Online Parameter Identification for Traffic Simulation via Eulerian and Lagrangian Sensing

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Abstract

This paper deals with parameter identification for traffic models. Two fundamentally different approaches are presented, which are combined to the best advantage. With Eulerian sensing the sensors are spatially stationary, i.e. fixed in place, while with Lagrangian sensing the sensors move with the traffic flow as sensor-vehicles. We present a method to identify model parameters from both methods individually and investigate identifiability via the Fisher Information Matrix. Additionally an approach to combine both methods is presented, which is of particular importance where the data quality of one method is not sufficient to identify all model parameters. Results for both Eulerian and Lagrangian sensing as well as the combined method are presented.

Keywords: Lagrangian sensing, Optimization, Fisher Information, Identification

1. Introduction

Large scale traffic simulations are based on a macroscopic traffic model, where the vehicle flow is described by the transport equation, treating traffic like a compressible fluid. The relationship between the speed of the flow and the local traffic density is described by a fundamental diagram (FD) [1]. We use a first order numerical scheme that works with an arbitrary, piecewise differentiable FD [3]. To identify the traffic model parameters, e.g. the parameters of the FD, we formulate an optimization problem based on a traffic model and data collected from sensors. Traditionally, stationary loop sensors are used to gather measurement data, representing the Eulerian sensing (ES) approach. The main drawback of this method is the expensive installation, especially for urban networks, where significantly more sensors are required than for highway applications. Hence, Lagrangian sensing (LS) via mobile sensors moving with the flow is an active research topic. Both especially equipped vehicles, so-called floating cars, and smartphones are used.

We use the aforementioned macroscopic traffic model in combination with ES, and a simple microscopic model in combination with LS to keep computational costs low. At this point the quality of the collected data is critical. Utilizing the Fisher Information Matrix (FIM) for ES optimum stationary sensor placement for a range of realistic traffic situations can be determined. Additionally, parameters that can not be identified via ES alone may be identified via LS to yield a sufficiently accurate set of model parameters for the macroscopic traffic model.

2. Macroscopic Traffic Model

The macroscopic description of traffic is based on the transport equation, a hyperbolic partial differential equation

$$\frac{\partial q(x,t)}{\partial t} + \frac{\partial \phi(q(x,t))}{\partial x} = 0, \quad x \in \mathbb{R}, t > 0 \tag{1}$$

where q represents the vehicle density in [veh./m], x and t represent the spatial and temporal coordinates respectively, and $\phi(q)$ represents the flux function

$$\phi(x,t,q) = f(q) = q(x,t) \cdot v(q, \boldsymbol{\theta}), \quad \text{[veh./s]} \tag{2}$$

based on the (arbitrary) FD $v(q)$. The vector $\boldsymbol{\theta}$ collects the parameters of the FD.

In order to solve Eq. (1) the spatial domain is discretized into cells along the length of a given road section. The vehicle balance for each cell i at a discrete time n is given by

$$\hat{q}_i^{n+1} = \hat{q}_i^n + (\phi_{i-1}^n - \phi_i^n)\frac{\Delta t}{\Delta x}, \quad i \in \mathcal{D}, n \geq 0 \tag{3}$$

where ϕ_{i-1}^n denotes the the flux entering cell i from the left, ϕ_i^n denotes the flux exiting cell i and $\mathcal{D}$ denotes the discrete spatial domain. The fraction $\frac{\Delta t}{\Delta x}$ accounts for temporal and spatial increments respectively that are restricted by the Courant-Friedrichs-Lewy (CFL) condition for convergence. Meeting the CFL condition ensures that the numerical domain of dependence includes the physical domain of dependence.

3. Microscopic Traffic Model

An auto-regressive car-following model where a floating car *follows* a reference vehicle is used. The position $\hat{x}$ of the floating car at time step $n+1$ is updated according to the model equation

$$\hat{x}^{n+1} = \hat{x}^n + v(h, \boldsymbol{\theta}) \cdot \Delta t \quad \text{with } h = q^{-1} = x_{ref} - \hat{x} \tag{4}$$

where $v(h, \boldsymbol{\theta})$ denotes the current velocity, which depends on the local traffic density represented by the measured headway h with respect to the reference vehicle at x_{ref} and the parameters $\boldsymbol{\theta}$ of the FD.

4. Parameter Identification

The set of model parameters $\boldsymbol{\theta}$ is identified via optimization. The model with its boundary conditions (BC) and initial conditions represents the *system*. The *optimization goal* is to minimize the error J_{OE} computed by the *objective function* for the given system via variation of the *decision variables* $\boldsymbol{\theta}$ while observing *constraints* on $\boldsymbol{\theta}$. For ES, the objective function evaluates the normalized output error defined by

$$J_{OE,ES} = \sum_{s,n} \left\| \frac{(\hat{q}_{j,s}^n - q_{j,s}^n)}{\max(q_{j,s})} \right\|_2^2 \tag{5}$$

where the sensor locations are denoted by s and the measurement data from a traffic sensor at a location (j, s) is denoted $q^n_{j,s}$.

Equivalently, for LS the cumulative prediction error of the model is defined by

$$J_{OE,LS} = \sum_{j,n} (\hat{x}^n_j - x^n_j)^2 \tag{6}$$

where x^n_j represents the *true* position of the floating car j at time step n.

4.1. Parameter Sensitivity

The parameter sensitivity vector $\boldsymbol{\psi}^n_{i,OE}$ is defined by the total derivative of the output with respect to $\boldsymbol{\theta}$,

$$\boldsymbol{\psi}^n_{i,OE,ES} = \frac{d\hat{q}^n_i(\boldsymbol{\theta})}{d\boldsymbol{\theta}} = \frac{\partial \hat{q}^n_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} + \frac{\partial \hat{q}^n_i(\boldsymbol{\theta})}{\partial \boldsymbol{x}^n(\boldsymbol{\theta})} \frac{d\boldsymbol{x}^n(\boldsymbol{\theta})}{d\boldsymbol{\theta}} \tag{7}$$

with $\hat{q}^n_i = f(\hat{q}^{n-1}_{i-1}, \hat{q}^{n-1}_i, \hat{q}^{n-1}_{i+1}, \boldsymbol{\theta})$ as defined by Eq. (3), and $\boldsymbol{x}^n(\boldsymbol{\theta})$ denoting the regressor vector containing the corresponding past model outputs $\boldsymbol{x}^n(\boldsymbol{\theta}) = \begin{bmatrix} \hat{q}^{n-1}_{i-1} & \hat{q}^{n-1}_i & \hat{q}^{n-1}_{i+1} \end{bmatrix}^T$.

For LS and a vehicle j, Eqn (7) and the chain rule yield

$$\boldsymbol{\psi}^n_{j,OE,LS} = \frac{d\hat{x}^n_j(q, \boldsymbol{\theta})}{d\boldsymbol{\theta}} = \frac{d\hat{x}^{n-1}_j(\boldsymbol{\theta})}{d\boldsymbol{\theta}} + \Delta t \frac{d\hat{v}^{n-1}_j(q(\boldsymbol{\theta}), \boldsymbol{\theta})}{d\boldsymbol{\theta}}. \tag{8}$$

4.2. Identifiability

To asses identifiability of parameters we utilize the FIM in the *output error* (OE) configuration [2] for both ES and LS defined by

$$\boldsymbol{\mathcal{I}}_{OE} = \frac{1}{\sigma^2} \sum_{i,n} \left(\boldsymbol{\psi}^n_{i,OE}\right)^T \left(\boldsymbol{\psi}^n_{i,OE}\right) \tag{9}$$

where σ^2 is the (minimum) standard deviation of the estimated parameters.

We use a scalar criterion $J_S = \sigma_{min}(\boldsymbol{\mathcal{I}})$ for evaluation where the smallest singular value σ_i of $\boldsymbol{\mathcal{I}}$ is maximized. A singular value $\sigma_i = 0$ of $\boldsymbol{\mathcal{I}}$ indicates a non-influential (hence, non-identifiable) right-singular vector $\boldsymbol{v_i}$, which in turn corresponds to non-identifiable parameter(s) of $\boldsymbol{\theta}$.

5. Mixed sensing

Ill conditioned singular values indicate that some parameters can not be estimated well from the data set at hand, hence, the data quality is insufficient. However, parameters that can be identified from a certain measurement data set may still be used. Utilizing both the estimated parameters found based on Lagrangian and Eulerian sensing allows for high accuracy while at the same time keeping sensor-costs low. Blending of parameter sets is performed via weights $w_{ES,LS} \in [0, 1]$, which depend on the corresponding (relative) σ_i and $\boldsymbol{v_i}$. Note here that blending algorithms must take spatial and temporal relevance into account as well.

The example illustrates insufficient Eulerian data, which is complemented by data form one floating vehicle, such that all parameters $\boldsymbol{\theta}$ can be estimated reliably.

6. Example

We consider a single-lane road with a total length of 2500m and a speed limit of 13.89m/s (50km/h). The spatial increment $\Delta x = 50$m, the temporal increment is $\Delta t = 1$s, with a total simulation time of $t_{end} = 600$s. The road is empty initially, vehicles enter at the left boundary and stop at the right boundary (red light). Results are validated via simulation with VISSIM®.

We use one sensor location at position $x = 2400$m, recording density values at every time step during the simulation. The collected data is used in Eq. (5). Evaluating Eq. (9) shows that not all parameters can be identified properly. Namely, parameters corresponding to the high-speed-low-density part of the FD are affected. Parameter identification through optimization itself is done by genetic optimization.

Utilizing one additional floating car from within the first vehicle platoon provides data to reliably identify the high-speed-low-density parameters, while, on its own, data from this floating car is insufficient to identify the remaining parameters of the FD. Combining both sets of parameters weighted by σ_i only, since both spatial and temporal relevance is given in this example yields a reliable parameter vector. Spatial and temporal relevance depend strongly on weather conditions and traffic incidents.

The resulting optimum parameter vector $\boldsymbol{\theta}_p$ is validated via simulation. The results together with the reference solution generated by VISSIM® at various time steps are depicted in Fig. 1.

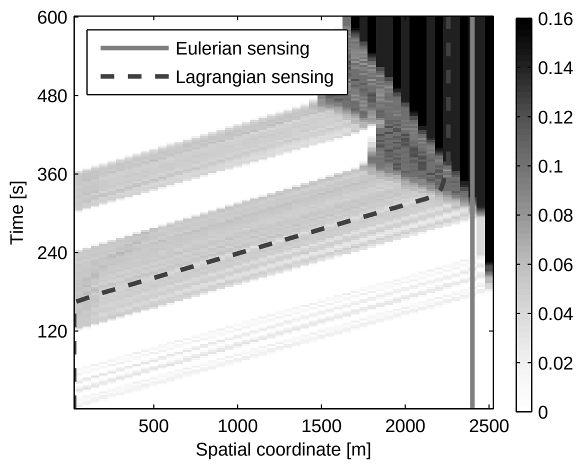

Figure 1: Traffic density over time, lines indicating ES and LS

7. Conclusion

We briefly introduced Eulerian and Lagrangian sensing and their relevant application in parameter identification for traffic simulations. Parameter sensitivity and identifiability are investigated and a blending method is proposed. Results are validated via simulation, however, real measurement data is currently collected for rigorous validation. LS data will become readily accessible as vehicles are equipped with ACC and navigations systems and Car2Infrastructure communication becomes standard equipment.

References

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