

MODELLING AND SIMULATION EXAMPLES BASED ON COMPUTER ALGEBRA SYSTEMS

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ABSTRACT

Nowadays education has to satisfy different requirements regarding the variety of demands of industry but also pedagogical. The students have to learn as well as apply different skills. Another important part is deep understanding of the environment. It is not enough to know how to apply a formula they should also be able to connect that formula to the mathematical background to enable connecting different topics. They should be able to apply the learnt methods in real-life scenarios. This all should be done in a minimum of time during the course. The lectures are forced to accomplish the impossible. Therefore universities are encouraged to use online environment to support their courses to enable individual learning. Using platforms and other e-tools the learning process can be translocated to leisure time. This also enables students to learn in their own speed and to their own conditions. In this paper three different e-learning environments will be presented and their usage and development at the TU Wien will be presented.

Keywords: b-learning, mathematical modelling, mathematics, education

1. INTRODUCTION

The research group for mathematical modelling and simulation (MMS) at the TU Wien started to use online tools in 2004. The first step was the usage of a moodle based platform. The moodle platform was initialized by the university and therefore also some modification or add-ons according to university's requirement were made. This webpage is mainly used to organize all the materials of the lectures. On the one hand the lecture materials but also additional documents and links are provided using moodle. On the other hand the administration of students can be done using this system. There are different tools which are useful to organize exercise tasks, dues and appointments. The person in charge of the system for TU Wien also programmed some plugins if needed. For example the administration of the exercises required a certain plugin. In the exercises organized by the MMS the students

have one week to prepare their homework. Task information is provided in form of pdf documents uploaded onto the moodle platform. At the end of the week the students have to tell the lecturer or tutor which examples he or she has prepared. This is realized using a special plugin where the students can tick off which examples they have prepared.

This plugin is mainly used in mathematical courses. Due to the fact that the research area of MMS covers as well different applications of modelling and simulation in various fields the group also organizes different lectures dealing with modelling and simulation. For these courses moodle cannot fulfil all requirements to ensure an adequate e-learning support. Therefore a combination of moodle and an e-learning system, called MMT, is used. The MMT server, which was developed by a spinoff company and the group itself in 2006, stores various modelling and simulation examples. It can be compared to a CMS for modelling examples. In the following section it will be described in detail.

In 2008 the vice rector for academic affairs initialized a refresher course for mathematics which should be supported by an e-learning tool. Due to the experience in this field this course idea was executed by MMS. The used e-learning system is called Maple T.A. which will be explained later. The refresher course is now held for nearly every first semester student at TU Wien which would not be possible without the usage of an additional online learning platform.

2. MMT

In the modelling and simulation courses the MMT, which stands for Mathematics, Modelling and Tools, is used. It was developed in 2006. Before that the research group used a server which was hosted by MATLAB itself. After MATLAB shut the server down the research group invented a new system by themselves.

2.1. Structure

MMT is partly a content management system. The content is organized in a tree structure. Additionally it is possible to execute MATLAB files on the system. There are basic mathematical examples realized as well

which are still used in mathematical courses for students of geodesy in the first and second semester.

The main advantage of this system is the parametrization of the examples. If the students face an example, it does not matter if it is a basic mathematical one or a simulation example, the parameter used for calculations of this example can be changed and the example can be executed again. In this way the students see the influence of the variables using different values. This online system is used in several modelling and simulation courses. The realized examples for these courses deal with different model approaches. In the beginning this simulation examples with variable parameters where only implemented in MATLAB. In the last year some improvements were made. There are as well examples implemented using other software for example open source Octave, Simulink, Anylogic or System Dynamics are now available. Nearly for the last 10 years this platform was used to support the learning process and understanding. In the lectures the examples are used to present the different modelling aspects. Using different application examples enables students to gain a connection between real-life problems and the mathematical definition of the according model.

2.2. Examples

In the following example the structure of the system as a CMS is shown.

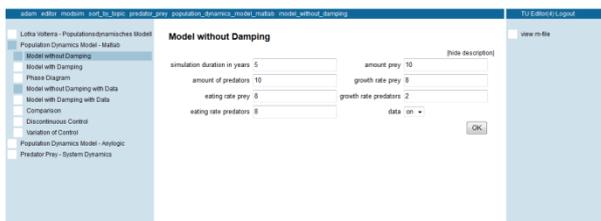


Figure 1: The structure of the MMT server is shown.

In Figure 1 the principle structure of the MMT server is shown. The left side displays the tree structure of the content. The students can see the examples connected to their course sorted by topic. In the middle of the picture the parameter section is shown. There the students can fill in the parameter values they want to explore. After submitting the values the simulation starts by clicking the OK button. Depending on the complexity of the example the calculations can take some seconds. In this example that is not the case. Then the result plot appears and the student can study the data.

In the right column there is a link to the source code which was used for the program. With this code the students have the possibility to execute the example after some modifications on their own MATLAB or Octave installation. Regarding the other program languages the model can be rebuilt using the representation provided by the MMT system.

Figure 2 shows the result of an implementation of the Lotka-Volterra equation. This differential equation system of first-order describes the dynamical interaction

of prey and predator. The following two equations characterize the change of the populations.

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

This predator prey model takes into account two populations, predators, for example foxes, and preys, for example rabbits. The existence of predators is dependent on the number of the prey, because their survival is dependent on how much they can eat (prey). The survival of the preys in return is also dependent on the amount and eating habits of the predators.

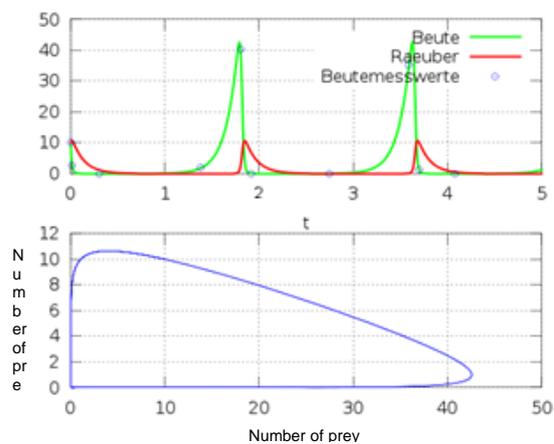


Figure 2: The results of the prey-predator model are shown.

In the Figure 2 in the upper graphic the red line gives the amount of predators and the green line shows the number of prey.

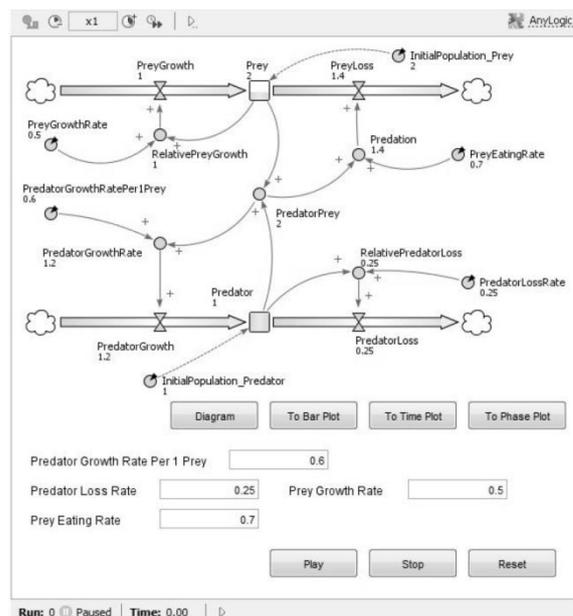


Figure 3: Implementation of the prey-predator model in AnyLogic.

The dependency mentioned before can be noticed easily. When the predators are less the preys can reproduce. Due to the increasing number of preys the predators can hunt again.

In Figure 3 the same example is developed in AnyLogic where Java-applets are used. AnyLogic is a simulator which can export the created models as Java-applets. Therefore it is possible to produce models using this software for the MMT.

Currently these examples are not only used to present them during the lecture to explain certain modelling aspects but also for practising the influence of variables in a model.

In the courses about modelling and simulation the students have to pass a test. As mentioned before this test was performed on the moodle platform. There are some randomized multiple choice questions realized in moodle itself. Other questions are dealing with the determination of certain parameter values in order to generate certain behaviour of the model. These parameter values have to be entered in moodle. The grading of the test is also realized in moodle.

In order to generate a great range of examples and also different applications more examples were gathered through cooperation with a research group in Ljubljana, Slovenia. They also use an e-learning tool to improve the learning process of the students. Due to their profession they were also able to provide some real-life experiments. The students have the possibility, using a step-by-step environment to prevent experimental errors, to set parameters for a real experiment and watch the results live. This principle we could not include directly up to now but the experiments gave us some new example ideas. For further cooperation an enlarging of the example pools of both groups is planned. An inclusion of their experiments in form of a webcam connection is also in mind.

3. MAPLE T.A.

In 2008 the vice rector for academic affairs asked for a so-called refresher course for mathematics. The necessity of such course is still controversial. Due to the fact that most of the academic institutions in Germany have already initiated such courses it seems to be necessary not only in Austria.

This kind of course should remind the students of their perhaps forgotten knowledge from school especially in mathematics. The refreshment should then help the students to surmount difficulties which can appear in the first year of study. In order to offer this course for nearly every newcomer at the TU Wien the course combines ordinary lectures with hands-on practice and e-learning tools.

3.1. Moodle Connection

Maple T.A. stands for Maple Teaching and Assessment. It is a computer algebra system which is based on Maple. Therefore it is a very powerful mathematical tool. In the provided interface the management of examples as well as user administration come in handy. The examples are organized in different classes which are related to the different courses in university. This system can not only be used for exercising but is also, as mentioned in the name, used to perform tests. All the test results are stored in the system and can also be accessed by the student. Since some years there is also the possibility to connect Maple T.A. to a moodle platform. As mentioned before TU Wien uses a moodle based website to organize course materials. Regarding modelling and simulation courses the moodle platform is also used to realize small tests.

In order to perform a test in the mathematical courses the students have to provide three different logins to connect to the different platforms. Using this point of intersection the Maple T.A. system uses the moodle platform to identify the students and to make sure that only registered students are using Maple T.A. Therefore the students need only access to the university platform to exercise examples on Maple T.A. The layout of moodle can be used to arrange a certain course structure and include the related examples from Maple T.A. directly to this structure. This ensures a more instructed learning process which should support students' progress.

3.2. In General

Maple T.A. offers different prepared question types as multiple choice, numerical graded, maple graded but also drop down menu questions. All these question types can be created separately but can also be combined in a single question. This range of question types enables a variability of examples focusing on different learning phases.

In order to create an example an algorithm is necessary. The algorithm which the example is based on can be realized in two different ways. On the one hand there are some special commands which are implemented for the use in Maple T.A. For example the randomization of integer values in a certain range is possible with a special command which is only working inside Maple T.A. There are other commands like if and switch which are different or unique in Maple T.A. On the other hand it is also possible to use commands and procedures programmed in Maple itself. In the algorithm Maple can be accessed in every step and for every variable. But in order to assure nice performance, part of the algorithm should be done in Maple T.A. itself. It is possible to integrate a whole Maple procedure into the algorithm as well if it is necessary. After programming the algorithm the next step is to define the right grading for the student's response. There are different predefined grading routines. For example a multiple choice question is automatically graded using partial grading in terms of multiple

possible answers. Other forms of grading are, for example, numeric, equations and maple-graded. The numeric routine enables the tutor to define a certain range of accuracy for the student's solution. Using the equation type allows the student to enter an algebraic equivalent answer as an equation. In the maple-graded section the grading routine can be an algorithm written in Maple. This last choice offers many different possibilities regarding the type of student responses.

3.3. Library

Most examples are graded using the maple-graded response. In the answer window of the question students enter their results which can be either a value or a whole function. If the example deal with Linear Algebra it is also possible that the result is a vector or matrix. These results can be graded using every available Maple command. The grading should depend on the purpose of the examples therefore the didactical aim of the example should be focused.

In order to guarantee a student friendly grading routine MMS created two different libraries. One library deals with grading of different types of answers. The main reason was an example dealing with partial fraction expansion. The calculation for students is complicated and prolonged but one small mistake ruins the whole result. The first step was to grade this example partially. Every coefficient should be considered and not only the whole answer. Furthermore also the grading routine of vector and matrix follows this principle. If the example asks for the inverse matrix the grading algorithm considers every single entry and not only the matrix as a whole. Additionally, grading routines for sets and integration results are included.

In Figure 4 the grading routine for vectors is applied. This example is a special one because the routine determines how many entries of the resulting vector of the matrix vector multiplication are zero. In this example only the multiplication of the first row is zero. Therefore the student receives 25 % of the points.

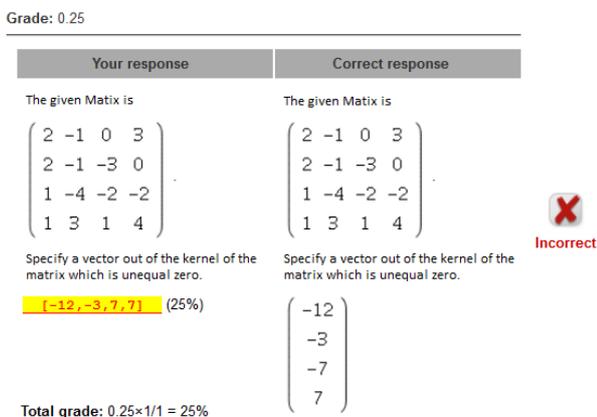


Figure 4: The grading routine for vectors is used in this example.

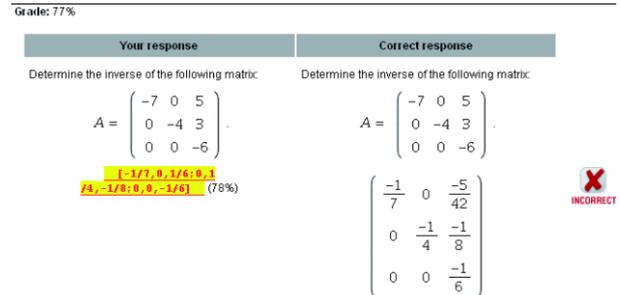


Figure 5: Using the matrix grading routine every entry is graded separately.

In Figure 5 shows the example mentioned before. The student has to determine the inverse of a certain matrix. Due to the fact that the last entry of the first row and the second entry in the second row are wrong the student gains 78% of the points.

Another advantage of using this online tool lies in the automatic generation of a various number of examples by creating only one. In the algorithm the used variables can be picked randomly out of a certain range. If the students choose one of these examples twice they get new actual numbers every time. Due to this fact, memorizing of example solutions is prevented. Sure they might remember how to calculate the solution but that is exactly one of the aims of this system. Students can practice as long as they want. Sometimes the numbers might repeat but in a sequence of several attempts. It is not only possible to vary the variables but also to randomly generate function with every attempt. Using the second developed library the function can be picked out of a defined set. This does not explain why the need of a separate library was necessary. But as seen before the examples dealing with Linear Algebra are in some aspects more difficult than others. Therefore this library also includes some creation routines which support the development of questions about matrix properties and eigenvalues. Some of the properties are size, rank and zerocount which defines how many zero entries the matrix contains. Another command defines the definition of the matrix which is generated. Randomization of integer vectors is also included in the library. This library eases the work of the developer but also guarantees a comparability of the different attempts and assures a greater variety of the examples because the different cases are not separated but randomized in one example.

The development of the libraries and the following increased randomization has two main consequences. The students have literally the possibility to practice examples all day long. The main goal of the examples remains but the calculations changes every time. For teachers and lecturers this fact changes the arrangement of tests and exams. The variable variation reduces the chances to cheat in examination. The neighbouring student gets examples with other numbers so he or she

cannot copy the results from him or her. The students have to find a way to calculate it.

Since usage of the online platform in the research group students attending mathematical courses had to pass two of three tests realized in Maple T.A. These tests were distributed throughout the semester. In the last two years, the experiment executing exams in Maple T.A. has started. Due to the fact that the environment requirements for online exams for around 300 students are not given at the university this form of exam is voluntary. The students can decide if they want to perform the exam in Maple T.A. or as an ordinary written exam. After this exam the second part is an oral examination of the professor in order to determine the theoretical knowledge of the student.

In the following two examples of the different mathematical levels are presented.

3.4. Examples

The first example is used in the refresher course as well as in the mathematical course in the first semester of electrical engineering. The question shows either a cosine or sine and the student has to fill in the right parameters. In Figure 6 two different question types available in Maple T.A. are used. This example is every year a challenge for the students. Additionally this example is very important for electrical engineers to learn how to read a graphic right.

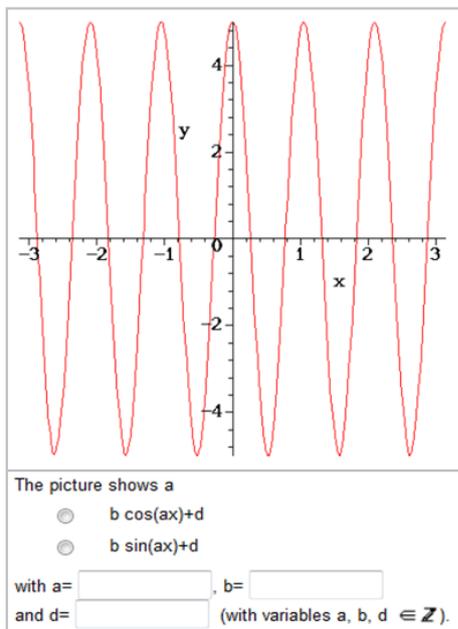


Figure 6: The example is used in the refresher course and basic mathematical course.

Question:

Let's observe the matrix

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}.$$

- Compute the characteristic polynomial:

$$p(\lambda) = \text{[input field]}$$

- Calculate the eigenvalues and insert them as set.

$$\text{[input field]}$$

- Compute the eigenvector v to the eigenvalue $\lambda = -2$

$$v = \text{[input field]}$$

- Choose the right answer. The matrix A is:

- positiv definit
- negativ definit
- indefinit

Figure 7: The example deals with the matrix properties and is used in the higher mathematical courses.

The question of Figure 7 shows a Linear Algebra example. Firstly, the student has to enter the characteristic polynomial to determine the eigenvalues for the second step. In the third step the eigenvector should be calculated. Due to the used library the last step is easy to develop and asks for the definition of the matrix.

All in all there are many different examples dealing with various topics which are part of the mathematical courses.

4. EXAMPLE TRANSFERATION

The testing tool of moodle is quite useable. In order to be more flexible regarding the creation and development of tests there are some unsatisfying gaps. In the description of the two used systems some differences can be outlined. On the one hand the MMT system offers many different possible programming languages and provides an interface where students can experiment with the modelling examples. A small disadvantage is the location of the examples. It is not possible to link them directly to the moodle platform. The students have to enter a certain login to get to the right course examples. The testing environment is realized in moodle and the students have to experiment meanwhile on the MMT System.

The testing and assessment environment of Maple T.A. is quite mature. There are many properties which can be adapted to the needs of students and lecturers. The example tool was up to now only dealing with mathematical tasks. The usability of these questions can be shown due to the fact that at the last exam more than 60% of the students chose the online exam over the

written exam. There are different reasons for this development. On the one hand the students can practice all examples as preparation for the exam. On the other hand the online exams provide the students' results immediately after finishing the exam. This enables the students to make the oral exam shortly after the written one.

Due to the new developments of Maplesoft, Maple T.A. is updated every year. The last update includes some nice features which led to a new step regarding the two e-learning environments. In the newest version it is possible to integrate MathApps into the Maple T.A. question. The main advantage of this feature is the possibility for students to explore and experiment with graphical outputs. Additionally this change leads to a completely different structure of the questions. The MathApp part can be used to convey the mathematical theory or background of the question.

This possibility enables the research group to merge both systems into one. The simulation examples are transformed into MathApp questions. The disadvantage of an external grading of the MMT system using moodle disappears.

Figure 8 shows one of the first examples transformed from MMT to Maple T.A. As seen before the students had to fill in their parameter values and generate the new image using an OK button. In the MathApp implementation the parameter variation is realized using sliders. The position of these sliders can be changed by the student. Immediately the system generates the plot again to show the student the modified output.

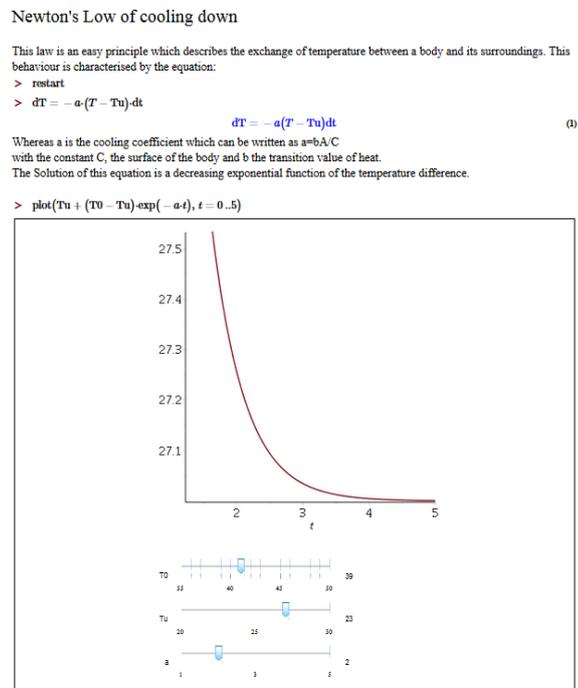


Figure 8: A modelling example realised as MathApp in Maple T.A. is shown here.

In the modelling and simulation courses a test is performed. The fact that the grading of these MathApps can now be done in Maple T.A. directly eliminates the usage of moodle for testing. But moodle is still used to organize the lecture dates and materials and links directly without further login to the according examples in Maple T.A.

5. CONCLUSION

The new features of Maple T.A. support the merging of the three e-learning systems. Unfortunately the transition of the MMT example to the Maple T.A. system will take more time than expected due to the fact that the examples have to be implemented again. This time span can be bridged using both systems until the transition is done. In the end all the existing examples will be gathered inside one system with an easy administration and assessment.

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Stefanie Winkler passed her bachelor study in technical mathematics in 2012 and her master study as well in technical mathematics in 2014. Both theses covered applications of mathematical modelling and simulation. The master thesis in particular covers a comparison of different approaches to simulate groundwater pollution. Her fields of activity include numerical aspects of simulation, comparison and evaluation, simulation of partial differential equations as well as applied mathematics in engineering. Since 2014 she is project assistant in the research group of Mathematical Modelling and Simulation at the Institute for Analysis and Scientific Computing at Vienna University of Technology. She is currently working on her PhD in mathematics.