FORMULATING THE PERFECTLY MATCHED LAYER AS A CONTROL OPTIMIZATION PROBLEM

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In many applications where an unbounded solution of a wave-like equation is desired, the problem occurs that due to limited computational capabilities the domain has to be truncated at some point. To let the solution of this confined domain approximate the free-wave propagation, boundary conditions with absorbing properties have to be applied. The work by Engquist and Majda [1] addressed this issue and absorbing boundary conditions (ABCs) were derived which worked well under certain circumstances. The technique to surround the computational domain with a perfectly matched layer was first described by Berenger [2] for the absorption of electromagnetic waves. The idea of the perfectly matched layer was later extended and applied to other wave propagation problems, both in a split or un-split field formulation [3] [4] and was interpreted in terms of a ”complex-coordinate stretching” [5]. The PML performs well even in dispersive media, but the design process involves a high mathematical effort due to a transformation of the spatial derivatives. With a high order of spatial derivatives, as it is the case for the Euler-Bernoulli beam, this becomes a tedious task. This work aims to find a constant state-feedback controller which is capable of emulating PML properties, thus avoiding the transformation of the partial differential equations.

PML AS A CONTROL OPTIMIZATION PROBLEM

It is assumed, that the partial differential equation for which the PML controller is to be designed, is linear, has a harmonic wave solution, and that the discrete dispersion relation can be obtained. The partial differential equation can then be discretized and aggregated into a state space system of the form

$$\boldsymbol{x}^{j+1} = \boldsymbol{A}\boldsymbol{x}^{j} + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}^{j} \quad (1)$$

where $\boldsymbol{x}^j$ denotes the displacement field at the time step j, $\boldsymbol{A}$ is the system matrix corresponding to the partial differential equation. $\boldsymbol{K}$ is the feedback controller and $\boldsymbol{B}$

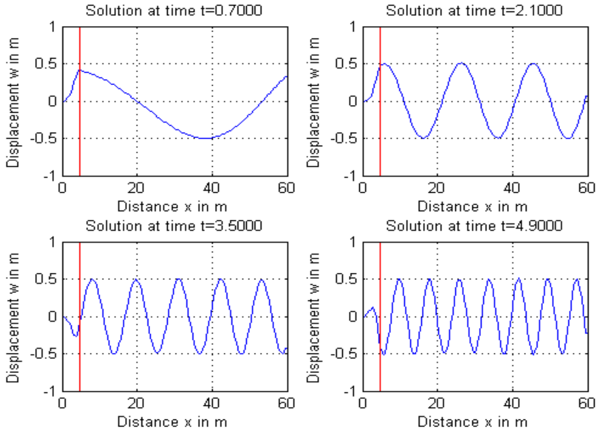

Fig. 1: The optimized feedback controller is applied on the left boundary. A frequency sweep excites the right boundary. No reflections back into the computational domain are visible

distributes the control action to the nodes of the PML region.

To find a suitable controller, an objective function is devised which evaluates the error between the controlled state-space system and a desired damped fundamental solution.

$$J(\boldsymbol{K}) = \sum_{\Omega} \sum_{j=2}^{j_{\max}} \left| \boldsymbol{x}_{\text{fund}}^{j}(\omega_x, \omega_t) - \boldsymbol{x}^{j}(\boldsymbol{K}) \right|_2^2 \tag{2}$$

ω_x and ω_t are the wavenumber and the angular frequency whose dependency is given by the dispersion relation. $\boldsymbol{x}_{\text{fund}}$ denotes a desired, damped, fundamental solution which can be analytically given for a certain frequency pair (ω_x,ω_t). The objective function is evaluated over a set of frequency pairs Ω and a finite number of time steps $j_{\max}$ and minimized with respect to the parameters of the feedback controller $\boldsymbol{K}$ using a genetic algorithm. The performance of such a controller designed for an Euler-Bernoulli beam is shown in Fig. 1.

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