Entanglement Entropy in Shock Wave Collisions

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Motivation

Central question:

How does a strongly coupled quantum system which is initially far-from equilibrium evolve to its equilibrium state? [see also talk by E. Lopez]
Quark-gluon plasma (QGP) is a deconfined phase of quarks and gluons produced in heavy ion collision (HIC) experiments at RHIC and LHC.
Why AdS/CFT?

The QGP produced in HIC's behaves like a strongly coupled liquid rather than a weakly coupled gas.
AdS/CFT correspondence

AdS/CFT correspondence: [Maldacena 97]

Type IIB string theory on $\text{AdS}_5 \times S^5$ is equivalent to $\mathcal{N}=4$ super symmetric SU($N_c$) Yang-Mills theory in 4D.

Supergravity limit:

Strongly coupled large $N_c$ $\mathcal{N}=4$ SU($N_c$) SYM theory is equivalent to classical (super)gravity on $\text{AdS}_5$.

Strategy:

- Use $\mathcal{N}=4$ SYM as toy model for QCD.
- Build a gravity model dual to HICs, like colliding gravitational shock waves.
- Switch on the computer and solve the 5-dim. gravity problem numerically.
- Use the holographic dictionary to compute observables in the 4 dim. field theory form the gravity result.

[see talks by Takayanagi, Lopez, Ammon, Riegler, ...]
Holographic thermalization

Thermalization = Black hole formation

$T_{\mu\nu}$  $g_{\mu\nu}$
Divide the system into two parts $A, B$. The total Hilbert space factorizes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

The reduced density matrix of $A$ is obtained by the trace over $\mathcal{H}_B$

$$\rho_A = \text{Tr}_B \rho$$

Entanglement entropy is defined as the von Neumann entropy of $\rho_A$:

$$S_A = - \text{Tr}_A \rho_A \log \rho_A$$
Entanglement entropy in a two quantum bit system

Consider a quantum system of two spin 1/2 dof's. Observer Alice has only access to one spin and Bob to the other spin.

A product state (not entangled) in a two spin 1/2 system:

$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle + |\downarrow_A\rangle) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$

$$S_A = 0$$

A (maximally) entangled state in a two spin 1/2 system:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle \otimes |\downarrow_B\rangle - |\downarrow_A\rangle \otimes |\uparrow_B\rangle)$$

$$S_A = \log 2$$

Entanglement entropy is a measure for entanglement in a quantum system.
Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method**.

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated**!

With the **replica method** one gets **analytic results** for 1+1 dim. CFTs. [Holzhey-Larsen-Wilczek 94]

One finds **universal scaling** with interval size:

\[ S_A = \frac{c}{3} \log \frac{L}{a} + \text{finite} \]

**3-sheeted Riemann surface**

**Notable generalization: 1+1 dim. Galilean CFTs** [Bagchi-Basu-Grumiller-Riegler 15]

**AdS/CFT** provides a **simpler method** that works also in higher dimensions.
Holographic entanglement entropy

Within AdS/CFT entanglement entropy can be computed from the area of minimal (extremal) surfaces in the gravity theory.

\[ S_A = \frac{\text{Area}(\Sigma)}{4G_N} \quad \text{[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]} \]

Within AdS/CFT, entanglement entropy is related to the area of extremal (minimal) surfaces in the gravity theory.
Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved. [ongoing work: CE-Grumiller-Kapetanoski-Khavari]
- Can we somehow simplify our lives?

Yes we can!

minimal surface for a starboundary region (red) in AdS4
Surface Evolver]
Entanglement entropy from geodesics

Consider a **stripe region** of **infinite extend** in **homogeneous directions** of the geometry. The **entanglement entropy** is **prop. to the geodesics length** in an **auxiliary spacetime**.

\[
S_A = \text{const.} \frac{\text{Length}(\Gamma')}{4G_N}
\]

\[
\tilde{g}_{\mu\nu} = \Omega(z, t, x)^2 g_{\mu\nu}
\]
Numerics: relax, don't shoot!

Geodesic equation as two point boundary value problem.

\[ \ddot{X}^\mu (\tau) + \Gamma^\mu_{\alpha \beta} \dot{X}^\alpha (\tau) \dot{X}^\beta (\tau) = 0 \]

BCs: \( (V(\pm1), Z(\pm1), X(\pm1)) = (t_0, 0, L/2) \)

- There are two standard numerical methods for solving two point boundary value problems.
  [see Numerical Recipes]

- **Shooting:**
  Very sensitive to initialization on asymptotic AdS spacetimes.

- **Relaxation:**
  Converges very fast if good initial geodesic is provided.
Isotropization of homogeneous plasma

A homogeneous but initially highly anisotropic (N=4 SYM) plasma relaxates to its isotropic equilibrium state. [Chesler-Yaffe 09]

The dual gravity model describes the formation of a black brane in an anisotropic AdS$_5$ geometry.
Geodesics in anisotropic $\text{AdS}_5$ black brane background

- Far-from equilibrium geodesics can go beyond the horizon.
- Near equilibrium geodesics stay outside the horizon.

[CE-Grumiller-Stricker 15]
Quasinormal ringdown of entanglement entropy

The late time dynamics of EE is captured by a single (complex) number:

\[ \frac{\omega_1}{\pi T} = \pm 3.1119452 - 2.746676i \]
Holographic shock wave collisions

HIC is modeled by two colliding sheets of energy with infinite extend in transverse direction and Gaussian profile in beam direction. [Chesler-Yaffe 10]
Wide vs. narrow shocks

Two qualitatively different dynamical regimes

- **Wide shocks (~RHIC): full stopping**
- **Narrow shocks (~LHC): transparency**

[Solana-Heller-Mateos-van der Schee 12]
Geodesics and apparent horizon

[CE-Grumiller-Van der Schee-Stanzer-Stricker 15?]
Two point functions

Two point functions for operators $O(t,x)$ of large conformal weight $\Delta$ can be computed from the length of geodesics. [Balasubramanian-Ross 00]

$$\langle O(t,x)O(t,x') \rangle \propto e^{-\Delta \text{Length}(\Gamma)}$$

Wide Shocks

Narrow Shocks

[CE-Grumiller-Van der Schee-Stanzer-Stricker 15?]
Entanglement entropy

\[ S_A = \text{const.} \frac{\text{Length}(\Gamma)}{4G_N} \]

Wide Shocks

Narrow Shocks

[CE-Grumiller-Van der Schee-Stanzer-Stricker 15?]
Summary

- The **near equilibrium dynamics** of holographic entanglement shows **quasinormal mode** behaviour. [CE-Grumiller-Stricker 15]

- In holographic **shock wave collisions** the **entanglement entropy** and the **two point function** may serve as **order parameter** for the **full stopping–transparency transition**. [CE-Grumiller-Van der Schee-Stanzer-Stricker 15?]

Ongoing Work

- **Going beyond supergravity**: string corrections, semi-holography, … [CE-Mukhopadhyay-Preiss-Rebhan-Stricker]

- **Better understanding of EE in HICs**: different shapes, higher dim. surfaces, other backgrounds, … [CE-Grumiller-Kapetanoski-Khavari-Stanzer]
Take home message

Complicated stuff in CFT often is very simple on the AdS side.

- **thermalization**
  - $T_{\mu\nu}$

- **entanglement entropy**
  - $\text{area of extremal surface}$
  - $\langle \mathcal{O}(t, x)\mathcal{O}(t, x') \rangle$

- **two point function**
  - $\text{length of geodesic}$

- **black hole formation**
  - $g_{\mu\nu}$

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Strong subadditivity

- A **fundamental property** of entanglement entropy is strong subadditivity.
- Hard to prove within QFT, very **intuitive** in the **dual gravity picture**.
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\[ S_A + S_B \geq S_{A \cup B} + S_{A \cap B} \]
Numerical check of strong subadditivity

\[ S_A + S_B - S_{A \cup B} + S_{A \cap B} \]

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