A Generalized Ray Model of 3-D Propagation on the Continental Shelf Including Shear Wave Effects in an Absorptive Bottom

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Outline of Presentation

- Overview of Generalized Ray Method Used
  - Wedge Model of the Continental Shelf
  - Integral Representations of Multi-Reflected Wave Fields
  - Propagation Environment
  - Attenuation and Generalized Rays
Outline of Presentation (cont.)

- Application to Large-Range Cross-Slope Propagation
  - Structure of Unattenuated Pressure Response at 40 km
  - Effects of Horizontal Refraction in the Present Environment
  - Analysis of Received Levels (RLs) of Acoustic Tones
- Conclusions


Acoustic Wave Field in a Wedge

- Wave Equation:
  \[ c^2 \nabla^2 p - \frac{\partial^2 p}{\partial t^2} = -f(t)\delta(x)\delta(y)\delta(z-z_0) \]

- Boundary Conditions:
  \[ p = 0 \text{ at } z = 0 \]

- Field Resolution:
  \[ p = \sum_{k=1}^{N} p_{\pm k}, \quad N = \frac{\pi}{\alpha} \]
Source and Multi-Reflected Wave Fields

- 121 Fields with up to 60 Bottom Reflections in a 3 deg Wedge

\[ p_0 = \frac{p_c}{R} f(t - \frac{R}{c}) = p_c H(t - t_0) \int_{t_0}^{t} \dot{f}(t - \tau) \left[ \frac{2}{\pi} \text{Re} \int_{0}^{q(\tau)} S \frac{dg_0^{-1}}{d\tau} dq \right] d\tau \]

\[ p_{\pm k} = p_c H(t - t_{\pm k}) \int_{t_{\pm k}}^{t} \dot{f}(t - \tau) \left[ \frac{1}{2\pi} \text{Re} \int_{0}^{q(\tau)} S \Pi_{\pm k} \frac{dg_{\pm k}^{-1}}{d\tau} dq \right] d\tau \]
## Propagation Environment

<table>
<thead>
<tr>
<th>Water Wedge</th>
<th>Seabed (limestone)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope (deg)</td>
<td>Compressional wave speed (m/s)</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
</tr>
<tr>
<td>Sound speed (m/s)</td>
<td>Compressional attenuation (dB/λ)</td>
</tr>
<tr>
<td>1500</td>
<td>0.10</td>
</tr>
<tr>
<td>Density (g/cm³)</td>
<td>Shear wave speed (m/s)</td>
</tr>
<tr>
<td>1.0</td>
<td>1460</td>
</tr>
<tr>
<td></td>
<td>Shear attenuation (dB/λ)</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Density (g/cm³)</td>
</tr>
<tr>
<td></td>
<td>2.40</td>
</tr>
</tbody>
</table>

### Cross-Slope Propagation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth (m)</td>
<td>200</td>
</tr>
<tr>
<td>Source depth (m)</td>
<td>100</td>
</tr>
<tr>
<td>Receiver depth (m)</td>
<td>200</td>
</tr>
<tr>
<td>Propagation range (km)</td>
<td>40</td>
</tr>
</tbody>
</table>
Sound-Pressure Level and Attenuation with Distance

- \( L_p = 10 \log \frac{p^2}{p_{\text{ref}}^2} \), \( p_{\text{ref}} = 1 \, \mu\text{Pa} \)

- \( p_{\text{att}} = pe^{-\alpha' r} \)

- \( L_p = 10 \log \left( \frac{pe^{-\alpha' r}}{p_{\text{ref}}^2} \right)^2 = 10 \log \frac{p^2}{p_{\text{ref}}^2} - \alpha r \)

\( \alpha' = \text{Intrinsic attenuation (Np/m)} \)

\( \alpha = \text{Intrinsic attenuation (dB/m)} \)
Response at 40 km; Source Pulse of Heaviside Form

- \[ p = p_0 + \sum_{k=1}^{N} p_{\pm k}, \quad N = \frac{\pi}{\alpha} = \frac{180}{3} = 60 \]

- 121 Pulses with up to 60 Bottom Reflections at 40 km

```
Lateral Waves Arrivals
```

```
p/p_c
```

```
100 120 140 160 180 200
```

```
t/t_c
```

```
↑ ↑
```

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Response at 40 km; Source Pulse of Heaviside Form (cont.)

Spherical Waves Arrivals

\[ \frac{p}{p_c} \]

\[ t/t_c \]

26.66 s

27.14 s
Response at 40 km; Source Pulse of Heaviside Form (cont.)

![Scholte Waves Pulse Graph]

- $p/p_c$ vs $t/t_c$
- Time markers: $33.06$ s and $34.00$ s
Response at 40 km (cont.); Effects of Horizontal Refraction

Line of Apex

Cross-Slope Direction

Down-Slope Direction

\( \phi \)

\( \phi_I \)

\( \phi_L \)

\( \phi_S \)
Source Signals

- Source Level of 171 dB re 1 \(\mu\)Pa at 1m
- Tones Centered at 10, 15, 20, 25, 30, 35, 40, 45, 50, 52.5 Hz
- Example: Tone Centered at 10 Hz

![Source Signal; \(f = 10\) Hz, \(RL = 125\) dB at 200 m](image-url)
Response at 40 km; $f = 10$ Hz

Lateral Waves; $f = 10$ Hz, $RL = 53$ dB
Response at 40 km; f = 10 Hz (cont.)

Spherical Waves; f = 10 Hz, RL = 57 dB

\[ \frac{p}{p_c} \]

\[ \frac{t}{t_c} \]
Response at 40 km; $f = 10$ Hz (cont.)

- $RL = 279 - 60 = 219$ dB

![Scholte Waves; $f = 10$ Hz](image)

$p/p_c$

$t/t_c$

$5.2 \times 10^{14}$

$-5.2 \times 10^{14}$

250 260 270 280
Response at 40 km; $f = 15$ Hz

Lateral Waves; $f = 15$ Hz, $RL = 64$ dB
Response at 40 km; f = 15 Hz (cont.)

Spherical Waves; f = 15 Hz, RL = 64 dB
Response at 40 km; $f = 15$ Hz (cont.)

- $\text{RL} = 267 - 90 = 177$ dB
Response at 40 km; $f = 20$ Hz

Lateral Waves; $f = 20$ Hz, $RL = 65$ dB
Response at 40 km; f = 20 Hz (cont.)

Spherical Waves; f = 20 Hz, RL = 61 dB
Response at 40 km; $f = 20$ Hz (cont.)

* $RL = 252 - 121 = 131$ dB

![Graph of Scholte Waves; $f = 20$ Hz](attachment:graph.png)
Response at 40 km; $f = 25$ Hz

Lateral Waves; $f = 25$ Hz, $RL = 60$ dB
Spherical Waves; $f = 25$ Hz, $RL = 61$ dB
Response at 40 km; $f = 25$ Hz (cont.)

- $\text{RL} = 243 - 151 = 92$ dB

Scholte Waves; $f = 25$ Hz
Response at 40 km; $f = 30$ Hz

Lateral Waves; $f = 30$ Hz, $RL = 61$ dB
Response at 40 km; f = 30 Hz (cont.)

Spherical Waves; f = 30 Hz, RL = 66 dB
Response at 40 km; $f = 30$ Hz (cont.)

- $\text{RL} = 239 - 181 = 58 \text{ dB}$
Response at 40 km; $f = 35$ Hz

Lateral Waves; $f = 35$ Hz, $RL = 51$ dB
Response at 40 km; $f = 35$ Hz (cont.)

Spherical Waves; $f = 35$ Hz, $RL = 60$ dB
Response at 40 km; $f = 35$ Hz (cont.)

- $RL = 231 - 211 = 20$ dB
Response at 40 km; f = 40 Hz

Lateral Waves; f = 40 Hz, RL = 38 dB
Response at 40 km; f = 40 Hz (cont.)

Spherical Waves; f = 40 Hz, RL = 66 dB

\[
p/p_c
\]

\[
t/t_c
\]
Response at 40 km; $f = 40$ Hz (cont.)

- $233 - 241 < 0$ and so $RL = 0$ dB
Response at 40 km; $f = 45$ Hz

Lateral Waves; $f = 45$ Hz, $RL = 54$ dB

\[ \frac{p}{p_c} \]

\[ t/t_c \]
Response at 40 km; $f = 45$ Hz (cont.)

Spherical Waves; $f = 45$ Hz, $RL = 37$ dB
Response at 40 km; $f = 45$ Hz (cont.)

- $221 - 271 < 0$ and so $RL = 0$ dB
Response at 40 km; $f = 50$ Hz

Lateral Waves; $f = 50$ Hz, $RL = 58$ dB
Response at 40 km; f = 50 Hz (cont.)

Spherical Waves; f = 50 Hz, RL = 67 dB

\[ \frac{p}{p_c} \]

\[ \frac{t}{t_c} \]
Response at 40 km; $f = 50$ Hz (cont.)

- $210 - 301 < 0$ and so $\text{RL} = 0$ dB
Response at 40 km; $f = 52.5$ Hz

Lateral Waves; $f = 52.5$ Hz, $RL = 52$ dB

$p/p_c$ vs $t/t_c$
Response at 40 km; f = 52.5 Hz (cont.)

Spherical Waves; f = 52.5 Hz, RL = 68 dB
Response at 40 km; $f = 52.5$ Hz (cont.)

- $210 - 316 < 0$ and so $RL = 0$ dB

![Scholte Waves; $f = 52.5$ Hz](image)
Summary of RLs at $r = 40$ km, $f = 10, 15, 20, \ldots, 52.5$ Hz

<table>
<thead>
<tr>
<th></th>
<th>10 Hz</th>
<th>15 Hz</th>
<th>20 Hz</th>
<th>25 Hz</th>
<th>30 Hz</th>
<th>35 Hz</th>
<th>40 Hz</th>
<th>45 Hz</th>
<th>50 Hz</th>
<th>52.5 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Head Waves</strong></td>
<td>53 dB</td>
<td>64 dB</td>
<td>65 dB</td>
<td>60 dB</td>
<td>61 dB</td>
<td>51 dB</td>
<td>38 dB</td>
<td>54 dB</td>
<td>58 dB</td>
<td>52 dB</td>
</tr>
<tr>
<td><strong>Spherical Waves</strong></td>
<td>57 dB</td>
<td>64 dB</td>
<td>61 dB</td>
<td>61 dB</td>
<td>66 dB</td>
<td>60 dB</td>
<td>66 dB</td>
<td>37 dB</td>
<td>67 dB</td>
<td>68 dB</td>
</tr>
<tr>
<td><strong>Scholte Waves</strong></td>
<td>219 dB</td>
<td>177 dB</td>
<td>131 dB</td>
<td>92 dB</td>
<td>58 dB</td>
<td>20 dB</td>
<td>0 dB</td>
<td>0 dB</td>
<td>0 dB</td>
<td>0 dB</td>
</tr>
</tbody>
</table>
Conclusions

- Dominant lateral- and spherical-waves arrivals are coming in along “direct paths.”

- Dominant Scholte-waves arrivals are due to multi-reflected fields coming in along “in-shore refracted paths.”

- RLs of Lateral and Spherical Phases Too High

- RL of Scholte Phase Significant Only for “Low” Frequencies

- Only “low-frequency” cross-slope propagation may indeed be 3-D.