Magnetic Catalysis in the Onset of Nuclear Matter

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July 17, 2014
SEWM 2014
Motivation

Cold dense matter in strong magnetic fields is found in . . .

- magnetars

  on the surface $B \sim 10^{15}$ G

  in the core up to $B \sim 10^{18}$ G

- binary neutron star mergers

  Gravitational wave signature sensitive to equation of state

  The magneto-rotational instability may increase the initial B-field by one order of magnitude
The eLSM is based on

- degrees of freedom of the QCD vacuum: hadrons
- symmetries of QCD: chiral and dilatation invariance

Relevant ingredients ($N_f = 2$):

- chiral partner of pions $\sigma$: a finite VEV breaks chiral symmetry fluctuations correspond to the state $f_0(1370)$
- the lightest scalar resonance $f_0(500)$ corresponds to an $SU(2)_L \times SU(2)_R$ singlet state $\chi$
- tetraquark or pion-pion resonance
- the $SU(2)_L \times SU(2)_R$ singlet vector meson $\omega_\mu$ corresponding to $\omega(782)$

Tree level free energy:

\[ \Omega_{\text{tree}}^{\text{mes}} = -\frac{1}{2} m \sigma^2 + \frac{\lambda}{4} \sigma^4 - \epsilon \sigma - \frac{1}{2} m_\omega \omega_0^2 + \frac{1}{2} m_\chi \chi^2 - g \chi \sigma^2, \]

with gap equation for \( \chi \) and \( \omega_0 \)

\[ \omega_0 = 0, \quad \chi = g \sigma^2 / m_\chi \]

\[ \sigma_{\text{min}} = Z f_\pi \approx 154 \text{ MeV} \]
Including nucleons in the mirror assignment:


Isospin-douplet spinor $\psi_1$ and its parity partner $\psi_2$ obeying

$$\psi_{1, R/L} \rightarrow U_{R/L}\psi_{1, R/L}, \quad \psi_{2, R/L} \rightarrow U_{L/R}\psi_{2, R/L},$$

allow for a chirally symmetric mass term

$$m_0(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2).$$

dilatation invariance $\rightarrow m_0$ generated dynamically, e.g. $m_0 \propto \chi$.

Interaction Lagrangian:

$$\mathcal{L}^{\text{int}} = - \sum_i \bar{\psi}_i \left( \frac{g_i \sigma}{2} + g_\omega \gamma^0 \omega_0 \right) \psi_i - a\chi(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2)$$
Effective fermion mass and chemical potential:

\[ M_{N,N^*} = \pm \frac{g_1 - g_2}{2} \sigma + \sqrt{\left(\frac{g_1 + g_2}{4} \sigma \right)^2 + (a \chi)^2} \]

\[ \mu^* = \mu - g_\omega \omega_0 \]

Free Energy in "no sea approximation"


\[ \Omega = \Omega_{\text{mes}}^{\text{tree}} + \sum_i \int \frac{d^3 k}{2 \pi^3} \left\{ -E_i(k) + [E_i(k) - \mu^*] \Theta [\mu^* - E_i(k)] \right\} \]

\[ E_{N,N^*}(k) = \sqrt{\vec{p}^2 + M_{N,N^*}^2} \]
Solving the gap equation - gas/liquid phase transition:

\[ M_N^{\text{vac}} = 939.12 \text{ MeV} \]
\[ \mu_c = 923.06 \text{ MeV} \]
\[ \Rightarrow E_B = 16.06 \text{ MeV} \]
\[ \rho_0 = 0.15 \text{ fm}^{-3} \]

\[ M_N^{\text{vac}} = 1535.55 \text{ MeV} \]
is identified with \( N(1535) \)
Magnetic catalysis

Fermions in a magnetic field

The dispersion relations for fermions with charge $q$ in a magnetic field are altered: Landau level quantization

$$E_{\ell}(k_z) = \sqrt{k_z^2 + M^2 + 2|q|B\ell}$$

Free energy for one charged fermion species $\psi$ at $\mu = T = 0$

$$\Omega_f = \frac{B^2}{2} - \frac{|q|B}{4\pi^2} \sum_{\ell} (2 - \delta_{\ell,0}) \int d k_z E_{\ell}(k_z) =$$

$$= -2 \int \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + M^2} + \frac{B_r^2}{2} - \frac{(|q_r|B_r)^2}{24\pi^2} \ln \frac{2|q_r|B_r}{\Lambda^2 A^{12}}$$

$$- \frac{(|q_r|B_r)^2}{2\pi^2} \left[ \frac{x^2}{4} (3 - 2 \ln x) + \frac{x}{2} \left( \ln \frac{x}{2\pi} - 1 \right) + \psi^{(-2)}(x) \right],$$

with $x = M^2 / (2 |q_r| B_r)$, $A$ being the Glaisher constant and $\Lambda$ is the renormalization scale.
Magnetic catalysis

Magnetic catalysis in the eLSM

This contribution gives the well known magnetic catalysis effect


\[
\frac{M_N(B)}{M_N^{\text{vac}}} \approx 1 + \left( \frac{|q| B}{3.06 \times 10^{19} \text{ G}} \right)^2
\]
Magnetic catalysis in the onset of nuclear matter

Phase diagram:
Magnetic catalysis in the onset of nuclear matter

Phase diagram and mass gap:
Magnetic catalysis in the onset of nuclear matter

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Phase diagram and mass gap:
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density and binding energy along the critical line:
Magnetic catalysis in the onset of nuclear matter

Comparison with results from the Walecka model:
Conclusion

So far the effect of the B-dependent vacuum terms has always been neglected:

M. Sinha, B. Mukhopadhyay, and A. Sedrakian, Nucl.Phys. A898, 43 (2013),
J. Dong, W. Zuo, and J. Gu, Phys.Rev. D87, 103010 (2013),

incorporation of magnetic catalysis in models of nuclear matter is the first important step
Outlook

For realistic applications we need to:

- include the anomalous magnetic moment
- allow for anisotropic condensates: (magnetic) chiral spiral
  A. Rebhan, S. Stricker, and A. Schmitt, JHEP 0905, 084 (2009),
  A. Rebhan, S. Stricker, and A. Schmitt, JHEP 0905, 084 (2009),
  FP, A. Rebhan, and A. Schmitt, JHEP 1103, 033 (2011),
- implose neutrality and $\beta$-equilibrium conditions
- analyze the chiral phase transition in the eLSM