Conversion Gain for Interference Combating

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Abstract—This paper is written to treat interference combating techniques in a general way to clear terms, present applications and emphasize that combining cooperative and non-cooperative techniques to improve system-performance. The investigations are done under the restriction to minimize resources and to keep the system-complexity low to enhance the efficiency. The figure of merit is the conversion-gain and explained in detail.

I. INTRODUCTION

We assume that in the near future correlation detection will still be the most frequently used concept for data-detection. We try to make as less assumptions as possible about the interference to construct a robust and reliable communication complex. We want to handle AWGN-channels as well as mobile wireless channels. In each situation the decision instant SNR dictates the performance. As long as that is the case, any enhancement on SNR prior to the decision is advantageous. So interference combating techniques are a very attractive way to save resources. Additional channel coding is always possible for reliable communication, because it corrects wrong decisions and therefore it is out of the scope of this paper. We focus on the SNR improvement prior to the decision for the following assumptions: We try to be very efficient, if we compare the SNR-gain we achieve to the resources we have to spent.

The paper is organizes as follows: We start with a motivation to enhance SNR, followed by a brief review of correlation detection for an optimum situation (Gaussian noise). Based on the optimum solution we show what happen when non-Gaussian noise is present. We do the investigation for the worst case and show that that is an assumption that frequently occur in wireless communications (multipath-interference). So we need a philosophy that can handle multipath-interference and we choose the spread-spectrum technology. To reduce the spread-spectrum bandwidth, we use a non-linearity prior to spread-spectrum detection and exploit a structure that combines cooperative and non-cooperative interference reduction schemes. The overall SNR-gain that is achieved is characterized with a unique figure of merit, the conversion gain. That implies that the conversion-gain, strictly speaking, gives no hint how to realize it. Nevertheless it is useful to study it in a general manner. That is done in the next section. After that we present an example how to divide the conversion gain between cooperative and non-cooperative schemes to achieve a maximum on SNR with low resources invested.

II. WHY TO MAXIMIZE THE SNR PRIOR TO DETECTION

In this section we show that the bit-error-probability (BEP) is directly related to the SNR and that a higher SNR reduces the BEP. For the derivative we assume a synthesized binary communication system, matched filter detection and hard-symbol-decision. The SNR at the output of the matched-filter is given in (1). Due to the assumption of a fair binary-source and the symmetry of the Gaussian noise ($N \sim$ Gaussian random variable) we calculate the BEP as $Pr[Z \leq A]$. An estimate of the BEP can be derived with the Chebyshev’s inequality. The bound of the BEP is given in (2). We used the fact that the SOI (NRZ rectangular pulse with amplitude $A$ and duration $T_0$) is deterministic and the Gaussian noise is random. So the probability-density of the random variable (decision variable $Z$) is shifted by the SOI. In other words a deterministic variable can not change the standard-deviation of a random variable ($\sigma_n = \sigma_z$).

$$ SNR = \frac{A^2}{E[N^2]} = \frac{A^2}{\sigma_n^2} \tag{1} $$

$$ BEP = Pr[Z \leq A/2] \leq Pr[|Z - A| \geq A/2] = Pr\left[|Z - A| \geq \frac{A}{2\sigma_z} \cdot \sigma_z\right] \leq \frac{4\sigma_z^2}{A^2} = \frac{4}{SNR} \tag{2} $$

From (2) we see that an enhancement on SNR reduces the BEP and conclude that interference reduction lowers the BEP. That is the reason, why we exploit interference reduction for performance improvement.

III. CONVERSION GAIN

In the following discussion we are faced with signals we are interested in. We refer to such signals as signals of interest (SOI) and indicate them with $s(t)$. On signals we are not interested, we refer to them as signals of no interest (SONI) and indicate them with $n(t)$ if they are broadband in nature and we use $j(t)$ for any arbitrary waveform.

The conversion-gain is the ultimate figure of merit for each system if we want to discuss performance issues. In general, a system is characterized with its output to input behavior (output reaction to an input). A complex system is split into subsystems to handle the system more easily. For each subsystem a conversion-gain can be calculated. The
conversion gain is defined on a signal-to-noise (SNR) power-basis (for simplicity we did not distinguish between noise and interference, if we want to emphasize the difference we use SIR or SINR). That is useful because the reliability of a binary decision is based on the SNR presented at the input to the decision device. The SNR at the input of the decision device dictates the performance measured in BEP (see (2)). An interference reduction (IR) processor, sketched in Fig. 1, could be implemented as an add on in any communication receiver, prior to detection, to enhance the performance as long as the reliability of the decision is dependent on the SNR.

\[
G_c = \frac{SNR_y}{SNR_x} = \frac{P_J(P_S - \Delta P_S)}{P_S(P_J - \Delta P_J)} = \frac{1 - \frac{\Delta P_J}{P_S}}{1 - \frac{\Delta P_J}{P_J}} \tag{3}
\]

In (3) the relevant parameters are included to discuss the conversion gain to find conditions and clarify statements like: interference combating, interference reduction, interference suppression and interference mitigation. At the input of the IR-processor the power \( P_X \) appear as sum of SOI-power \( P_S \) and SONI-power \( P_J \). At its output the powers are modified indicated with \( \Delta \)-notation. In general an interference combating strategy degrades not only the SONI and may also corrupt the SOI. So interference combating is used in a general way without specifying the nature. The goal is to remove the SONI without significant distortion of the SOI, seen in (4). In our notation the condition for interference suppression is noted with: \( \Delta P_J \equiv P_J \) regardless of the damage of the SOI. The condition for interference reduction is: \( 0 < \Delta P_J < P_J \) and the reduction in SONI-power must overcome the power-loss of the SOI quantified with \( \Delta P_S \ll \Delta P_J \).

\[
\Delta P_J \equiv P_J \text{ and } \Delta P_S \ll \Delta P_J \quad \ldots \text{ IR goal} \tag{4}
\]

Now we switch to the discussion of remarkable IR-states, referring to (3). First we define the useless IR-state. The useless IR-state is characterized with \( \Delta P_J = 0 \). If \( \Delta P_S = 0 \) (SOI undistorted) we have effort without achieving an IR-advantage resulting in a conversion-gain of 1. Dependent on the damage of the SOI (\( \Delta P_S > 0 \)) it get even more worth (\( G_c = 1 - (\Delta P_S/P_S) < 1 \)). For the useless IR-state we gain no interference reduction and the SOI may be corrupted. The second state is the perfect IR state, the interference suppression. Interference suppression is characterized with \( \Delta P_J = P_J \), resulting in an infinite conversion-gain. For perfect interference removal it can be accepted that the SOI is corrupted (to a certain degree). The next state is termed useful IR-state and is characterized with: \( \Delta P_J \approx P_J \) and \( \Delta P_S \ll \Delta P_J \) hopefully \( \Delta P_S \approx 0 \) resulting in a conversion gain ranging from \( 1 < G_c < \infty \).
The efficiency of the conversion-gain, stimulated by the previous discussion is visualized in Fig. 2. A significant conversion-gain can be noted when the IR-scheme approaches the useful IR-state. In Fig 3 the area of useless (white area, \( G_c < 1 \)) and useful (\( G_c > 1 \)) conversion-gain is divided by the line indicated with \( G_c = 1 \). Fig. 4 shows in a clear manner the IR-states.

The conversion-gain is by its nature a unique measure (figure of merit, definition) and contain no information how it can be realized.

In the following we encounter that the spread-spectrum technology offers a conversion-gain based on a sophisticated bandwidth-spreading technique and it appears in the literature as processing-gain. The comparison of (3) with (18) shows that both are defined in the same way. A completely different way to achieve a conversion-gain is to exploit memoryless non-linearities prior to signal detection [3, 4].

In general we can combine processing-gain with the conversion-gain achieved with nonlinear preprocessing. That leads to the combination of cooperative and non-cooperative interference reduction, achieving a total conversion-gain \( G_T \). Due to the serial connection of the subsystems the total conversion-gain equals the product of the individual conversion-gains.

\[
G_T = \prod_{m=1}^{M} G_c^{(m)} \Rightarrow G_{T, dB} = \sum_{m=1}^{M} G_c^{(m)}
\]

We refer to the combination of cooperative and non-cooperative interference reduction as efficient interference reduction (EIR). The EIR concept can handle arbitrary jamming waveforms and offers flexibility in bandwidth assignment.

V. THE WAY TO ROBUST INTERFERENCE REDUCTION

As mentioned previously, we want to achieve a conversion-gain for changing interference conditions. So we are looking for robust communication concepts that deliver always a conversion-gain greater than 1. In the derivative we start with an optimum binary communication system and highlight the properties. The study of the detection process shows the way to NOMAC a robust communication system. From NOMAC we specify the spread-spectrum concept.

A. Optimum binary communication system

\[
G_p = T_0 \cdot B = G_c
\]

In the matched-filter example we have assumed that the interference is stationary AWGN and known in advance. So it is an optimum solution and the processing-gain is fixed and guaranteed. Now we show, that it is possible to maintain a processing-gain that the communication system is robust against any interfering waveform which offer a conversion-gain significantly greater than one, but not fixed. The gain is achieved by exploiting the degree of freedom to structure the SOI. That leads to the NOMAC concept (see beyond). The NOMAC concept allows also to distinguish between the multipath components and therefore it is able to cope with multipath-interference.

In the previous examples (matched-filter in Gaussian noise, NOMAC in non-Gaussian noise) we exploit the time-bandwidth-product of unstructured and structured SOIs. Now we want to save bandwidth and/or data-rate. That is achieved with nonlinear preprocessing prior to detection, exploiting the conversion-gain offered by memoryless nonlinearities.

Fig. 5. Model for binary baseband-transmission (antipodal signaling). Matched-filter implemented as correlator. \( D_n \in \{ \pm 1 \} \).

\[
r(t) = d(t) + n(t) = D_n \cdot s(t) + n(t)
\]

\[
x(t) = r(t) \cdot s(t) = D_n \cdot s^2(t) + n(t) \cdot s(t) = x_s(t) + x_j(t)
\]

\[
Z_n = [r, s] = D_n \cdot [s, s] + [s, n]
\]

We emphasize the detection procedure to gain the insight to be able to design a robust detection philosophy. The first step during detection is to reduce the waveform to the decision variable \( Z \), sketched in Fig. 5. The corresponding equation is (9) in which we use an operator for the inner-product (scalar-product) of functions \( Z = [x, y] \) with vector notation for the functions \( x(t) \) and \( y(t) \). For optimal detection we need a perfect knowledge about the interference, which we assume as AWGN. White Gaussian noise (WGN) has a constant power-density spectrum (PDS) \( G_n(f) = N_0/2 \) over the entire frequency range. The Wiener-Khintchine-theorem relates PDS to auto-correlation function (ACF) using Fourier-transformation. Applied to WGN we identify the ACF of WGN as Dirac impulse (delta-function) with strength \( N_0/2 \) in (10).
like ACF indicates that the energy of the signal appears only for a perfect match of the signal with itself and for any time-shift the energy drops to zero (pulse energy compression).

\[ \phi_{nm}(\tau) = \frac{N_0}{2} \delta(\tau) \Rightarrow G_n(f) = \frac{N_0}{2} \quad (10) \]

The ultimate goal of a binary communication system is to minimize the BEP, so we assume matched-filter detection. The matched-filter performance is equal to the performance of a synchronized (synchronization is maintained between the SOI in the received signal with the SOI as reference signal) correlation detector. The decision variable \( Z \) consists of two terms (11). One is related to the auto-correlation of the SOI and the other is related to the cross-correlation between SOI and SONI. The cross-correlation term should vanish, putting the energy of the SOI into the decision variable.

\[ Z = D_n \cdot [s, s] + [s, n(t)] \quad (11) \]

The performance of matched-filter detection is given in (12).

\[ \text{BEP} = Q \left( \frac{A}{\sigma_n} \right) = Q \left( \sqrt{\frac{2E}{N_0}} \right) \quad (12) \]

We conclude, as usual for matched-filter detection, that the BEP is only dependent on the energy of the SOI. More specifically: equal energy SOI-signals achieve the same interference environment (AWGN) the same performance, regardless of their specific shape. So we gain the matched-filter advantage, that is the freedom to design the SOI to achieve the desired attributes for a certain application. We will come back and exploit that degree of freedom later.

So far we knew that correlation detection is optimum in AWGN. Before we continue our derivation to robust communications we want to raise a legal question from the designers point of view. What is the worst case interference? To evaluate that we use variation calculus \((\alpha_1, \alpha_2 \ldots \text{variation variables})\) in the sums of the decision variable (13). The worst case interference is, by its nature, a situation that fools the detector.

\[ Z = \alpha_1 \cdot D_n \cdot [s, s] + \alpha_2 \cdot [s, n] \quad (13) \]

\[ n = -D_n \cdot s \quad \ldots \text{worst case} \quad (14) \]

Observations form (13) lead to the conclusion that the detector is fooled, when the interfering waveform is SOI like but from opposite data-bit polarity (14). Mathematically speaking a wrong binary decision appear when \(\alpha_2 > \alpha_1\). The important question now is, is that a situation that occur in real communication scenarios? If yes how frequently? So the physical interpretation of the worst case situation is a multipath situation (14), sketched in Fig.6, that occur frequently in wireless terrestrial mobile applications. Now we knew the worst case scenario and we must find a solution to it, if we want to design a robust communication scheme. Why is it impossible for the correlation detector to resolve multipath components?

We approach the solution by inspecting the detection process with sketched waveforms to visualize what is going on during the correlation detection procedure to gain insight into the mechanisms working on the SOI and the SONI. That is sketched in Fig.7. Before we focus on that, we briefly recapitulate the tasks of the modem-section. In the transmitter the binary information is (embedded) mapped to a waveform with a time duration of one symbol \(T_0\). The most simplest mapping (coding) for carrying binary information is to map the information to the polarity of the waveform. That is achieved with a simple rectangular waveform (rectangular pulse \(p(t/T_0)\), NRZ). Within the pulse duration the waveform is constant, reflecting the polarity-information. The SOI is completely specified by the DC level which is in contrast to the interference which is (assumed) DC less. So the information was embedded into the SOI with an attribute (parameter) the SONI don’t have. The reason for that is, that the detector needs only to extract the polarity of the DC level from the received signal to recover the information because he assumes that it is something that is related to the SOI. Generally speaking: As long as the SONI has no possibility to appear at the input to the integrator (input to the detector if integrate and dump is used) with a DC level we expect no problem. The SOI is feed to the channel and corrupted by the SONI, modeled as white Gaussian noise. The mechanism the two waveforms combine in the channel is additive (AWGN model). As theory states the optimum solution to recover the binary information is correlation/matched-filter detection. The correlator operates on signals and the result is a measure (real number) of the similarity between the signals. The correlator multiplies the received signal with the SOI (the signal he want to detect) and integrate the result to deliver a real number (match between SOI and the received signal) to the decision device. By the way, the absolute value of the real number (correlator output at the sample time) indicates the probability of the decision variable, but is not exploited in the hard-decision strategy. The hard decision map the polarity of the real number to a binary number (reversing process, extracting the binary information from the waveform). Due to the linearity-property of integration we can formally split the correlation into two independent sub-correlation processes, sketched in Fig.7 and in accordance with (11). One sub-correlation processes the SOI (left column) and the other sub-correlation processes the SONI (right column). That is practically impossible, but for the understanding how the detector handles the SOI and the SONI extremely useful (see (8)).

The processing of the SOI (NRZ-pulse) shows that the multiplication with unity (reference) don’t change the SOI
waveform and the subsequent integration turns the DC-level into a ramp-function. That makes the SOI-part (polarity-information) in the decision variable linearly stronger with time. The processing of the SONI, a DC-less white Gaussian noise waveform drawn as a sample function from a Gaussian random process, appears also unchanged after multiplication with the reference (SOI). The nature of the noise waveform is purely random. So many small positive and negative areas change frequently in time and the summation behavior of integration averages the resulting area under the SONI to zero, its DC-level. So the essence of the SNR-maximization behavior is based on the averaging out of the SONI. Now it is evident that the SONI has to be DC-less and from random nature. In other words the nature of the SONI itself achieves the interference reduction capability. Once more: To achieve the IR capability the SOI must appear at the integrator input with a strong DC-component and the SONI must appear as DC-less random signals. That is the global behavior of IR in correlation detection.

B. NOMAC-Concept

Now the previously discussed global behavior is applied in such a way that it is always possible to achieve (enforce) a conversion-gain greater then one, independent of the SONI-waveform [1]. In principle the SOI/SONI-relation is changed and the freedom of matched-filter detection is exploited to be free with the structure of the SOI. The straightforward implementation is to use a random waveform as SOI (sample function of a white Gaussian noise source). Related to the random nature of the SOI this type of modulation is termed: Noise modulation and correlation (NOMAC).

We derive the NOMAC-system, sketched in Fig.8 from Fig.7 when we exchange in the right column (SONI) the two top waveforms (input and reference), which leaves the result in the right column unchanged. So the global behavior of IR is preserved. Additionally we have solved the problem with a DC-component originated form an interfering waveform. That enables the NOMAC-receiver to combat echos in a multipath environment. That is the reason why this type of receiver is called a multipath-rejection receiver (MRR). In general the MRR copes with any type of interference, so it is a blind IR-scheme and termed robust communication complex.

For the NOMAC-concept we raise the question about the gain of robustness against unspecified waveforms. From the previous explanation it is obvious that the gain is not constant for all possible interfering waveforms. We are especially interested on two interfering-waveforms: (a) WGN and (b) multipath. The first question is easily answered. WGN is the prototype of randomness, so it could not be further enhanced and the introduced additional polarity changes from the SOI do no significant harm to the randomness. So in AWGN the NOMAC-concept can not gain any advantage compared to an optimized system in AWGN (for equal signal conditions and equal symbol-rates). So AWGN serves as a baseline for comparison. The multipath interference can be handled superior (10). The wanted direct-path results in a DC-value, corresponding to the product of the SOI, prior to integration (after integration we have the mean value of the product) and the echo (SONI) is chopped and averaged out by the subsequently following integration.

The previous question is connected to the power-density relations in the NOMAC-system. We make the following assumptions: The channel-bandwidth is set equal to the SOI bandwidth \( B_{\text{spread}} = B_{\text{SOI}} \). The information-bandwidth is set to the reciprocal of the data symbol-duration \( B_{\inf} = T_{\text{s}}^{-1} \). The structure and signaling of the NOMAC-system is the same as for the optimized system in AWGN, sketched in Fig.5. The essential difference is that \( s(t) \) is a noise-like signal and the interference is not restricted to AWGN, it can be any jamming-waveform \( j(t) \). From the structure we derive, that the information signal is multiplied twice with the SOI and the arbitrary jamming-signal \( j(t) \) is multiplied once by the SOI. In the frequency domain the SOI (transmitter) spread out the information-bandwidth \( (B_{\text{SOI}} / B_{\inf} \gg 1) \). The NOMAC-receiver compresses it back to its origin bandwidth \( B_{\inf} \) without loss. That means that the SONI, if it is confined to the information-bandwidth, spreads out roughly to the SOI bandwidth (more precisely \( B_{\text{SOI}} + B_{\inf} \)). That results in a
SONI-power reduction corresponding to the bandwidth ratio (15) defining the processing-gain. We exploited the matched-filter freedom, to construct the signal as we like, as long as the energy is the same it has the same BEP. We structured the SOI and achieved a time/bandwidth-product, the processing-gain.

\[ G_p = \frac{B_{\text{spread}}}{B_{\text{inf}}} = T_0 \cdot B_{\text{spread}} \quad \text{... processing-gain} \quad (15) \]

If the processing-gain is large, then the information-bandwidth of the spread-signal is roughly constant and the resulting power-density of the SONI can be modeled as an approximation of WGN (16). The BEP depends on the product of the processing-gain with the SNR at the detector input shown in (18).

\[ J_0 = \frac{P_J}{B_{\text{spread}}} \quad \text{... SONI-PSD} \quad (16) \]

\[ \text{BEP} = Q\left(\sqrt{G_p \cdot \text{SNR}_{\text{in}}}\right) \quad (17) \]

Inserting (15) in (18) shows the equivalence in defining conversion-gain and processing-gain.

\[ \text{SNR}_{\text{out}} = \frac{E}{N_0} = \frac{P_{\text{SOI}}/B_{\text{inf}}}{P_{\text{SONI}}/B_{\text{ss}}} = G_p \cdot \text{SNR}_{\text{in}} \quad (18) \]

From a theoretical point of view the derivative of the spread-spectrum concept is based on the matched-filter degree of freedom about the SOI waveform. We use a structured waveform to achieve the following two goals: (a) Interference reduction for almost any type of SONI-waveforms (blind IR) and (b) the accurate time resolution to resolve multipath components. That can only be achieved with a noise-like waveform. The Dirac-like ACF shows that only for perfect alignment the energy of the SOI appears. That indicates that the SOI detection-process is a pulse compression technique.

We think that the robustness-property is responsible for the more than 60 years use of the NOMAC-concept and is still attractive today in his commercialized form, the spread-spectrum technology. We can also say that the spread-spectrum concept is the digital version (quantized SOI-waveform) of NOMAC.

VI. EFFICIENT INTERFERENCE REDUCTION

Efficient interference reduction combines cooperative and non-cooperative interference reduction schemes as indicated in Fig.9 and is a representative of robust and reliable communication. The cooperative part belongs to the spread-spectrum implementation and the non-cooperative part is implemented with a memoryless nonlinearity. The processing-gain as well as the conversion-gain are dependent on the actual interfering waveform and therefore not fixed, but each of them are always greater than one. The total conversion-gain in (19) is the product of the processing-gain and the conversion-gain. It is possible to trade bandwidth-reduction with conversion-gain.

\[ G_T = G_c \cdot G_p \quad (19) \]

More information and performance-curves are provided in [2], [3], [4], [5].

VII. CONCLUSION

We have presented and defined frequently used terms related to interference combating. We have shown that the conversion-gain is a suitable measure to quantify the quality of any interference combating scheme. We have derived the conversion-gain of a robust communication system. We have presented applications of different interference combating philosophies and that the combination of cooperative and non-cooperative schemes improves the conversion-gain and offers flexibility in the bandwidth assignment.

REFERENCES