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Calculation of gravity terrain effects for the correction of sub-surface gyroscope observations

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Introduction

Geodetic zenith distance and horizontal angles measurements generally refer to the vertical to the local equipotential surface (plumb line). To correct these measurements for deflections of the vertical, both long-wave and short-wave contributions have to be considered. The long-wave contributions are modeled by means of a recent global potential SH-model (EGM2008) and the short-wave contributions are calculated from local mass attractions with the help of regional topography models. With this procedure topographic components of the vertical deflections can be calculated with an accuracy of ± 1". This technique is suited for surface points but also for establishing deflections of the vertical of sub-surface points (e.g for correcting gyro measurements in tunnels).

TOPOGRAV working scheme:

1. Import station file (surface or sub-surface points)
2. Import/ Select DEM (global or national)
3. Compare DEM height with station height (quality check for the height grid).
4. Select calculation method: prism method/DFT
   a. Surface points: use DEM height as station height
   b. Sub-surface points: use station height
   → calculate VD-components and gravity acceleration
5. Output is stored in ASCII format

TOPOGRAV

The software TOPOGRAV (see Fig.1) was established at TU-Vienna for the calculation of gravity terrain effects, such as:

- Vertical deflections (VD) - high and low frequency
- Gravity acceleration
- Bouger anomaly
- Orthometric correction

High frequency components are accounted for by using a digital elevation model (DEM), low- and middle frequency components by evaluating the 'Earth Gravitational Model 2008'.

Program structure:

- *Input*: Station file, DEM, parameters (e.g. density)
- *Visualization*: station coordinates and DEM
- *Output*: calculated values

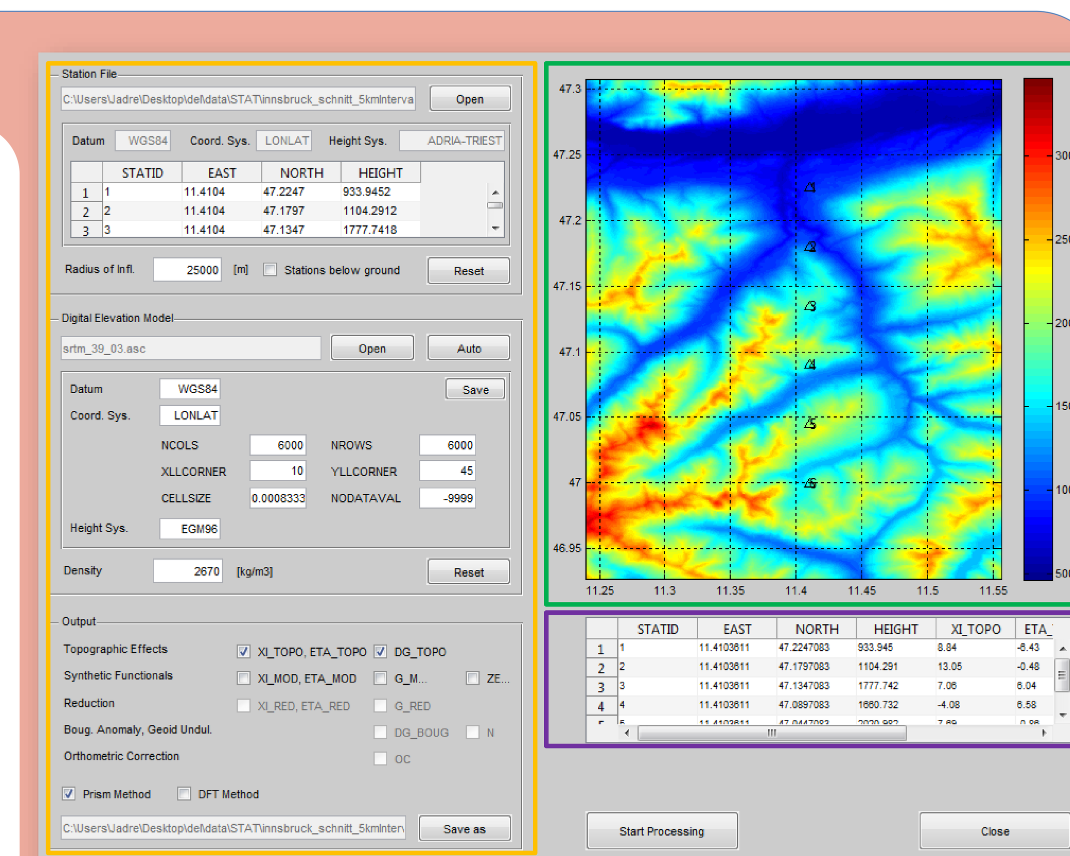

Figure 1: TOPOGRAV overview

Shuttle Radar Topography Mission (SRTM)

The Space Shuttle Endeavour launched on 11th of February 2000 collected in a 10-day mapping mission enough data to derive an almost complete global elevation model. The vertical error of the DEM's is reported to be less than 16m.

Specifications:

- produced by NASA
- combined solutions from satellites observations and free air anomalies
- with a spatial resolution of 30m or 90m (equator)
- extraction of terrain elevation by using interferometry
- offered in 5° x 5° tiles
- can be downloaded free of charge

TOPOGRAV allows the use of SRTM and local terrain models.

Calculation of sub-surface vertical deflection components

The software has been tested by introducing approximate station coordinates of an ongoing tunnel project located in the Alps of southern Austria (see Figure 2 and 3). For validation of TOPOGRAV surface VDs, we compared our results to observed VDs available in the Austrian VD database as well as VDs established by the EURALP project. Finally we estimated VDs for the tunnel trajectory. Figure 5 shows the topography and the location of the tunnel trajectory together with some calculated gravity terrain effects. The sub-surface VDs are depicted in Figure 4. They show a variation of up to 20" in the η-component and a negative tendency in the ξ-component induced by the geological masses located north of the tunnel trajectory. In order to validate TOPOGRAV, the vertical deflections at eight surface points were measured. The real measurements were then compared to values calculated by TOPOGRAV, see Figure 6. All Differences (except for 1) are within the error limits of ± 1,5". The RMS-error of the ξ- and η-component is 1,9" and 0,98" respectively.

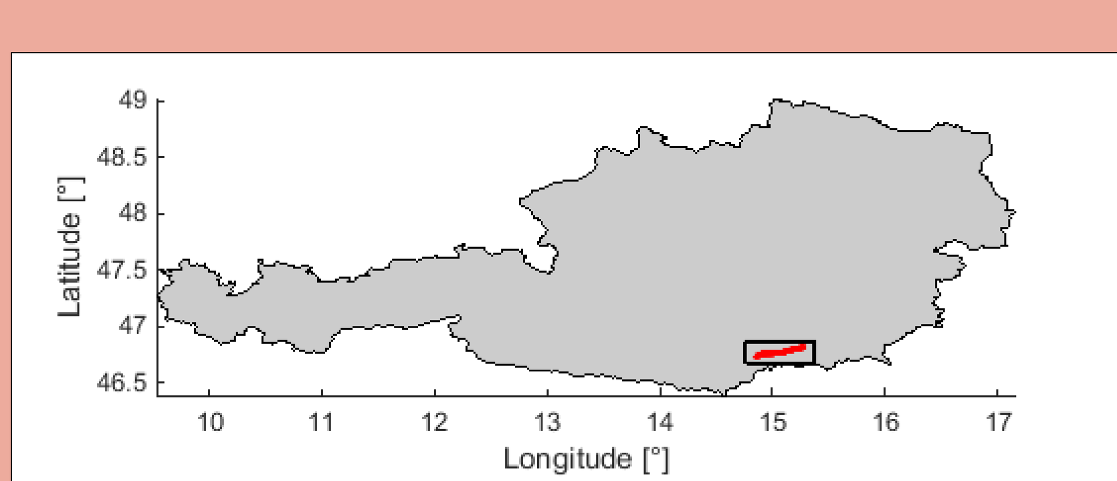

Figure 2: Location of the tunnel project in Austria with the DEM

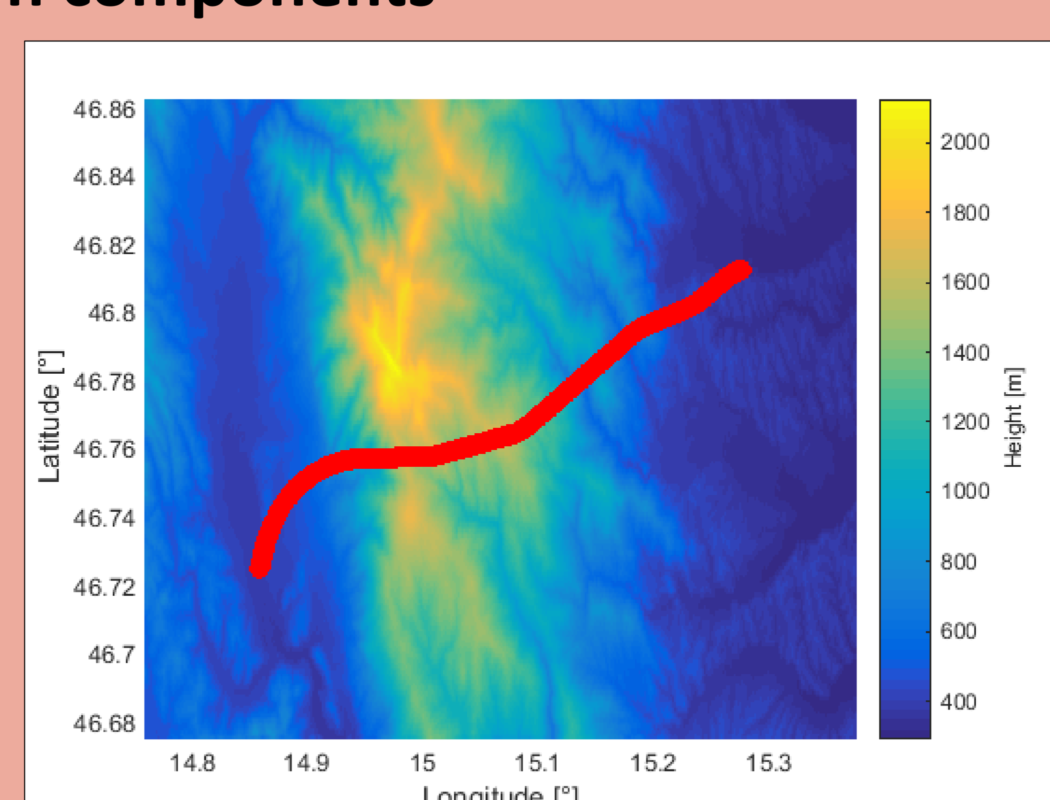

Figure 3: Trajectory of the investigated tunnel

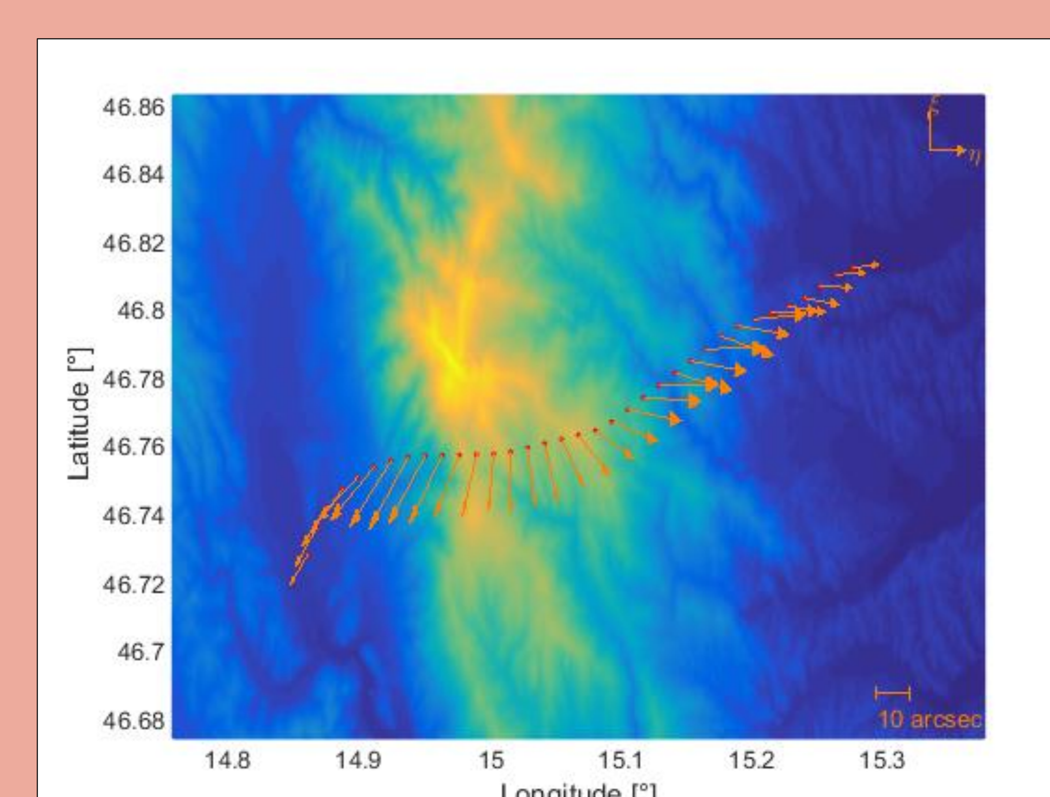

Figure 4: Direction vectors of the calculated VD for sub-surface points

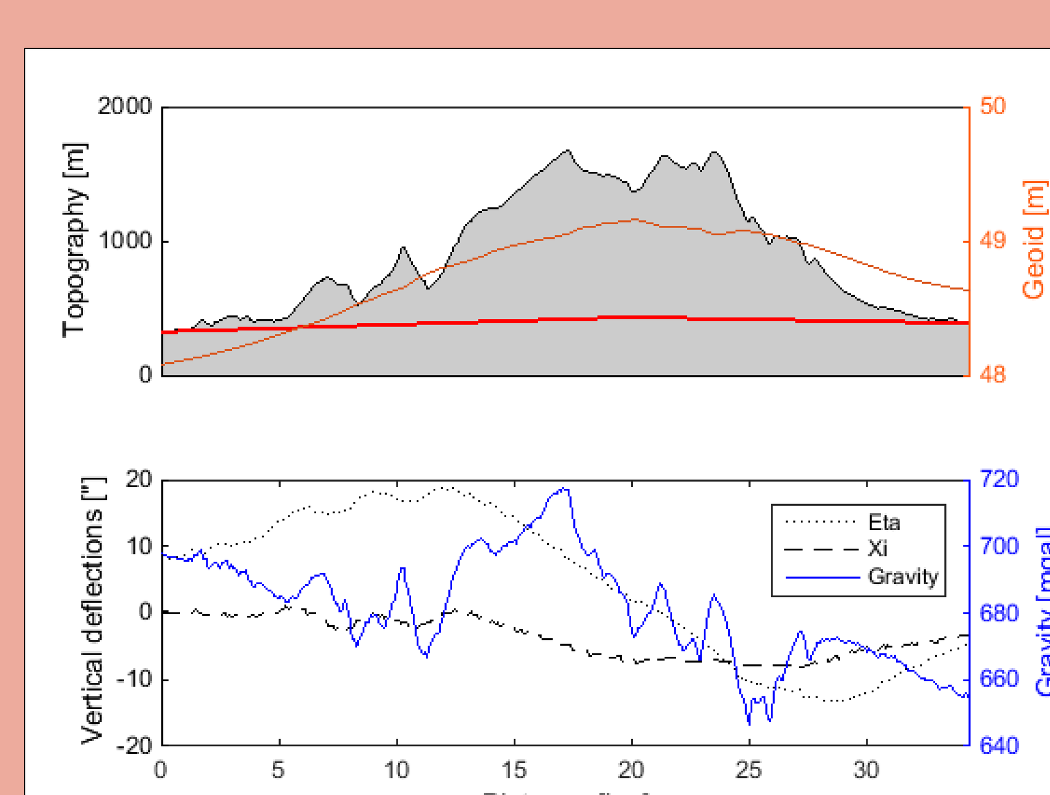

Figure 5: a) Topography at the project area with calculated geoid
b) Calculated sub-surface VD and gravity acceleration (-980 gal)

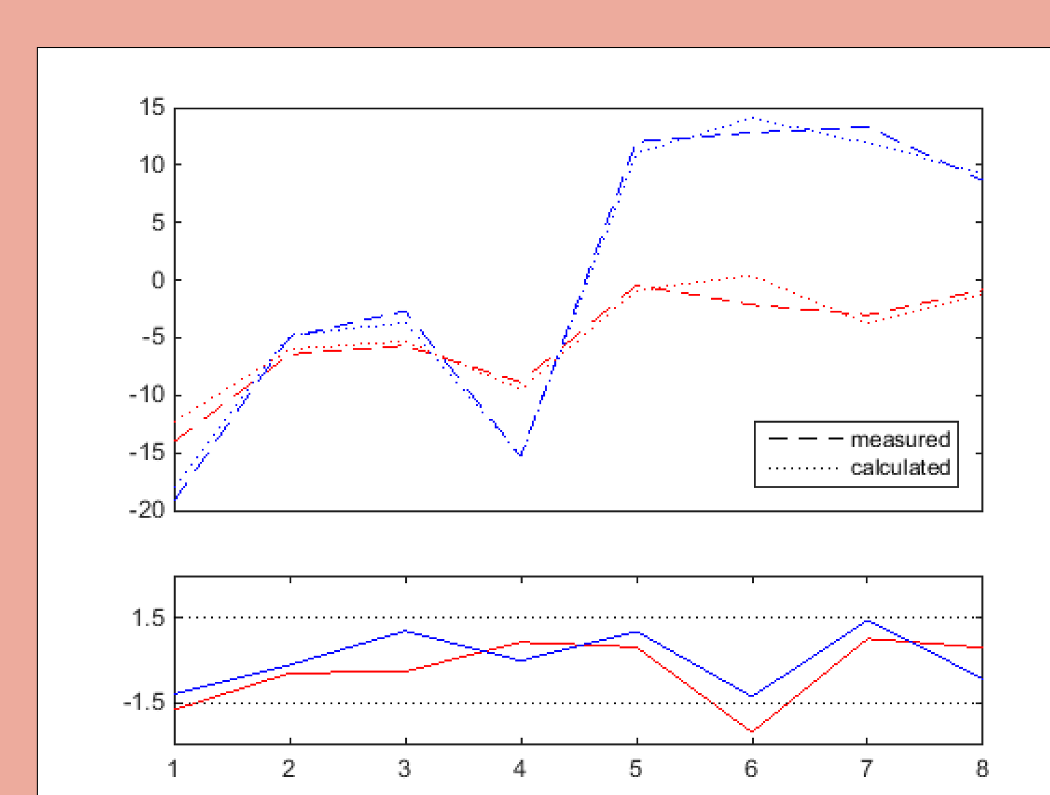

Figure 6: a) Comparison between calculated and measured VD (ξ in red and η in blue)
b) Difference between calculated and measured VD

Relation between perturbation potential and height anomaly, gravity anomaly and deflections of the vertical

The perturbation potential T at point P is defined as **T (P) = W (P) - U (P)** (difference from the gravitational + centrifugal potential and the normal gravity potential) and can be obtained from a spherical harmonic expansion. From the perturbation potential for a specific station height the parameters below can be derived:

- **Height anomaly** $\zeta = \frac{T}{\gamma_Q}$
- **Gravity anomaly** $\Delta g = -\frac{dT}{dr} - \frac{2}{r}T$
- **Deflections of the vertical** $\xi = -\frac{1}{\gamma r}\frac{\partial T}{\partial \varphi}$ $\eta = -\frac{1}{\gamma r \cos\varphi}\frac{\partial T}{\partial \lambda}$

These can be used to reduce tachymetric measurements, as well as for the correction of gyroscopic measurements.

References

- W. Torge, J. Müller (2012). Geodesy. De Gruyter , 4th Edition
- R.Weber (2014). Skriptum zur LV 'Theorie und Beobachtung des Erdschwerefeldes'
- http://srtm.csi.cgiar.org/