ML ESTIMATION OF POPULATION SIZE WHEN OBSERVING MULTIPLE FILL LEVELS IN SLOTTED ALOHA

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ABSTRACT

An open problem in slotted Aloha protocols is to optimally estimate the number of participants as such knowledge is crucial to select the optimal frame length. First results are known in literature based on observing the slot fill levels in case of empty slots and single occupancies (singleton slots). Advances in signal processing allow now also to decode successfully slots with higher fill levels, for example, due to multiple antennas. In this paper we derive the maximum likelihood estimator when arbitrary occupancies up to a maximal fill level R have been observed. Due to our novel approach, the derivation is rather simple and its implementation is of low complexity.

***Index Terms*—** RFID tags, ML-estimation, collision mitigation

1. INTRODUCTION

In slotted Aloha communications, N participants are accessing randomly F slots in a frame. We are interested in estimating the number of N in a frame by counting the numbers m_r of particular fill levels r. Such fill level refers to the number of participants found in a particular slot. This is a typical problem appearing in Radio Frequency IDentification (RFID) technology where a multitude of tags (e.g., hundreds or even thousands) are to be interrogated. Standard compliant [1] readers typically observe the number m_0 of empty slots (fill level zero) and the number m_1 of so-called singleton slots (fill level one) and deduce an estimate of the tag population N. As it is well known for slotted Aloha protocols that the maximum throughput depends on a proper selection of the frame size F, the reader selects such number [2]. A very similar problem now occurs at Intelligent Transport Systems (ITS) in the context of Wireless Access in Vehicular Environments (WAVE), where 802.11p, a wireless communication system similar to Wireless Fidelity (WiFi), has been proposed to offer ad hoc communication between vehicles as well as from cars to infrastructure [3, 4]. Here as well many participants share the wireless medium and their access is controlled by a slotted Aloha protocol. If N is much larger than F, it can be difficult to come up with a good estimate based on empty and singleton slots as most slots are filled with collisions of unknown fill level.

1.1. Relation to Prior Work

The strategies to increase throughput in RFID are commonly addressed at physical or at MAC layer. In the former, novel receiver structures for RFID readers are proposed to extract the exact number of colliding tags in a slot as well as the acknowledgment of these colliding tags [5–9]. In the latter [10–17], the reader adjusts the number of slots per round depending on the number of tags to identify, by estimating the number of tags competing per round. Such procedure is recently being called *Cardinality Estimation* [18] or *Capture-Aware Estimation* [19]. These estimators are typically designed to use as inputs the number of slots with no responses, with one tag response and with more than one tag response, corresponding to empty, singleton and collision slots, respectively. Here, [15] derives the first true Maximum Likelihood (ML) estimator based on the prior observation of $\{m_0, m_1, m_{>1}\}$. With advances on the physical layer [2], it became possible to resolve two tag collisions by single antennas. Recently with slight modifications of the standard, it is possible to resolve up to eight tag collisions [20–24] when employing readers with four antennas. Similarly in car2car communications, several antennas can be employed at each car [25, 26]. This gives the need to define optimal estimators when observing different fill levels in the slots. Furthermore, the estimate of the tag population becomes better. These new advances at physical layer provide new inputs that can improve the tag population size estimation: the number of slots with exactly $2, 3, \cdots, R$ tags colliding in a slot.

1.2. Notation

The paper is written in terms of combinatorial results. Some variables are used frequently and -for the convenience of the reader- are summarized in Table 1. We refer to $\mathbb{N}^+$ as the positive integers and to $\mathbb{N}^0$ as the non-negative integer numbers.

1.3. Organization of the Article

After this introduction in which the problem is motivated and the state of the art is presented, we derive the new ML estimator in Section 2. Section 3 presents examples and simulation results, validating the improved performance of the new estimator. Finally, Section 4 concludes the article.

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variable	support	meaning
N	$\mathbb{N}^{+}$	size of population
$\hat{N}_{\mathrm{ML}}$	$\mathbb{N}^{+}$	ML estimate for size of population
F	$\mathbb{N}^{+}$	number of available slots
r	$\mathbb{N}^{0}$	fill level in a slot
m_r	$\mathbb{N}^{0}$	number of slots with fill level r
$m_{>R}$	$\mathbb{N}^{0}$	number of slots with fill level larger than R
R	$\mathbb{N}^{+}$	maximal fill level that can be recognized

Table 1. Table of often used variables.

2. ML ESTIMATOR

The idea of an ML estimator is that the probability of a certain event is maximized with respect to the desired parameter, in our case the number N of the tag population. To this end we have to compute several combinatorial results first.

Lemma 2.1 *Given the number N of tags and the amount of slots F, there exist exactly*

$$S(N,F) = \begin{pmatrix} N+F-1 \\ F-1 \end{pmatrix} \tag{1}$$

possible distribution patterns.

Proof: We show this by induction. Assume we have a number of tags k from zero to N on the first slot and the rest, $N-k$ tags, on the remaining $F-1$ slots, as depicted in Figure 1, then we find:

$$S(N,F) = \sum_{k=0}^{N} S(N-k, F-1). \tag{2}$$

At the other end of the inductions, we find $S(n,1) = 1$, n denoting

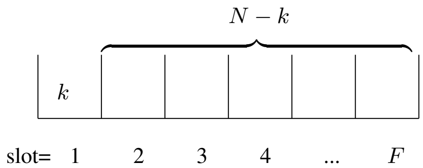

Fig. 1. Consider F slots with k tags in the first slot.

the remaining tags for the last slot F. Taking two last slots, with n tags left, we find $S(n,2) = n+1 = \begin{pmatrix} n+1 \\ 1 \end{pmatrix}$ possibilities and then $S(n,3) = \sum_{k=0}^{n} S(n-k,2) = \sum_{k=0}^{n}(n-k+1) =$ $= (n+1)(n+2)/2 = \begin{pmatrix} n+2 \\ 2 \end{pmatrix}$. Due to the general relation

$$\sum_{k=0}^{m} \begin{pmatrix} n+k \\ n \end{pmatrix} = \begin{pmatrix} n+m+1 \\ n+1 \end{pmatrix}$$

we find (1).

In the following Lemma 2.2 we want to restrict the number of distribution patterns by excluding those that have fill levels of maximally R tags. For this we repeat the previous procedure but constrain the fill levels, starting at $R+1$.

Lemma 2.2 *Given the number $N \geq N_{\min} = (R+1)F$ of tags and the amount of slots F, there exist exactly*

$$S_R(N,F) = \begin{pmatrix} N-FR-1 \\ F-1 \end{pmatrix} \tag{3}$$

possible distribution patterns that exclude the fill levels $r = 0, 1, .., R$.

Proof: Reconsider Figure 1. Assume we have $k > R$ tags in the first slot. Then we find by induction that the number of permutations for the remaining $N-k$ tags

$$S_R(N,F) = \sum_{k=R+1}^{N-(F-1)(R+1)} S_R(N-k, F-1), \tag{4}$$

where we excluded all fill levels from $k = 0, 1, ..., R$ in the summation. As we have a fill level of at least $R+1$ in all remaining $F-1$ slots, the minimal number possible is $N = N_{\min} = (R+1)F$. On the other hand the upper bound for the sum is also restricted as we have to fill at least all remaining $F-1$ slots with fill level $R+1$. Working through the induction, we finally find $S_R(n,2) = n-2R-1$ and $S_R(n,3) = \sum_{k=R+1}^{n-2(R+1)} n-k-2R-1 = (n-3R-2)! = \begin{pmatrix} n-3R-1 \\ 2 \end{pmatrix}$, in general we find (3).

Note that these two lemmas already offer a simple estimator for the tag population number. If we count the number of collision slots (slots that have colliding tags, i.e., a fill level larger than R) $m_{>R}$ and set this in relation to the known number F of slots, then this ratio should be identical to $S_R(N,F)/S(N,F)$. We can simply find an intuitive estimate by

$$\hat{N}_{\mathrm{I}} = \arg\min_{N} \left| \frac{m_{>R}}{F} - \frac{S_R(N,F)}{S(N,F)} \right|. \tag{5}$$

Being a simple estimate, we do not expect it to be optimal. Its performance will be evaluated in Section 3 in the context of the optimal ML estimate.

We now consider modern detectors that are able to identify certain numbers of collisions in a slot, that is, they know what fill level per slot is present up to a limit, say R. Identical fill levels are then counted and the final number (frequency) is stored in the $R+1$ fill level parameters $m_r; r = 0, 1, ..., R$.

Lemma 2.3 *Given the amount F of slots and the frequencies $\{m_0, m_1, ..., m_R\}$ with which the fill levels $0, 1, ..., R$ are observed, the number of possible outcomes is given by*

$$T_R(N,F) = \frac{F!}{F_R! \prod_{r=0}^{R} m_r!} S_R(N_R, F_R). \tag{6}$$

with

$$F_R = F - \sum_{r=0}^{R} m_r \tag{7}$$

$$N_R = N - \sum_{r=0}^{R} r m_r. \tag{8}$$

Note that, although not shown explicitly, the variable N_R is dependent on N, a fact that will be used later.

Proof: The number of permutations to find m_0 slots with fill level zero in F slots is

$$\begin{pmatrix} F \\ m_0 \end{pmatrix} = \frac{F!}{(F-m_0)!m_0!}. \quad (9)$$

Now that we have found already m_0 empty slots, only $F-m_0$ slots remain for the next step. Thus the number of permutations to find m_0 slots with fill level zero and m_1 slots with fill level one is:

$$\begin{pmatrix} F \\ m_0 \end{pmatrix}\begin{pmatrix} F-m_0 \\ m_1 \end{pmatrix} = \frac{F!}{(F-m_0-m_1)!m_0!m_1!}. \quad (10)$$

Continuing this argument until fill level R, we find $\frac{F!}{F_R!\prod_{r=0}^{R} m_r!}$. Now we have to consider that for each of these patterns, the remaining N_R tags can have $S_R(N_R, F_R)$ patterns in the remaining F_R slots. Taking this also into account we finally find (6).

Note that the number F_R in (7) coincides with the previously used variable $m_{>R}$, i.e., the number of slots that have a fill level higher than R, or in other words, those slots for which the tags cannot be detected. When comparing with previous results using the set $\{m_0, m_1, m_{>1}\}$, we realize that $m_0 + m_1 + m_{>1} = F$. Thus, knowing F and counting $\{m_0, m_1\}$ is equivalent. Our formulation of the ML estimator will not use the number of collision slots, as this information is equivalently given by the number F of slots.

Theorem 2.4 *Given the number F of slots and the frequencies $\{m_0, m_1, ..., m_r\}$ with which the fill levels $0, 1, ..., r$ are observed, F_R and N_R defined in (7), and (8), respectively, the ML estimator $\hat{N}_{\rm ML} \geq N_R$ for N is given by:*

$$\hat{N}_{\rm ML} = \arg\max_{N\geq N_{\min}} \frac{\begin{pmatrix} N_R - F_R R - 1 \\ F_R - 1 \end{pmatrix}}{\begin{pmatrix} N+F-1 \\ F-1 \end{pmatrix}}, \quad (11)$$

where the search is being conducted, starting with

$$N_{\min} = \sum_{r=0}^{R} r m_r + F_R(R+1). \quad (12)$$

Proof: By definition, the ML estimate maximizes the given probability:

$$\hat{N}_{\rm ML} = \arg\max_{N\geq N_{\min}} \frac{T_R(N,F)}{S(N,F)}, \quad (13)$$

Inspecting the definition of $T_R(N,F)$ in Lemma 2.3, we find that only the term $S_R(N_R, F_R)$ is dependent on N. We can thus drop the remaining terms and obtain (11). We restricted the search for N in (13) to the feasible values $N \geq N_{\min}$. The minimal number $N_{\min}$ in (12) is found by adding up all observed fill levels and assuming at least a fill level of $(R+1)$ for the remaining (collision) slots.

3. EXAMPLES

We will provide three examples, first a simple one, for which one can write easily down all possible outcomes, count them and by this validate the result. Secondly, we compare with standard compliant results, known from literature and thirdly a more elaborate numerical example is presented in which a larger tag population is to be estimated by various estimators.

Example 1: Let us assume $F = 3$ slots and $R = 1$, allowing us to observe two fill levels, e.g., $m_0 = 1, m_1 = 0$. Consequently $m_{>1} = F - m_0 - m_1 = 2$. We find from Lemma 2.1 that we have $S = (N+2)(N+1)/2$ possibilities. The smallest possible N is $N_{\min} = 1m_0 + 0m_1 + 2m_{>1} = 4$, that is two remaining slots are filled by at least fill level 2. As we have observed m_0 and m_1, we have to check for all combinations that occur with fill levels larger than $R = 1$. For $N = 4$ we find $S_1(4,2) = 1$, for $N = 5$ we find $S_1(5,2) = 2$ and in general we find $S_1(N,2) = N-3$. Following Theorem 2.4 we compute

$$\hat{N}_{\rm ML} = \arg\max_{N} \frac{6(N-3)}{(N+1)(N+2)} \quad (14)$$

starting with $N = N_{\min} = 4$ for which we find that $\hat{N}_{\rm ML}$ takes on the values 7 and 8 with equal probability (1/3).

Example 2: In a standard-compliant setup, only the fill levels $\{m_0, m_1, m_{>1}\}$ are observed and based on those a decision for optimal N given the frame length F is made. We find for such case $N_{\min} = 2(F - m_0) - m_1$ and the desired N by searching through:

$$\hat{N}_{\rm ML,R=1} = \arg\max_{N\geq N_{\min}} \frac{\begin{pmatrix} N-F+m_0-1 \\ F-(m_0+m_1+1) \end{pmatrix}}{\begin{pmatrix} N+F-1 \\ F-1 \end{pmatrix}}. \quad (15)$$

The obtained estimator corresponds to the result in [15].
In [2] a method is explained and shown to work by experiments that with a single antenna even two tag collisions can be detected. In this case we observe the fill levels $\{m_0, m_1, m_2, m_{>2}\}$. The optimal estimator in this setup is found by

$$\hat{N}_{\rm ML,R=2} = \arg\max_{N\geq N_{\min}} \frac{\begin{pmatrix} N-2F+2m_0+m_1-1 \\ F-(m_0+m_1+m_2+1) \end{pmatrix}}{\begin{pmatrix} N+F-1 \\ F-1 \end{pmatrix}}. \quad (16)$$

for $N_{\min} = 3(F - m_0) - 2m_1 - m_2$.

Example 3: We estimated $N = 40$ tags under various values of slot number F and maximal fill level R. Figure 2 depicts the standard deviation of the estimators obtained after 10 000 Monte Carlo (MC) runs parameterized in (F, R). We ran the experiments for $F = 15, 16, ..., 50$ and $R = 1, 2, ..., 8$. We recognize the following behavior

- The last line in the background corresponds to the conventional ML estimator ($R = 1$) that uses fill levels zero and one as well as the remaining number of collision slots.
- As expected the performance of the ML estimator increases monotonically with growing R and F.

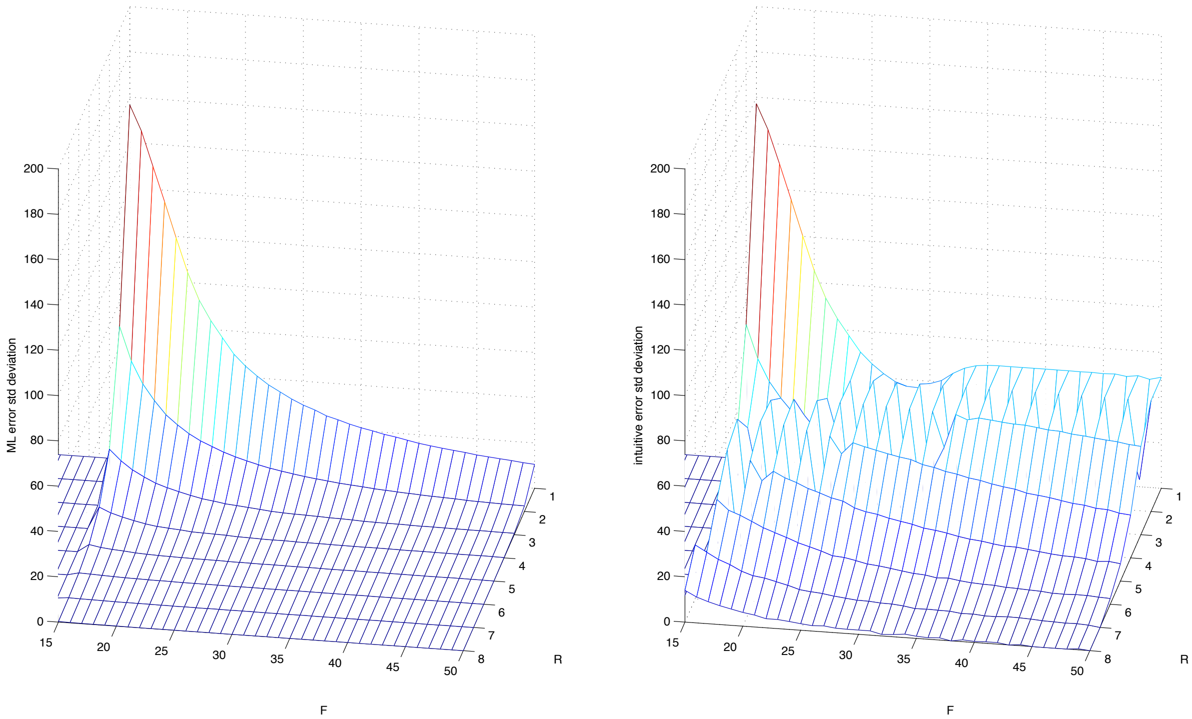

Fig. 2. *Estimation error performance in dependence to slot number F and maximal fill level R. Left: ML-estimator, right: intuitive estimator from (5).*

- Surprisingly, the simple intuitive estimator from (5) delivers a similar quality for a certain limited range (small F and R) but can become quite poor for larger values of F. The reason for this that if $N \gg F$ and R small, mostly higher fill levels are occupied, thus most information is in $m_{>R}$ and then both estimators behave practically equivalent.

4. CONCLUSIONS

We revisited the problem of estimating the optimal number of tags in a frame of F slots by observing particular identifiable slots of fill levels from zero to R. Differently to previous results in which only fill levels of zero and one were taken into account, we generalized the result for arbitrary fill level patterns. Due to our new combinatorial approach, we found that the ML estimator is surprisingly simple to obtain. The particular simplicity of our result also relies on the fact that we assumed a continuous range of fill levels starting with zero ending at R. Although the method can be extended to include also non-continuous ranges, it is not expected to obtain compact results. The behavior of the ML estimator offers to estimate the tag number more accurately with relatively short frames $F \ll N$, speeding up significantly the initial estimation problem. For such cases an alternative much simpler estimator of less complexity is provided that –in some cases– can even exceed the quality of the ML estimator.

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