate of 1.1% during the 90's to 3% between 2000 and 2004 (Raupach et al. 2007), aggravating the global warming phenomenon.

Along with increased activity and deterioration of the global economy's carbon intensity, one main driver of this recent acceleration in carbon emissions is the decreased efficiency of natural sinks (Canadell et al. 2007). Over the last 50 years, the efficiency of natural carbon sinks has decreased by 5%, and will continue to fall. One illustration of this trend is the oceanic carbon sink, the decreased efficiency of which has contributed substantially to the increasing growth rate of atmospheric carbon for the 2000–2006 period (Canadell et al. 2007).

This trend should quickly become worse with global warming as the terrestrial biosphere is expected to switch from a carbon sink to a source within this century (Cox et al. 2000; Schaphoff et al. 2006; Lewis et al. 2011). Such an extreme scenario would give rise to uncontrollable consequences due to an increase of exceptional magnitude of atmospheric carbon. For instance, the stock of carbon stored in the permafrost, forests, wetlands and oceans that would potentially be released in the atmosphere due to global warming is estimated at more than 3,700 Gt.

These general observations have critical theoretical implications. Broadly speaking, the pollution stock reduces the environmental absorption capacity, which may progressively switch from a pollution sink to a source. Since a reduction in the environmental absorption capacity increases in return the pollution stock, a 'mere' reduction in the current emissions rate seems insufficient to counteract the positive feedback between pollution and environmental absorption capacity. It is thus essential that an additional policy instrument be considered for directly controlling changes in the environmental absorption capacity, to prevent it from switching from a sink to a source. It is exactly these fundamentals that we want to implement in our representation of pollution dynamics.

A first goal of this paper is to identify the optimal paths of production and efforts spent on reducing emissions that are compatible with non-explosive damages from pollution. Whether such pair paths are chosen depends on the trade-off between revenues drawn from the production rate, on the one hand, and the cost of emissions reduction efforts plus the net cost of negative externalities from pollution, on the other hand.

As shown in Table 1, two kinds of strategies for reducing current emissions may be envisioned, that is, strategies based either on substitutability or on complementarity between the production rate and the technological effort level for reducing emissions.

Substitutability-based strategies, which combine low (resp., high) production revenues and pollution costs and high (resp., low) emissions reduction costs, do not actually result from any trade-off. They merely reflect a situation where either environmental or economic sustainability, that is, either the environmental or economic ability to meet the needs of the present and future generations (Pezzey 1992), prevails while the other fails, which is likely to be hardly manageable in the long run.

Conversely, *complementarity-based* strategies are part of the trade-off. The first one is consistent with the idea that large technological efforts for the reduction of emissions may reconcile a high level of production activity with the preservation of the environment (World Bank 1992; Grossman and Krueger 1995), while the second one is more in line with the general premise that mass production activity is destructive for the environment, and therefore not viable in the long run, including from an economic viewpoint (Georgescu-Roegen 1971; Daly 1997). The first type of strategy widely reflects the industrialist viewpoint,[1] and will be referred to hereafter as the *industrialist strategy*. The second strategy reflects an ecologically-oriented analysis of the impact of mass production on the environment, i.e., ecological economics (Harris 2002), and will be designated as the *ecologist strategy*. We seek to introduce a simple analytical framework to evaluate both the economic and environmental viability of these two types of strategies.

A second goal of this paper is to determine how the development of the environmental absorption capacity can contribute to better control the stock of pollution. As Dasgupta and Mäler (2004) wrote, "in the popular literature, the morals that would appear to have been drawn from the finding are [...] that resource degradation is reversible: degrade all you want now, Earth can be relied upon to rejuvenate it later should you require it". Following this principle, the analysis of optimal pollution accumulation rested, until recently, on the assumption of positive, constant environmental absorption capacity. But, as noted by Wirl (1999), constant environmental absorption capacity "attributes implicitly an advantage to large pollution stocks [...] because a large stock of pollution implies in this case large cleaning."

With increasing evidence of "nature's non-convexities", typically exemplified in the phosphorus discharge into a shallow lake (Holling 1973), the representation of the environmental absorption capacity as an instantaneous nonlinear function of the stock of pollution has become prevalent (Tahvonen and Salo 1996; Dasgupta and Mäler 2003). This change in perspective indeed allowed a more accurate handling of potential irreversible environmental degradation. However, the extension of the nonlinear specification to complementary essential considerations, e.g., investment in the environmental absorption capacity, inherent self-restoring capabilities of the environmental absorption capacity, generalization to non-cooperative game framework, would add substantial mathematical complexity to current difficulties related to non-concave representation.

In regard to these restrictions, we propose to model the environmental absorption capacity as a state variable. Beside the destructive influence of the pollution stock, we assume that the environmental absorption capacity is susceptible to be developed through a specific policy instrument. Such additional control of the environmental absorption capacity should make it possible to counteract the fatal interdependence between pollution and environmental absorption capacity, as described above. In addition, it should alleviate the exclusive, weighty constraint traditionally devolved upon production activity, and then prevent the requirement of an unmanageable zero production terminal state.

[1] For instance, "economic growth is good for the environment, because countries need to put poverty behind them in order to care" (Editorial, *The Independent*, December 4th, 1999); or "trade improves the environment, because it raises incomes, and the richer people are, the more willing they are to devote resources to cleaning up their living space" (*The Economist*, December 4th, 1999, p. 17).

This paper is organized as follows: The next section presents a two-state-variable model in which a social planner can control the stock of pollution by reducing the rate of current emissions and/or by developing the environmental absorption capacity. The social planner is supposed to maximize over an infinite time horizon the discounted social utility resulting from the difference between production revenues, the cost of pollution, and the respective costs of efforts to reduce emissions and to improve the environmental absorption capacity. In the third section, the optimal emissions reduction and environmental absorption capacity development strategy is identified. In the fourth section, steady state values associated with the optimal emissions reduction strategy are computed, and their stability is analyzed. In the fifth section, the transition paths associated with the optimal strategy are determined with numerical means, and a sensitivity analysis to the parameter values is conducted. Section 6 concludes this paper.

2 Model formulation

As noted by Wirl (1999), "dynamic considerations of environmental policies (...) are by large restricted to one state variable control models". According to the most employed formulation, the pollution stock increases with the rate of current emissions and depreciates at a decay rate reflecting the environmental absorption capacity. Let $P(t) \geq 0$ denote the pollution stock at time t, its evolution is given by:

$$\dot{P}(t) = e(t) - \kappa(P(t)) \qquad P(0) = P_0 > 0 \tag{1}$$

where $e(t) \geq 0$ is the rate of current emissions, and $\kappa(.) \geq 0$ is the environmental absorption rate as a function of $P(t)$. Most often, the key specification of $\kappa(.)$ is that of a linear environmental absorption rate, i.e., $\kappa(P(t)) \equiv \rho P(t)$, $\rho > 0$. This specification was introduced in Keeler et al. (1972) and extensively used in the existing literature, notably to determine an optimal economic growth path compatible with environmental preservation (Nordhaus 1991; Stokey 1998), an optimal cleanup policy of a pollution stock (van der Ploeg and Withagen 1991; Caputo and Wilen 1995), and an optimal exploitation of a natural resource whose consumption inflicts negative externalities on the environment (Ulph and Ulph 1994; Farzin 1996).

Although a linear environmental absorption rate is mathematically more tractable, only few natural processes, such as radioactive decay, can be characterized in this way. Further, the linear specification implies that the environmental absorption capacity is increasing with respect to the pollution stock, and disregards the empirical evidence that high pollution stock levels may destroy the environmental absorption capacity (Holling 1973).

A significant refinement of this model assumed that the environmental absorption function is nonlinear, concave with low pollution stock and convex when pollution is higher, with a switching point in between. More precisely, there exists $\bar{P} > 0$ such that $\kappa(P(t)) > 0$, for all $P(t) \in [0, \bar{P}[$ and $\kappa(P(t)) = 0$ for all $P(t) \geq \bar{P}$. This formulation was suggested by Forster (1975) and notably used by Tahvonen and Withagen (1996), Chevé (2000), Brock and Dechert (2008), and Kossioris et al. (2008). Interestingly, this formulation gives rise to multiple equilibria, typically optimal current emissions paths leading to reversible and irreversible pollution equilibria. The trajectories obtained may be discontinuous and non-monotonic in the case of irreversible pollution (Tahvonen and Salo 1996; Tahvonen and Withagen 1996). An extension of this model to include an adjustment process for reducing the current emissions level results in even more complex trajectories, i.e., limit cycles, including in the simpler case of reversible pollution (Wirl 2000).[2]

It is well known that the assumption of the nonlinear environmental absorption function generates challenging technicalities, notably related to the non-differentiability absorption function at the critical pollution level beyond which the environmental absorption capacity is zero. Further, the environmental absorption function is represented as an instantaneous process, with initial conditions similar to those of the pollution stock. As a result, this representation limits the possibility of considering complementary influences, among which are activities aiming to develop the environmental absorption capacity, and inherent self-restoration capabilities of the environmental absorption capacity. These considerations, if they were also taken into account, would add significant complexity to current difficulties related to the non-concave representation.

The limitations associated with the pollution accumulation model in (1) justify that the environmental absorption capacity be treated as a state variable rather than just an instantaneous response function.[3] This paper presents the evolution of pollution accumulation and environmental absorption capacity as separate but interlinked dynamic processes. In this respect, we extend the pollution accumulation model in (1), introducing two distinguishing features. First, the rate of current emissions is controlled twofold, that is, through a production rate and an emissions reduction technological effort. Second, the environmental absorption capacity is defined as a stock variable.

To do so, we consider an economy with an instantaneous production activity. The production rate, denoted by $u(t) \geq 0$, determines the current emissions rate, $e(t) \geq 0$. Emissions can be reduced thanks to a technological reduction effort, the level of which is denoted by $v(t) \geq 0$. The reduction effort may consist in *de-carbonization* initiatives, that is, limiting energy consumption, using renewable energies, etc. The current emissions rate is then given as:

$$e(t) = \alpha u(t)(1 - \beta v(t)) \tag{2}$$

where $\alpha, \beta > 0$. According to (2), there are no emissions when production activity is nil. Further, reduction effort efficiency is such that, for a given positive output, the current emissions rate reaches a maximum value in the case where the reduction effort is inexistent, but decreases proportionally with a marginal increase in the emissions reduction effort. Given thermodynamic restrictions to the extent to which emissions can be reduced, the reduction effort should be bounded, that is, $v \leq 1/\beta$. A similar formulation to (2) can be found in van der Ploeg and Withagen (1991).

To describe the evolution of the pollution stock, we use the following formulation:

$$\dot{P}(t) = e(t) - \delta A(t)P(t) \qquad P(0) = P_0 > 0 \tag{3}$$

According to (3), the stock of pollution increases with $e(t)$ as defined in (2), and decreases linearly with the environmental absorption capacity $A(t)$ at the rate $\delta A(t)$, $\delta > 0$.

[2] An alternative formulation assumes that the evolution of environmental quality is similar to that of a renewable resource. Along with extensions where the environmental regeneration function depends on other variables, such as the stock of manufactured capital, the stock of natural resources or labour input, this formulation was used by Krautkraemer (1985), Mäler (1991), and Tsur and Zemel (1996), to derive an optimal strategy to protect the environment. The extension of this model to include an adjustment process for reducing the current emissions level gives dampened oscillations and limit cycles as possible optimal strategies (Wirl 1999).

[3] A similar idea is found in Leandri (2009) where the pollution accumulation is nevertheless disregarded.

The environmental absorption capacity obeys the following transition equation:

$$\dot{A}(t) = w(t) - \gamma P(t) \qquad A(0) = A_0 > 0 \tag{4}$$

where $\gamma > 0$. According to (4), the environmental absorption capacity is decreasing with the stock of pollution (Holling 1973; Scheffer 1997). For tractability, we assume that the destructive impact of pollution on the rate of change of the environmental absorption capacity is linear. It will be shown below that linearity does not imply any loss of generality compared with the nonlinear environmental absorption function in (1).

The natural absorption capacity in (4) can also be restored and/or developed by an absorption development activity, the instantaneous effort level of which is denoted by $w(t) \geq 0$. The rationale for this effort relies on the premise that investing in natural capital is economically beneficial (Carpenter and Cottingham 1997; Costanza et al. 1997; Costanza 1998). The numerous programmes and initiatives implemented at national and international levels (e.g., European centre for River Restoration, United National Environment Programme) toward environmental restoration and development illustrate the importance of such policy option. It may consist in maintaining and/or developing wetlands, promoting reforestation, restoring rivers, erosion reduction, combating desertification, expanding marine protected areas, promoting biodiversity conservation, carbon sequestration, etc.

Finally, we assume that the inherent capabilities of self-restoration of the environmental absorption capacity are negligible in (4). Yet, we believe that extending our model to the environmental self-restoration capabilities assumption is an important topic for future research.

Though the initial value of A is strictly positive in (4), an insufficient absorption development effort may cause it to decrease so that it becomes strictly negative. In this case, the natural absorption capacity turns to a source, that is, a natural rejection capacity. Since a return from source to sink would be difficult, switching from a positive to a negative value of A would then correspond to an irreversible situation causing the pollution stock to increase over time independently of the current emissions level.

To illustrate this evolution, we consider the differential system given by (3)–(4) and assume that there is no human interference (that is, no emissions and no absorption development activity) over an infinite horizon, i.e., $e(t) = w(t) = 0$, for all $t \geq 0$. Compared with the real-world situation, in which emissions are still largely irreducible and the environmental absorption capacity is massively degraded by deforestation (Canadell et al. 2007), assuming that there is no human interference reflects a utopic situation. In this case, the pollution-absorption dynamics is represented by the planar dynamical system:

$$\begin{aligned} \dot{P}(t) &= -\delta A(t) P(t) \qquad & P(0) &= P_0 > 0 \\ \dot{A}(t) &= -\gamma P(t) \qquad & A(0) &= A_0 > 0 \end{aligned}$$

At the steady state, $P^\infty = 0$, which yields $A^\infty \gtreqless 0$. The corresponding Jacobian matrix is:

$$\hat{J} = \begin{bmatrix} -\delta A^\infty & -\delta P^\infty \\ -\gamma & 0 \end{bmatrix}$$

For $A^\infty > 0$ (resp., $A^\infty < 0$), $\hat{J}$ has one zero eigenvalue, and one eigenvalue, $-\delta A^\infty$, which is either negative or positive. This gives rise to a line of either stable or unstable equilibria. For $A^\infty = 0$, $\hat{J}$ has two zero eigenvalues, which reflect a non-hyperbolic steady state.

The phase diagram below (Fig. 1) shows that the horizontal axis $\dot{A} = 0$ is a line of possible steady state points, where points on the positive axis are sinks, with stable equilibria

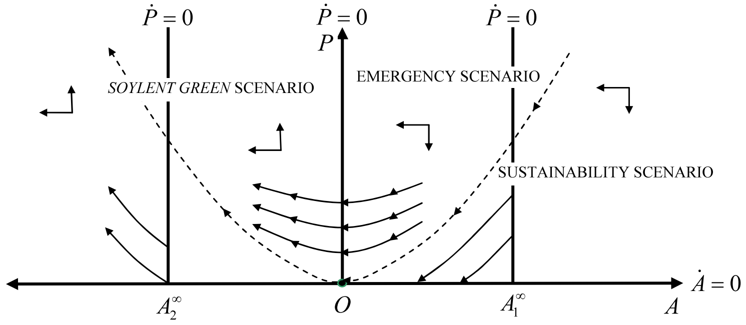

Fig. 1 Pollution–absorption dynamics without human interference

(e.g., A_1^∞), and the negative axis is a source with diverging equilibria (e.g., A_2^∞). Origin O separates two branches of the saddle-node bifurcation curve.

The phase diagram displays similar properties to those of the nonlinear environmental absorption function in (1), i.e., the environmental absorption capacity suffers from *hysteresis* as well as an *irreversibility effect*, since the system is unable to return to where it had been in the beginning. Also, the phase diagram depicts the possibility of switching from a pollution sink to a source.

Though the usual partition of the state space into a reversible and an irreversible region is easily obtained (e.g., Tahvonen and Withagen 1996), the phase diagram can be alternatively divided into three regions, that is:

– The *Sustainability scenario*, in which any initial pollution stock in the right side of the dashed line in the positive quadrant can be totally eliminated while preserving a positive environmental absorption capacity. In this sense, paths in this scenario should end in a sustainable steady state which ensures environmental resilience, conceived of as stability of the ecological-economic system that does not threaten sustainability (Common and Perrings 1992). Multiple degrees of sustainability can occur along the positive horizontal axis, where the highest sustainable states depict the pre-industrial period, characterized by a natural steady state without current emissions and thus no damage related to the pollution stock.
– The *Emergency scenario*, between the dashed line in the positive quadrant and the P-axis, in which any pollution path, although declining, inexorably tends to an exhaustion of the environmental absorption capacity. Paths in this region may provide early warning signals thus averting an imminent pessimistic evolution, notably the critical slowing down of the pollution absorption process, which is reflected in the rapidly increasing negative eigenvalue to zero (Scheffer et al. 2009). A final alternative to prevent breaching the threshold beyond which natural systems will lose resilience and suffer catastrophic collapse would consist of an urgent and massive effort to develop environmental absorption capacity, which would then eventually allow for a shift toward the sustainability region.[4]

[4] An estimate of the disruption threshold is given by the cumulative emissions level of CO_2, that is, 1,000 Gt, above which there is a 50% probability of global mean warming exceeding 2°C (Board on Atmospheric Sciences and Climate 2011). This limit of 2°C adopted by the European Union "stands a strong chance of provoking drought and storm responses that could challenge civilized society, leading potentially to the conflict and suffering that go with failed states and mass migrations. Global warming of 2°C would leave the Earth warmer than it has been in millions of years, a disruption of climate conditions that have been stable for

Table 2 Various configurations of the environmental absorption capacity

Pollution impact on the environmental absorption capacity	Existence of environmental absorption capacity development activity	
	No	Yes
Neutral	*Production-pollution dynamics with constant absorption capacity*	*Production-pollution dynamics with developing absorption capacity*
Positive	*Production-pollution dynamics with exhaustible absorption capacity*	*Production-pollution dynamics with controllable absorption capacity*

– The *Soylent green scenario*, on the left side of the $\dot{P} = 0$ vertical axis, which corresponds to a lasting pollution stock in an exhausted absorption capacity environment, irreversibly leads to a self-sustained evolution of the pollution path due to an increasing rejection of the past absorbed pollution by the environment. In this scenario, irreversibility stems from the impossibility for the environmental absorption capacity to recover a positive state, and then for the social planner to avoid an infinitely increasing disutility of pollution.[5]

One merit of the formulation proposed in the differential system (3)–(4) lies in its relative ‘simplicity’, that is, the potentially irreversible degradation of the environmental absorption capacity can be taken into account while avoiding some of the cumbersome technicalities implied by the assumption of the non-convex absorption function. There is however a difference in the way the decay of pollution is modeled in this paper compared to the existing literature using a nonlinear environmental absorption function. Tahvonen and Withagen (1996), for instance, assume that once a threshold of the pollution stock is reached, the natural rate of decay of pollution becomes zero. In our formulation, once the pollution stock reaches a certain threshold, the absorption capacity of the environment becomes negative and pollution grows even in the absence of anthropogenic emissions. Nature adds to the stock of carbon. This is in line with scientific evidence that the biosphere may switch from a carbon sink to a source (e.g., Cox et al. 2004). Also, in terms of the problems addressed, there is a difference between the existing literature using a nonlinear environmental absorption function and this paper. While the existing literature considers only pollution emissions as a policy instrument (e.g., Tahvonen and Withagen 1996), in our paper the policy maker has two instruments available, i.e. pollution emissions and the efforts to build up the natural absorption capacity.

The formulation in (3)–(4) encompasses the pollution accumulation model with constant absorption capacity as a special case, i.e., when the impact of pollution on the environmental absorption capacity is neutral ($\gamma = 0$), and there is no environmental absorption capacity development activity ($w(t) = 0, \forall t$). The table below (Table 2) depicts the various configurations encompassed by the differential system (3)–(4).

Another configuration that can be obtained as a special case of this model is the *developing absorption capacity* configuration, i.e., absorption capacity insensitive to pollution ($\gamma = 0$) and effective absorption capacity development activity ($w(t) > 0$ for all $t \geq 0$).

longer than the history of human agriculture. Given the drought that already afflicts Australia, the crumbling of the sea ice in the Arctic, and the increasing storm damage after only 0.8°C of warming so far, calling 2°C a danger limit… [may even seem] pretty cavalier” (Allen et al. 2009).

[5]This pessimistic scenario has been labelled in reference to the American film *Soylent green* directed by Richard Fleisher in 1973, in which mankind can do nothing except miserably endure a world-wide ecological catastrophe.

A far more realistic configuration is the *exhaustible absorption capacity* configuration, which combines the negative impact of pollution on the absorption capacity ($\gamma > 0$) and no development activity of absorption capacity ($w(t) = 0$ for all $t \geq 0$). This configuration may be seen as an actual reflection of the real-world situation, which is actually more in line with a reduction of the environmental absorption capacity due to deforestation, that is, $w(t) < 0$. According to the Stern review (Stern 2006), the exhaustible absorption capacity configuration is highly desirable because "curbing deforestation is a highly cost-effective way to reduce emissions". This configuration remains nevertheless consistent with the fact that a large enough pollution stock may destroy the environmental absorption capacity.

Finally, a more general configuration is the *controllable absorption capacity* configuration, where the absorption capacity development activity ($w(t) > 0, \forall t$) can be leveraged to overwhelm the negative impact of pollution on the absorption capacity ($\gamma > 0$).[6]

We now turn to the definition of the social planner's objective function. The economy's instantaneous net utility function is supposed to be separable additive in its arguments. That is, the economy's revenue function is defined as an increasing concave function of the production rate. For simplicity, we use $a \ln u(t)$, $a > 0$ (e.g., Chevé 2000).

At each time period, the economy incurs a social cost of pollution given as an increasing convex function of the pollution stock, that is, $cP(t)^2/2$, $c > 0$. The reduction effort generates an increasing quadratic cost denoted by $dv(t)^2/2$, $d > 0$. Finally, the absorption capacity development effort generates an increasing quadratic cost denoted by $fw(t)^2/2$, $f > 0$.[7]

Denoting by $r > 0$ the social planner's discounting rate, and assuming that the planning horizon is infinite, the social planner's unconstrained optimal control problem then is:

$$\underset{u(.),v(.),w(.)}{\text{Max}} W = \int_0^{+\infty} e^{-rt} \left\{ a \ln u(t) - \frac{cP(t)^2}{2} - \frac{dv(t)^2}{2} - \frac{fw(t)^2}{2} \right\} dt$$

subject to (2)–(3), and $v(t) \geq 0$, $w(t) \geq 0$, $\forall t$.

3 Optimality conditions

The current-value Hamiltonian is:

$$H = a \ln u - \frac{cP^2}{2} - \frac{dv^2}{2} - \frac{fw^2}{2} + \lambda_1[\alpha u(1 - \beta v) - \delta AP] + \lambda_2(w - \gamma P) \tag{5}$$

where $\lambda_j \equiv \lambda_j(t)$ are costate variables, $j = 1, 2$.

Necessary conditions for optimality write:

$$H_u = \frac{a}{u} + \lambda_1 \alpha(1 - \beta v) = 0 \quad \Rightarrow \quad u = -\frac{a}{\lambda_1 \alpha(1 - \beta v)} \tag{6}$$

$$H_v = -dv - \lambda_1 \alpha \beta u = 0 \quad \Rightarrow \quad v = -\frac{\lambda_1 \alpha \beta u}{d} \tag{7}$$

$$H_w = -fw + \lambda_2 = 0 \quad \Rightarrow \quad w = \frac{\lambda_2}{f} \tag{8}$$

[6] As the environmental absorption capacity can be enhanced by human intervention rather than self-restoration capabilities, it is controllable rather than renewable.

[7] Note that the limit conditions characterizing the functions composing the objective criterion are expected to prevent boundary solutions from occurring.

We will focus on the case of an interior solution. In theory, it is possible to clean up all emissions, i.e., $v = 1/\beta$ and enjoy an infinitely large rate of production. However, this is an unrealistic outcome that we do not investigate. Given a feasible value of the production rate, i.e., $u \geq 0$, a feasible emissions reduction effort value in (7), i.e., $v \geq 0$, requires $\lambda_1 \leq 0$. Since the pollution stock has a negative marginal influence on the social planner's objective criterion, the implicit price for the pollution stock should be non-positive, i.e., $\lambda_1 \leq 0$, and can be interpreted as a disincentive to pollute more. However, for the production rate to have a feasible value in (6), i.e., $u \geq 0$, it is both required that $\lambda_1 < 0$ and $v < 1/\beta$. If these conditions are fulfilled, a strictly positive production rate should be obtained over the whole planning horizon, which in return implies a strictly positive value of the emissions reduction effort. Finally, a feasible value for the absorption capacity development effort, i.e., $w \geq 0$, in (8) requires a non-negative value of the implicit price of the absorption capacity rate, i.e., $\lambda_2 \geq 0$, which can be interpreted as an incentive to develop the environmental absorption capacity.

The costate equations are given by:

$$\dot{\lambda}_1 = r\lambda_1 - H_P = cP + (r + \delta A)\lambda_1 + \gamma\lambda_2 \tag{9}$$

$$\dot{\lambda}_2 = r\lambda_2 - H_A = r\lambda_2 + \lambda_1 \delta P \tag{10}$$

Substituting the expression of v in (7) into that of u in (6) gives:

$$(\alpha\beta\lambda_1)^2 u^2 + (\alpha d\lambda_1)u + ad = 0$$

which can be solved with:

$$u_i = -\frac{h_i}{\lambda_1} > 0_{|d \geq 4a\beta^2} \tag{11}$$

where $h_i \equiv [d \pm \sqrt{d(d - 4a\beta^2)}]/2\alpha\beta^2, i = 1, 2$.

Given a sufficiently large cost coefficient of the emissions reduction effort, that is, $d \geq 4a\beta^2$, the production rate should have a strictly positive value over the whole planning horizon. In the remainder of the paper we shall assume that $d \geq 4a\beta^2$. Note that, for $d \geq 4a\beta^2, u_2 \leq u_1$.

Plugging the RHS of (11) into the expression of v in (7) gives, respectively:

$$v_i = \frac{\alpha\beta h_i}{d} > 0_{|d \geq 4a\beta^2} \tag{12}$$

$i = 1, 2$. Provided $d \geq 4a\beta^2$, it can be shown that $v_2 \leq v_1 < 1/\beta$. Note also that $\dot{v}_1 = \dot{v}_2 = 0$, i.e., whatever the production rate, the emissions reduction effort should remain constant over time.

Substituting the RHS of (11) and (12) into (1), gives:

$$e_i = g_i u_i > 0_{|d \geq 4a\beta^2} \tag{13}$$

where $g_i \equiv \frac{\alpha}{2d}[d \mp \sqrt{d(d - 4a\beta^2)}] = \frac{a}{h_i}, i = 1, 2$, are the marginal levels of reduced current emissions from production respectively associated with u_i.

We refer to the solution that corresponds to $u_1(u_2)$ as the industrialist (the ecologist) strategy.

Proposition 1 *The industrialist strategy entails a greater production rate and emissions reduction effort than the ecologist strategy, but generates lower current emissions per unit produced over time.*

Proof Since $g_1 \leq g_2$, we should have $e_1 \leq e_2$ for any fixed production rate u_i. □

Checking the concavity property of the Hamiltonian for the two strategies to determine whether they correspond to globally or locally optimal solutions, it can be shown that neither strategy satisfies any of the standard sufficiency conditions for optimality, that is, the Mangasarian or Arrow conditions. Therefore, the existence of a globally optimal solution can not be guaranteed, but only assumed.

To determine which strategy satisfy the concavity requirements, we check the Legendre-Clebsch necessary conditions of concavity of the Hamiltonian with respect to the control vector (u, v, w) (e.g., Grass et al. 2008). To do so, the necessary conditions (6), (7), (8) as well as (11) are used to compute the Hessian matrix, that is:

$$H_H^i = \begin{bmatrix} -\frac{a}{u_i^2} & \frac{\alpha\beta h_i}{u_i} & 0 \\ \frac{\alpha\beta h_i}{u_i} & -d & 0 \\ 0 & 0 & -f \end{bmatrix} \tag{14}$$

$i = 1, 2$, from which the principal minors are:

$$|H_H^i|_1 = \left\{-\frac{a}{u_i^2}, -d, -f\right\}, \qquad |H_H^i|_2 = \left\{\frac{ad - (\alpha\beta h_i)^2}{u_i^2}, \frac{af}{u_i^2}, df\right\},$$
$$|H_H^i|_3 = \frac{f[(\alpha\beta h_i)^2 - ad]}{u_i^2}$$

$i = 1, 2$. Given $u_i(t) > 0, \forall t$, the principal minors of dimension 1 are all strictly negative. The second and third principal minors in $|H_H^i|_2$ are strictly positive. Moreover, we see that:

$$|H_H^i|_3 = -f\left[\frac{ad - (\alpha\beta h_i)^2}{u_i^2}\right],$$

$i = 1, 2$, which implies that a necessary and sufficient condition for the negative semi-definiteness of the Hessian matrix is that the first principal minor of $|H_H^i|_2$ is non-negative. To check this requirement, we plug the expression of h_i into the first principal minor of $|H_H^i|_2, i = 1, 2$, respectively, which yields:

$$-\frac{d[d - 4a\beta^2 + \sqrt{d(d - 4a\beta^2)}]}{2\beta^2} \leq 0|h_1$$
$$\frac{d[d - 4a\beta^2 - \sqrt{d(d - 4a\beta^2)}]}{2\beta^2} \geq 0|h_2$$

Provided $d \geq 4a\beta^2$, the first principal minor of dimension 2 is non-positive for h_1, and non-negative for h_2. Hence, the Hessian matrix is negative semi-definite only for h_2. This implies that the Hamiltonian is concave with respect to the control vector (u, v, w) in the neighborhood of (u_2, v_2, w), but not in the neighborhood of (u_1, v_1, w). Only the path of controls (u, v, w) that converges to (u_2, v_2, w) is a (global) maximizer of the Hamiltonian with respect to (u, v, w).

The preceding results are summarized in the subsequent proposition.

Proposition 2 *The ecologist strategy is optimal while the industrialist strategy is not.*

Given that the technological effort for the reduction of current emissions lies outside the control region, the larger emissions reduction effort cost involved by the industrialist strategy exceeds the additional benefits drawn from larger production rate. As a result, the industrialist strategy to mitigating emissions is clearly not an advisable solution for the preservation of the environment.

4 Steady state analysis

The main issue now is to determine whether the ecological strategy for reducing emissions leads to a stable steady state located in the sustainability region, as depicted in Fig. 1.

To do so, we first form the canonical system related to the controllable absorption capacity configuration in the state-costate variable space. Without loss of generality, we set $f = 1$. Substituting the optimal value of e_2 from (13) in (3) and plugging the optimal expression of w in (4) results in (15) and (16). Finally, recalling (9) and (10) gives (17) and (18).

$$\dot{P} = -\frac{a}{\lambda_1} - \delta A P \tag{15}$$

$$\dot{A} = \lambda_2 - \gamma P \tag{16}$$

$$\dot{\lambda}_1 = cP + (r + \delta A)\lambda_1 + \gamma \lambda_2 \tag{17}$$

$$\dot{\lambda}_2 = r\lambda_2 + \lambda_1 \delta P \tag{18}$$

We first compute the steady state of the system. From (16), we get:

$$\lambda_2^\infty = \gamma P^\infty \tag{19}$$

Considering (18), we use (19) to get:

$$\lambda_1^\infty = -\frac{r\gamma}{\delta} \tag{20}$$

which is strictly negative.

Considering (17), we use (19) and (20) to obtain:

$$P^\infty = \frac{r\gamma(r + \delta A^\infty)}{\delta(c + \gamma^2)} \tag{21}$$

From (15), we use (20) and (21) to obtain:

$$r^2\delta\gamma^2 A^{\infty 2} + r^3\gamma^2 A^\infty - \delta a(c + \gamma^2) = 0 \tag{22}$$

Solving (22) and imposing $P^\infty \geq 0$, we find:

$$A^\infty = \frac{k - r^2\gamma}{2r\delta\gamma} \tag{23}$$

where $k \equiv \sqrt{r^4\gamma^2 + 4\delta^2 a(c+\gamma^2)} > 0$, as the only valid root.

Substituting for λ_1^∞ from (20) in (11), for A^∞ from (23) in (21), for the resulting expression of P^∞ in (19), and for the final expression of λ_2^∞ in (8), respectively, leads to characterizing the system's steady state, which is formalized in the subsequent proposition.

Proposition 3 *The controllable absorption capacity configuration admits a unique steady state, given by*:

$$[u^\infty \ v^\infty \ w^\infty \ P^\infty \ A^\infty]^T = \begin{bmatrix} \frac{\delta h_2}{r\gamma} & \frac{\alpha\beta h_2}{d} & \frac{\gamma(k+r^2\gamma)}{2\delta(c+\gamma^2)} & \frac{k+r^2\gamma}{2\delta(c+\gamma^2)} & \frac{k-r^2\gamma}{2r\delta\gamma} \end{bmatrix}^T \tag{24}$$

Corollary 1 *The ecologist strategy for reducing emissions reduction allows for a unique non-trivial steady state production rate compatible with the preservation of a strictly positive environmental absorption capacity.*

As the steady state values are all interior, the variables should remain interior by continuity for initial values close enough to the steady state.[8] In contrast with the existing literature which models the environmental absorption capacity as an instantaneous nonlinear function of the pollution stock (e.g., Tahvonen and Withagen 1996, Kossioris et al. 2008), our model has a single steady state, which then implies no trade-off between 'reversible' and 'irreversible' state. Since it should be located either in the *sustainability region* or the *emergency region*, as illustrated in Fig. 1, that is, $A^\infty, P^\infty > 0$, the *Soylent green* scenario of a self-sustained pollution path can be avoided if the steady state of the current emissions rate can be reached and sustained.

Compared to the controllable absorption capacity configuration, the constant absorption capacity configuration (Table 2) also admits one steady state, i.e., $u^\infty_{|w(t)=\gamma=0} = h_2\sqrt{\rho(r+\rho)/ac}$ and $P^\infty_{|w(t)=\gamma=0} = \sqrt{a(r+\rho)/\rho c}$, which is also a saddle point, the determinant of the corresponding Jacobian matrix being strictly negative ($-2\rho(r+\rho) < 0$). However, given the corresponding production rate at the steady state, the constant absorption capacity is lower than the steady state absorption capacity of the controllable absorption capacity configuration ($\rho < \delta A^\infty_{|w(t)>0,\gamma>0}$), despite a lower steady state pollution stock, i.e., $P^\infty_{|w(t)=\gamma=0} < P^\infty_{|w(t)>0,\gamma>0}$.

In contrast with the controllable absorption capacity configuration, the developing absorption capacity configuration (Table 2) has a single, diverging steady state, which is characterized by an infinite output level and a zero absorption capacity development effort. Accordingly, the steady state pollution stock is positive, $P^\infty_{|w(t)>0,\gamma=0} = \sqrt{a/c}$, while the environmental absorption capacity is infinite. That is, if the pollution stock is not destructive to the absorption capacity (i.e., the environmental absorption capacity is strongly resistant), a positive, decreasing absorption capacity development effort over time results in an infinite absorption capacity, which in turn allows for an infinite production rate. Indeed, the steady state pollution stock in this configuration is lower than that in the controllable absorption capacity configuration.

In comparison with the controllable absorption capacity configuration, the exhaustible absorption capacity configuration (Table 2), which reflects the Stern review's advocated configuration (Stern 2006), admits a single steady state characterized by both a zero pollution stock and production rate. The reason is that the process here is similar to that of

[8]The subsequent numerical analysis shows that interior solutions are granted also for initial values far enough from the steady state.

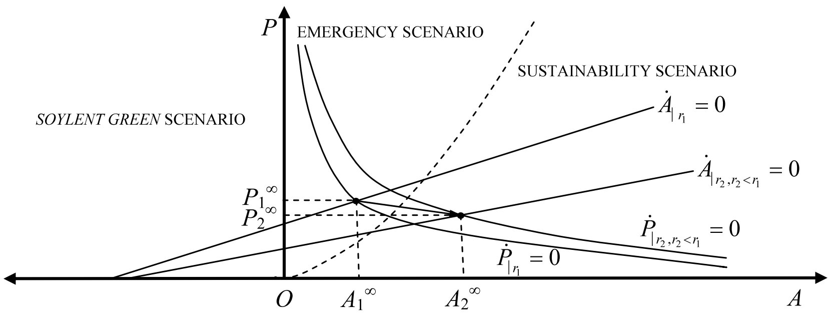

Fig. 2 Discounting rate and the steady state

the exploitation of an exhaustible natural resource, i.e., the production rate should be initially large and decreasing over time for a strictly positive initial environmental absorption capacity. As a result, the steady state absorption capacity is negative, and corresponds to a source fixed point. In the long run, the exhaustible absorption capacity configuration leads to an unmanageable zero production terminal state, while the *Soylent green* scenario can not be avoided. This result clearly shows that stopping deforestation is not sufficient to prevent from irreversible evolution to a self-sustained pollution path.

Corollary 2 *All things being equal*, *at the steady state*, *a smaller discounting rate should result in a higher production rate and environmental absorption capacity*, *a lower pollution stock and absorption development effort*, *and a similar emissions reduction effort*.

Choosing a near-zero *pure* time preference rate, that is, a near-zero rate at which future generations are discriminated against, is basically justified by intergenerational fairness principles. *All things being equal*, sensitivity analysis confirms that it is indeed the case both for economic and environmental reasons.

In the figure above (Fig. 2), it is shown that a substantial decrease in the discounting rate can eventually allow a shift of the steady state from the emergency scenario (A_1^∞, P_1^∞) to the sustainability scenario (A_2^∞, P_2^∞). In other words, a lower discounting rate can contribute to strengthen the environmental resilience. This result confirms the contentious suggestion of the Stern review (Stern 2006) about the virtue of a small discounting rate.

In the case of infinite discounting rate, $r \mapsto +\infty$, the unique steady state is characterized by an infinite pollution stock, and zero emissions rate and environmental absorption capacity, which corresponds to the ‘irreversible policy’ of Tahvonen and Withagen (1996). However, this ‘irreversible’ steady state resulting from a totally myopic social planner would require here an infinite effort in developing the environmental absorption capacity.

We now analyze the stability of the steady state. Given that the stability property available in two-dimensional state models is at best the saddle-point property (e.g., Dockner 1985), we check the conditional stability of the steady state of the controllable absorption capacity configuration. The Jacobian matrix associated with the canonical system (15)–(18) is:

$$J = \begin{bmatrix} -\delta A & -\delta P & \frac{a}{\lambda_1^2} & 0 \\ -\gamma & 0 & 0 & 1 \\ c & \delta\lambda_1 & r + \delta A & \gamma \\ \delta\lambda_1 & 0 & \delta P & r \end{bmatrix} \tag{25}$$

from which the determinant is:

$$|J| = \frac{k(k + r^2\gamma)}{2(c + \gamma^2)} \tag{26}$$

and, using Dockner's formula (Dockner 1985), the sum of the principal minors of J of order 2 minus the squared discount rate, denoted by K, is:

$$K = -\frac{\delta^2 a(2c + \gamma^2)}{r^2\gamma^2} - \frac{\gamma(k + r^2\gamma)}{c + \gamma^2} \tag{27}$$

where λ_1, P, and A are evaluated at their steady state value.

According to Dockner and Feichtinger (1991), a necessary condition for the occurrence of a Hopf bifurcation is $K > 0$. Given that $K < 0$, the existence of limit cycles is then ruled out. The necessary and sufficient condition for the steady state to be a saddle-point is fulfilled, since $K < 0$ and $|J| > 0$, which implies that two eigenvalues must have negative real parts.

Corollary 3 *The limiting transversality conditions are satisfied since*:

$$\lim_{t\mapsto+\infty} e^{-rt}\lambda_1(t)P(t) = \lim_{t\mapsto+\infty}\left[-\frac{r\gamma(k + r^2\gamma)e^{-rt}}{2\delta^2(c + \gamma^2)}\right] = 0 \tag{28}$$

$$\lim_{t\mapsto+\infty} e^{-rt}\lambda_2(t)A(t) = \lim_{t\mapsto+\infty}\frac{ae^{-rt}}{r} = 0 \tag{29}$$

As a result, the stable manifolds of the saddle-point of the canonical system (15)–(18) *yield the catching up optimal solution to the problem.*

To determine whether the optimal path is monotonic or follows transient oscillations, we put:

$$\Omega = K^2 - 4|J| = \left[\frac{\delta^2 a(2c + \gamma^2)}{r^2\gamma^2} + \frac{\gamma(k + r^2\gamma)}{c + \gamma^2}\right]^2 - \frac{2k(k + r^2\gamma)}{c + \gamma^2} \tag{30}$$

The sign of Ω can be either negative or positive. In the first case, the steady state is a saddle-focus with transient oscillations. Otherwise, it is a saddle-node and the optimal solution monotonically converges to the steady state. We highlight below the role played by the parameters r, δ, γ, a, and c in the type of convergence to the steady state.

From (31), it can be shown that: $\lim_{\gamma\mapsto 0^+}\Omega = +\infty$. On the other hand, we have:

$$\lim_{\gamma\mapsto+\infty}\Omega = \frac{\delta^2 a}{r^2}\left[\frac{\delta^2 a}{r^2} + 2(\sqrt{r^4 + 4\delta^2 a} - r^2)\right] > 0 \tag{31}$$

A similar analysis is made for r, a, c, δ. The final results are given in the table below (Table 3).

These results are summarized in the subsequent propositions.

Proposition 4 *For any given r, a, c, and γ, there exists a threshold $\tilde{\delta} > 0$ such that for any $\delta < \tilde{\delta}$ we have $\Omega < 0$ and thus an oscillatory convergence to the steady state.*

Remark As we also have $\lim_{a\mapsto 0}\Omega < 0$, the behaviour of the system around $a = 0$ is similar to its behaviour around $\delta = 0$.

Table 3 Limit value analysis

	$L_1 = 0^+$	$L_2 = +\infty$
$\lim_{r \mapsto L_j, j=1,2} \Omega$	$+\infty$	$-\infty$
$\lim_{a \mapsto L_j, j=1,2} \Omega$	$-\frac{4r^4\gamma^2 c}{(c+\gamma^2)^2}$	$+\infty$
$\lim_{c \mapsto L_j, j=1,2} \Omega$	$\frac{\delta^2 a}{r^4}[\delta^2 a + 2r^2(\sqrt{r^4 + 4\delta^2 a} - r^2)]$	$+\infty$
$\lim_{\delta \mapsto L_j, j=1,2} \Omega$	$-\frac{4r^4\gamma^2 c}{(c+\gamma^2)^2}$	$+\infty$
$\lim_{\gamma \mapsto L_j, j=1,2} \Omega$	$+\infty$	$\frac{\delta^2 a}{r^2}[\frac{\delta^2 a}{r^2} + 2(\sqrt{r^4 + 4\delta^2 a} - r^2)]$

Table 4 Baseline parameters

Parameter	r	a	c	d	α	β	δ	γ
Value	0.1	0.01	0.01	0.01	1	0.5	0.001	0.01

Proposition 5 *For any given a, c, δ, and γ, there exists $\tilde{r} > 0$ such that for any $r > \tilde{r}$ we have $\Omega < 0$ and thus an oscillatory convergence to the steady state.*

This result emphasizes another feature of a small discounting rate that was not initially envisioned by the Stern review (Stern 2006; Nordhaus 2007), which is the fact that the monotonicity property of the optimal path to sustainable steady state is ensured by a small discounting rate. In the converse case (i.e., large discounting rate), an oscillatory behaviour results even in the case of a strongly resistant environmental absorption capacity.

5 Transition behaviour

An important issue suggested in the previous section is to know if a transient convergence to the steady state falls in the *Soylent green area* for a transient time period, thus imposing temporarily potentially damaging consequences. To check this, we examine the transition of the economy from the initial conditions to the steady state.

To generate numerically the state variables optimal paths, we use the baseline parameters reported in Table 4. For a sensitivity analysis, the parameters r, a, δ and γ are varied in the bounds $r \in [0.05, 0.1]$, $a \in [0.001, 0.01]$, $\delta \in [0.0005, 0.01]$, $\gamma \in [0.002, 0.05]$. For each of the parameter values, the solution paths are calculated for the lower, upper and intermediate parameter value. Solution paths are calculated for 4 initial state values, that is, $(A_0, P_0) = \{(0, 0), (40, 0), (0, 80), (40, 80)\}$.

The figure below (Fig. 3) confirms that, whatever the initial state, a low discounting rate results in a monotonic path to the steady state, which results in a low pollution stock and a strictly positive environmental absorption capacity. This suggests that the steady state could fall into the *sustainability region*. Conversely, in the case of a large enough discounting rate, transient paths are observed which overflow the *emergency region*. As a result, the *Soylent green scenario* is unavoidable during a certain time period. Afterward, the steady state is characterized by a large pollution stock and a zero environmental absorption capacity, which corresponds to the emergency scenario.

According to the figure below (Fig. 4), small values of the production rate utility function coefficient result in paths oscillating between the *emergency region* and the *Soylent green*

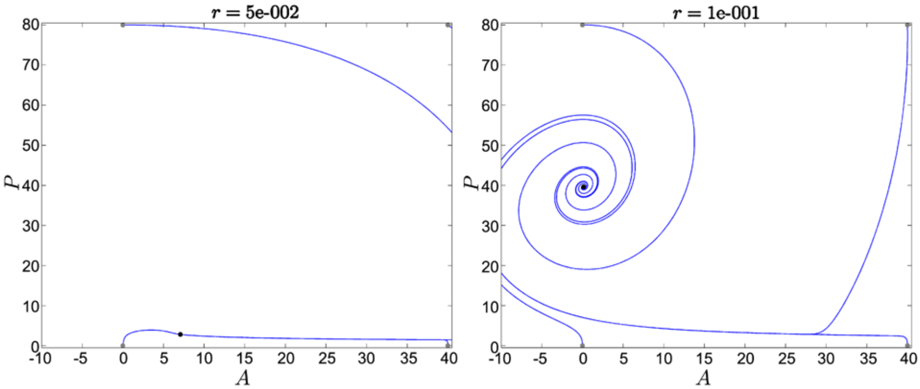

Fig. 3 Steady state path and the discounting rate

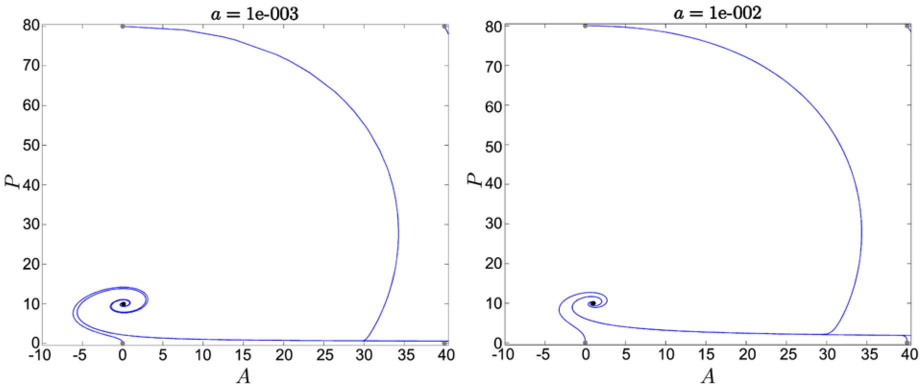

Fig. 4 Steady state path and the production rate utility function coefficient

scenario. An increase in the production rate utility function coefficient does not affect the steady state pollution stock, but slightly increases the environmental absorption capacity.

In the figure below (Fig. 5), it is shown that, whatever the initial state, a small enough value of the marginal efficiency of the absorptive capacity results in transient paths which encroach upon the *Soylent green region*. Here also, the steady state lies in the *emergency region*. In the converse case of a large enough marginal efficiency of the absorptive capacity, a monotonic path to the steady state is obtained which ends up in the *sustainability region*.

According to the figure below (Fig. 6), the path to the steady state can also be affected by the marginal impact of pollution on the absorption capacity. In the case of a strongly resistant environmental absorption capacity, a monotonic path to the steady state is observed which ends up in the *sustainability region*. On the other hand, though we showed that $\lim_{\gamma\mapsto 0^+}\Omega = +\infty$ and $\lim_{\gamma\mapsto+\infty}\Omega > 0$ (cf. Table 3), it appears that for some interval of γ, Ω can be negative, which is the case in our example for $\gamma = 0.05$. As a result, a spiralling behaviour is observed. That is, a vulnerable environmental absorption capacity gives rise to widely transient paths which largely overflow the *emergency region*, notably from an initial state characterized by a low enough pollution stock. Here also, the *Soylent green scenario* can not be avoided during a certain time period, and the steady state finally lies in the *emergency region*.

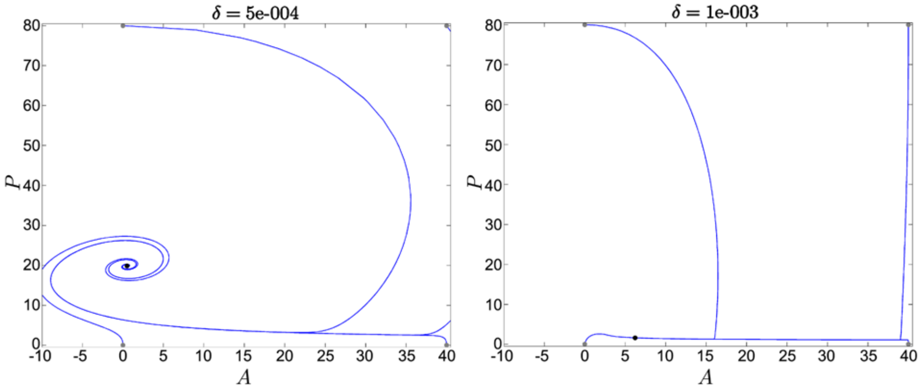

Fig. 5 Steady state path and the marginal efficiency of the absorptive capacity

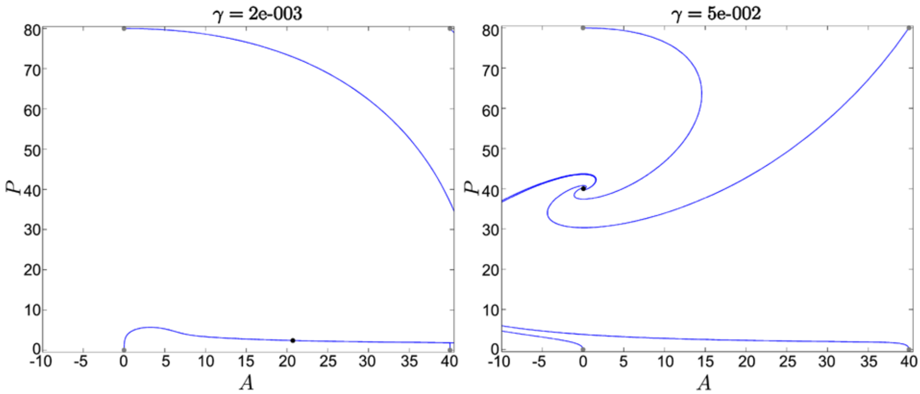

Fig. 6 Steady state path and the marginal impact of pollution on the absorption capacity

As pollution can be controlled by two policies, production reduction and absorption capacity development, the fact that an oscillatory path can be optimal implies that they may well evolve differently over time. Indeed, the sign of Ω allows to determine the nature of the convergence to the steady state (monotone *versus* oscillatory) in the neighbourhood of the steady state. For instance, when $\Omega > 0$, the convergence to the steady state from initial conditions that are 'close enough' to the steady state is monotonic, but a non-monotonic approach (over-shooting) to the steady state from initial conditions that are 'far enough' from the steady state is not precluded. To investigate this, we compute the control time paths of the control variables according to the baseline parameters reported in Table 4.

According to the computations, the signs of the co-state variables are such that $\lambda_1(t) < 0$, and $\lambda_2(t) > 0$, $\forall t$, as expected, which ensures that the control variables always have interior values. The typical patterns of an optimal environmental policy depending on the initial conditions are summarized in the table above (Table 5).

Whatever the pattern of convergence to the steady state, when the initial stock of the environmental absorption capacity is small and the initial stock of pollution is large, which reflects the real-world situation, the optimal policy consists of initially choosing a small and increasing then decreasing production rate and a large and decreasing absorption capacity development effort. As a result, the environmental absorption capacity is increased and the

Table 5 Initial conditions and optimal environmental policy

Initial environmental absorption capacity level	Initial pollution stock level	
	Low	Large
Low	*Initially high and decreasing production rate Initially low and increasing then decreasing absorption capacity development effort*	*Initially low and increasing then decreasing production rate Initially high and decreasing absorption capacity development effort*
Large	*Initially high and decreasing production rate Initially low and increasing then decreasing absorption capacity development effort*	*Initially low and increasing then decreasing production rate Initially high and decreasing then increasing absorption capacity development effort*

Table 6 State and costate steady state values

	r		a		δ		γ	
	0.05	0.1	0.001	0.01	0.0005	0.01	0.002	0.05
λ_1^∞	−0.500	−2.000	−1.000	−1.000	−2.000	−0.100	−0.200	−5.000
λ_2^∞	0.028	0.396	0.099	0.100	0.199	0.016	0.005	2.001
A^∞	7.078	0.126	0.101	1.000	0.504	6.225	20.718	0.050
P^∞	2.826	39.629	9.911	10.000	19.852	1.606	2.413	40.020

pollution stock is substantially reduced in the case of monotonic convergence to the steady state. In the case of oscillatory convergence to the steady state, the environmental absorption capacity remains saturated, but with a significantly lowered pollution stock.

The graphs for the cases where we do not have an oscillatory convergence to the steady state show that, for any initial level (A_0, P_0), we have monotonic convergence to the steady state with either for A and P both decreasing or both increasing and converging to their steady values. It is interesting to note an interesting situation in these cases where the optimal path implies overshooting. That is, in the left figure of Fig. 6, for instance, starting from initial values of A and P that are both below (above) their steady state values, the stock of pollution is built-up (brought down) to levels above (below) its steady state value.

On the whole, the numerical results suggest that the occurrence of transient paths might be interpreted as a prediction that the steady state will end up in the *emergency region*. In the table above (Table 6), the steady state values of the state and costate variables are reported.

It clearly appears that the monotonic path to the steady state is more favourable than the oscillatory path as it results in both lower pollution stock and larger environmental absorption capacity, and it also allows for larger production rate and requires less effort to build-up the environmental absorption capacity. Another important result lies with the fact that the transient occurrence of the *Soylent green scenario*, which gives rise to a self-sustained evolution of the pollution path, is not irreversible. That is, not only the development of the environmental absorption capacity can contribute to better control the stock of pollution, but it also allows the environmental absorption capacity to recover from a negative transient state.

6 Conclusion

To date, the political debate about pollution has two main opposing approaches: the industrialist approach and the ecologist approach. The opposition between the two approaches is explained in the simplest terms by van der Ploeg and Withagen (1991), that is: "the optimists [i.e., industrialist approach] say production must go up in order to be able to afford to clean up the environment, whilst the pessimists [i.e., ecologist approach] argue that production must go down, *even* if this leaves less scope for cleaning-up activities, as pollution as a by-product of production dominates all else".

According to our model, which accounts for a trade-off between revenues from production, and the respective costs of emissions reduction effort and absorption capacity development effort plus the net cost of negative externalities from pollution, the technological effort cost needed to reduce the emissions resulting from a higher production rate exceeds the corresponding net revenues. In other words, the industrialist strategy is not optimal. This conclusion is confirmed by a number of studies that pointed out the oversized nature of the technological challenge associated with carbon emissions reduction (e.g., Hoffert et al. 1998; Caldeira et al. 2003; Pielke et al. 2008).

Our results clearly give support to the ecologist approach. Nevertheless, taking into account the natural limitations of the absorption capacity, a positive level of effort to build up the absorption capacity of the environment is necessary to prevent from the switching of the environment from pollution sink to a source. Merely stopping deforestation is not enough, a more active effort to regenerate the absorption capacity is warranted, not because it is economically beneficial, but basically to avoid the *Soylent green* scenario. The possibility for a win-win situation combining both a higher production rate and technological effort of emissions reduction as described in Table 1 is then dominated by a strategy that complements the environmental absorption capacity building effort with a lower production rate.

In contrast with models representing the environmental absorption capacity as an instantaneous nonlinear function of the pollution stock (e.g., Tahvonen and Withagen 1996), our model has a single, conditionally stable steady state, compatible with both positive environmental absorption capacity and emissions rate. As a result, the possibility of a concomitant optimal 'irreversible policy' along with this sustainable steady state, as obtained in Tahvonen and Withagen (1996), is ruled out.

The optimal path to the steady state can be either monotonic or oscillatory. It is noteworthy that the possibility of oscillating convergence to the steady state that we have within our framework contrast with the monotonic convergence to the reversible steady state that is obtained within a framework such as the one of Tahvonen and Withagen (1996), where the limitation of the environmental absorption capacity is captured through the use of a non-linear natural rate of decay of pollution.

The conditions under which the optimal solution involves a monotonic or an oscillatory convergence of the pollution stock and the environmental absorption capacity to their steady state values were identified. It turns out that the monotonic optimal path to the steady state results mainly from a farsighted policy, reflected in a low discounting rate, and/or favourable environmental conditions, induced by large marginal efficiency of the absorptive capacity or strongly resistant environmental absorption capacity. Conversely, an oscillatory optimal path to the steady state is essentially due to a myopic policy, and/or unfavourable environmental conditions. Given such prerequisites, the monotonic optimal path predicts a relatively clean environment with both positive environmental absorption capacity and production activity level, while the oscillatory optimal path should lead to expect a relatively polluted environment with exhausted environmental absorption capacity and low production rate.

The important question of whether pollution should be controlled by simultaneously building up absorption capacity and curbing emissions by reducing production, or by building absorption capacity and affording an increasing rate of production was also considered. Under realistic initial conditions characterized by a large initial pollution stock and a low initial environmental absorption capacity (relative to their steady state levels), our model prescribes (i) starting from a production rate smaller than the steady state, increasing the production rate and approaching the state level from above, and (ii) starting from a higher absorption capacity development effort than its steady state level and approaching the steady state from above . These prescriptions, regrettably, depart strongly from the observed environmental policies reflected in the global trends of large emissions and deforestation levels (Canadell et al. 2007). As the combination of initial conditions considered mirrors the emergency scenario depicted in Fig. 1, which calls for urgent, appropriate effort to allow for a shift to the sustainability region, one can expect a pessimistic evolution.

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