

COUPLING BUILDING MORPHOLOGY OPTIMIZATION AND ENERGY EFFICIENCY – A PROOF OF CONCEPT

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ABSTRACT

This contribution focuses on recent efforts towards coupling building morphology optimization and energy efficiency computation in the context of early stage planning of complex buildings, using the hospital domain as a showcase. In more detail, a NetLogo model was conceived to generate typological design variations based on a grid grammar. Each resulting design was subjected to a fitness test based on a coupled energy performance model, which is unprecedented for early stage form finding to the best of our knowledge. However, given that hospitals require a large amount of energy for operation, energy efficiency considerations should indeed be implemented as early as possible within the design process. The given paper presents methodology and early results of these efforts.

INTRODUCTION

Problem. Automated floor planning algorithms transform a given space programme into a (two- or three-dimensional) layout, using prescribed areas and adjacency relations as requirements. The quality of a solution - i.e. its *fitness*, is determined by comparing a generated layout against the specification. Repeated generation and evaluation of this fitness allows for optimization of the floor plan and thus of the spatial arrangement. Such an approach is especially helpful in case of complex buildings - e.g. hospitals, airports, industrial facilities, in which there are too many spaces governed by adjacency relationships to be satisfied manually.

The generation of building morphology, on the other hand, does not necessarily deal with optimization in the previous sense - the overall goal lies in the generation of form. We wish to argue that complex buildings may equally well benefit from an *optimization of the building envelope*, since this is directly connected to factors such as energy efficiency, extensibility and visibility within the urban context. Furthermore, establishing an optimized building envelope before applying automated floor planning is beneficial for the overall workflow, since the building shape can act as boundary condition for space layout.

Contribution. We wish to showcase a *technique for coupling morphology generation with energy performance evaluation* in a manner employable for a wide range of applications. In more detail (see 'Simulation and Experiment'),

1. our approach generates different building typologies given an intended building volume, using a three-dimensional cell-space grammar implemented in NetLogo (see Subsection 'Typological Cell Grammar')
2. for each individual solution, the energy performance is computed on the fly, by communicating with an energy performance model written Excel (see Subsections 'NetLogoExcelBridge' and 'Energy Performance Model')
3. the calculated performance forms part of a solution's fitness value, which may additionally take measures calculated inside the morphology generator into account (see Subsection 'Fitness Calculation')
4. repeating generation and evaluation over the whole solution space, we may find the most suitable typologies for the specific building spot and intended building volume in question (see Subsection 'Optimization')

Our showcase is performed in the context of early stage hospital planning, since hospitals require a large amounts of energy for their operation. The analysis of generated morphologies shows that a rating by energy performance alone is not enough for producing "interesting" forms (see 'Analysis of Results'); an additional prescription of a volume in which building should proceed improves the quality of results in that respect (see 'Discussion') and can furthermore be used to honor adherence to zoning regulations and/or design intent.

RELATED WORK

Automated floor planning based on pre-existing space programs has been investigated since the late 1960ies (eg. Buffa et al. 1964, Mitchell and Dillon

1972 in two dimensions; Weinzapfel et al. 1971 in three dimensions). The general task in all of these approaches was to distribute spaces such that the required areas were met, also considering their relative locations prescribed by an adjacency matrix. In doing so, authors would get a single solution having a certain degree of fitness (i.e. congruence between requirements and generated spatial arrangement). Optimizing floor plans based on the fitness of many solutions became feasible with growing computing power, using Genetic/Evolutionary algorithms as means (e.g. Gero and Kazakov 1998; Elezkurta and Franck 1999).

In parallel, *generation of building morphology* was being made popular by work on Shape Grammars (e.g. Stiny and Gips 1972, Stiny and Mitchell 1978 for the two-dimensional case; Duarte 2003 in three dimensions). However, fitness testing and/or optimization of the generated designs was rarely done in that context. Notable exceptions were Gero and Louis (1995) as well as Chouchoulas (2003), who worked on evolutionary shape grammars, and Rosenman (1997), who published about growing polyominoes by adding edges iteratively using an evolutionary approach.

To put these approaches into a more recent perspective, the ubiquity of parametric design tools such as Grasshopper - which has its own genetic solver Galapagos, would seem to suggest that optimization could easily be added to *generative* design (for shape grammars e.g. GRAPE by Grasl and Economou 2011, for which a plugin exists). However, this is currently not reported in literature, to the best of the authors' knowledge. It is, however, certain that designers actively use optimization in *parametric* design, e.g. in the form of the GECO plugin (Grabner and Frick 2013) that links Grasshopper to Ecotect for performance-based design based on environmental simulation.

Another recent research effort that can be considered as related is the SEMERGY project (Pont et al. 2014). In this research effort, building energy evaluation was coupled with semantic web technologies for building envelope optimization. However, to limit the space of potential solutions, the SEMERGY environment focused on applied building materials and products rather than on geometry/morphology optimization.

SIMULATION AND EXPERIMENT

Typological Cell Grammar

In previous work (Wurzer and Lorenz 2016) a simple cell-space grammar was developed that can generate 'typological' designs - designs that resemble classical typologies as depicted in Figure 1, but are three-dimensional.

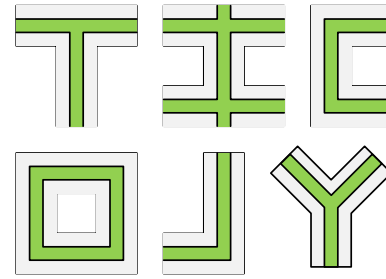


Figure 1: Typologies

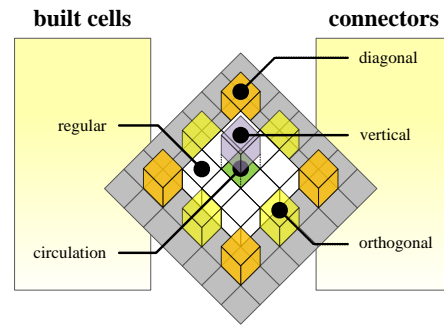


Figure 2: Megacell

The approach consecutively places "Megacells" consisting of 3x3 *built cells* and surrounding connectors (see Figure 2). Built cells can either be of *regular* (white) or *circulation* type (green), while connector can be *orthogonal* (yellow), *diagonal* (orange) or *vertical* (purple). Our shape derivation proceeds in the following steps (refer to Figure 3):

Start Step. The algorithm is given a budget (B_C) that states how many cells it should place. It then chooses an arbitrary position within the base level of the three-dimensional lattice ($z=0$) and places a Megacell, subtracting $3 \times 3 = 9$ cells from the budget. As long as this is greater or equal to 9, it execute steps A, B and C.

Step A. A connector is chosen by probability (P_o , P_d and P_v corresponding to preference for orthogonal, diagonal or vertical connectors). If it is not possible to build in that direction (e.g. building spot boundary hit), we choose an arbitrary free connector.

Step B. Retrieves the new center of the 3x3 built cells lying in the direction of the connector.

Step C. Places a Megacell around this center. The center becomes a circulation cell, the surrounding cells become regular built cells lest they are connectors, in which case they become circulation cells. 9 cells are deducted from the budget.

Finalization Step. All connector cells are cleared.

Since we are dealing with the three-dimensional case, additional criteria apply for the vertical axis: We may only place a Megacell if it is (a) situated at base level $z=0$ or (b) if there is already a Megacell at most k levels below. k is called the *cantilever setting* that

specifies over how many floors may be left empty between protrusions.

Figure 4 shows an example output of our grammar. Depending on the cell budget given and the three probabilities, we can either get "simple" typologies as in Figure 1, or "complex" ones as shown.

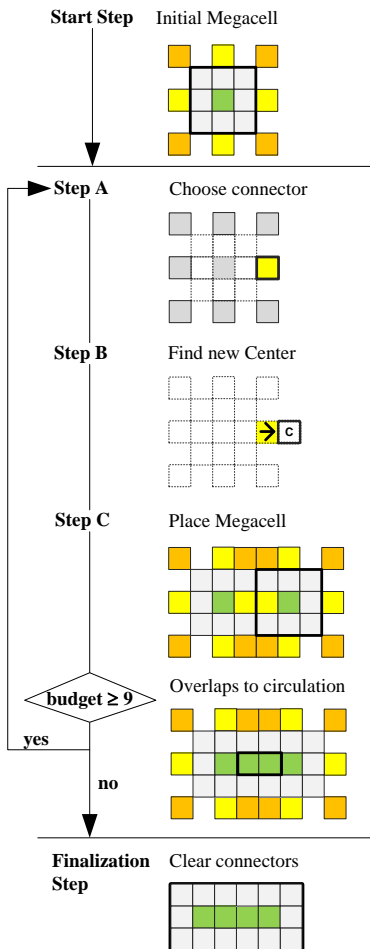


Figure 3: Typological Cell Grammar

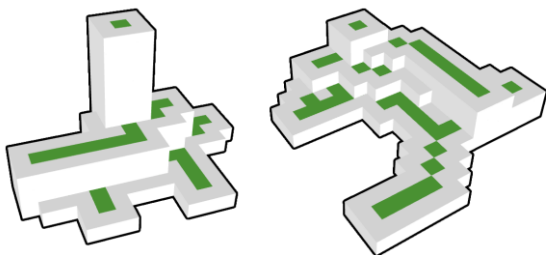


Figure 4: Example Output

The grammar was implemented using the NetLogo Simulation Platform (Wilensky 1999). In the next section, we show how to link NetLogo to Excel, in order to showcase how off-the-shelf spreadsheet calculations such as our energy performance model can be harnessed for computing part of a solution's fitness.

NetLogoExcelBridge

In hospital planning, spreadsheets are commonly used by planners for assessment tasks. Furthermore, some authors have argued for spreadsheets as own kind of simulation platform (Seila 2006). In order to harness pre-existing calculation aids, a connection between NetLogo and Excel was conceived under the name NetLogoExcelBridge (a NetLogo Extension for Windows available freely with source under <http://www.iemar.tuwien.ac.at/processviz/NetLogoExcelBridge/NetLogoExcelBridge.zip>). Its details are as follows: It makes NetLogo available from within Excel (Button in upper part of Figure 5). On startup, NetLogo establishes the bridge and is now able to read and write from Excel. technically, this is achieved via the Dynamic Data Exchange (DDE) Protocol.

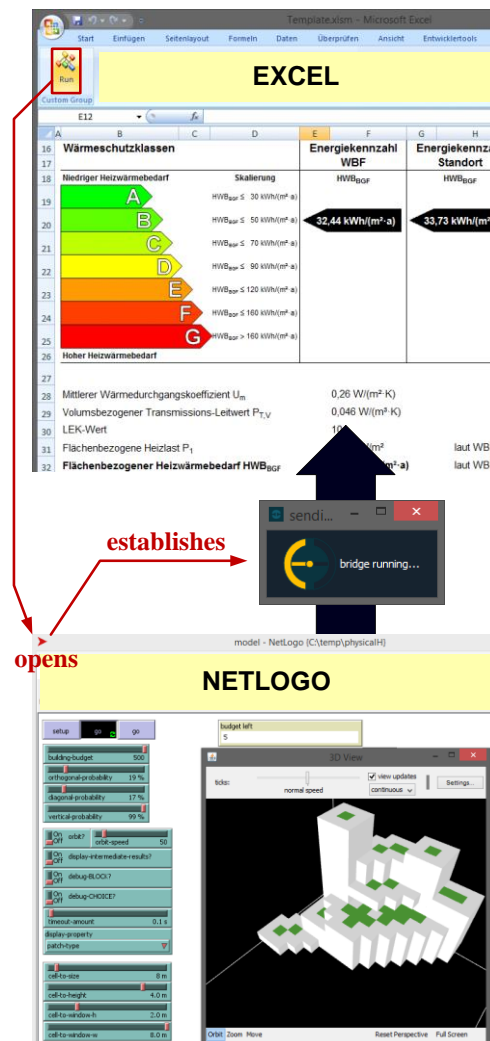


Figure 5: Parts of our simulation setup. (upper part) Excel showing the energy performance model (middle) NetLogoExcelBridge Extension (bottom part) NetLogo running our grammar model

Energy Performance Model

The building energy performance model utilizes a simple spreadsheet-based tool for calculating energy performance certificates formerly used in Austria (Demacek 1999; see upper part of Figure 5).

For every solution that is generated by the grammar, we map cells to square meters or cubic meters (one cell represents 8 m x 8 m and is 4 m high) and fill in the required values of the energy performance model. Parameters derived from the grammar include the heated gross area, the heated gross volume, and the building envelope. As a result, we receive a number of key performance indicators (KPI) from the spreadsheet. These KPIs include the heating demand (in kWh.m².a⁻¹), and its constitutive parameters (transmission losses, ventilation losses, solar gains, internal gains).

The resulting KPIs of the energy performance calculation, are retrieved from Excel after a short timeout of 0.8 s.

Fitness Calculation

Energy performance related indicators are only one of several aspects domains that can be explored to calculate a fitness value for a single solution. In previous work (Wurzer and Lorenz 2016 [in press]), we have e.g. conducted an assignment of departments to different levels of the generated building and done a subsequent evaluation of adjacencies between these as contribution to the fitness value. We have also calculated a factor for the extensibility of the design, as ratio between the perimeter of the building and the area left for development (i.e. unclaimed space around the building).

For this paper, we initially take the calculated fitness solely from the heating demand (in kWh.m².a⁻¹) in order to correlate that to the building budget B_C and the three probabilities for growth P_o , P_d and P_v . We later add the possibility to specify a "desirable volume" in which the building should progress, which also enters the fitness calculation (see 'Discussion' for more details).

Optimization

We conduct a parameter sweep experiment according to Table 1. Keeping the mapping of cells to cubic volume and window dimensions constant, we vary the cell budget and all three probabilities. The goal is to find, for every building budget, the probabilities at which the optimum (respectively lowest) heating demand (in kWh.m².a⁻¹) is achieved. These three values are also linked to the typology of the generated buildings.

As further parameter, we have the option whether or not to employ very energy efficient building material properties (expressed as U-values), which translates into writing certain constants into the Excel sheet (see Table 2).

Table 1:
Parameter Sweep Experiment in NetLogo

PARAMETER	VALUE
cell-length	8 m (constant)
cell-height	4 m (constant)
window-height	2 m (constant)
window-length	8 m (constant)
B_C (cell budget)	90, 180, 360, 450
P_o (orthogonal probability)	5, 25, 50, 75, 95
P_d (diagonal probability)	5, 25, 50, 75, 95
P_v (vertical probability)	5, 25, 50, 75, 95
passive (= use of very energy-efficient building materials)	yes, no

Table 2:
Parametrization in Excel

PARAMETER	VALUE
building type	hospital
construction type	22° C, $q_i = 5.0 \text{ W/m}^2$
heated gross volume	heavy, $\text{ETA} = 1$
heated gross area	(from NetLogo)
solar gains of windows	(from NetLogo)
ventilation	$\text{Ag} = 0.7 * \text{Aw}$ mechanical, change rate $2 * \text{h}^{-1}$, 75% heat recovery
exterior wall	(from NetLogo; "passive": U-value 0.15; else 0.35)
exterior wall area	(from NetLogo; "passive": U-value 0.10; else 0.20)
exterior ceiling	(from NetLogo)
exterior ceiling area	(from NetLogo)
ceiling to underground car park	(from NetLogo; "passive": U-value 0.20; else 0.40)
ceiling to undergd. car park area	(from NetLogo)
windows	U-value 0.90
window area	(split into north, east, west and south facade area; from NetLogo)

ANALYSIS OF RESULTS

The conducted parameter sweep has exactly 1000 different parameter settings. We repeated each experiment run 10 times in order to account for stochasticity, giving a total of 10000 outcomes which had to be analyzed (see Figure 6). In more detail, we wanted to find the *causal relationship* between the setting of the input parameters and the resulting fitness. As means for conducting our analysis we chose *parallel coordinates* - a technique often used on multivariate data (also see Appendix):

- A dataset is first filtered so it contains only "interesting" values of a chosen target variable. In the present case, were interested in the best

and worst fitness values as shown in Figure 6 (top 1000 results having the lowest heating demand and worst 1000 having the highest heating demand).

- The filtered dataset is then input into a diagram containing parallel axes, one for each variable (see Figure 7). Each row of data is depicted as line joining the data points on the different axes.
- The order of the axes is important: (see e.g. relationship between "passive/not passive and "Fitness" in Figure 7). Causal relationships between consecutive axes can be shown by rearrangement.

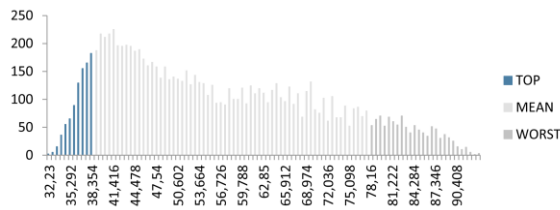


Figure 6: Fitness of all generated cell arrangements when using energy performance certificate calculation methods, as frequency distribution of the calculated heating demands. Top 1000 solutions (lowest heating demand) shown blue, worst 1000 (highest heating demand) shown dark gray, intermediate light gray.

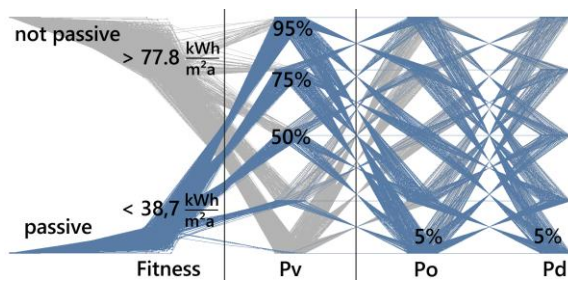


Figure 7: Results when using the energy performance certificate calculation, as parallel coordinates diagram. Top 1000 results shown blue, worst 1000 dark gray.

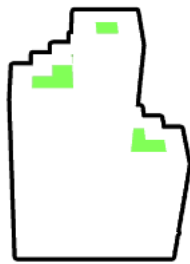


Figure 8: A thin, high building along the east-west axis is "fittest" ($P_v=75\%$, $P_o=5\%$, $P_d=5\%$, passive)

The fitness of our generated solutions was in the range $[32.23, 93.47]$ kWh.m².a⁻¹. The most influential factor for being in the top 1000 class was the adherence to energy efficient building material properties (all of the top 1000 were based on highly efficient building constructions, labeled as "passive" in Figure 7). The next-important influential factor was the vertical probability P_v , which 93% of the top 1000 results had equal or higher than 50%. The third-most influential factor was a low setting on the tendency to grow in the plane: 38% of the top 1000 had a setting of 5% for diagonal probability P_d , 30% for orthogonal probability P_o .

In considering why the fitness is best for all buildings of vertical type, it is important to reconsider the calculation method: Having a higher building increases the wall area, which can be beneficial for solar gains. This in turn leads to a preference for "high towers" oriented in the west-east axis (see fittest result in Figure 8). Lower buildings, on the other hand, tend to be bulkier: Cells within the bulk do not add to achievable solar gain, leading to the additional preference for "thin" structures. This preference, however, is strongly dependent on the glazing percentage; High solar gains can outbalance the higher transmission losses which need to be considered in "non-compact" structures. The overall bias ("build thin and high") is independent of cell budget used, which we have found not to influence results in the given case.

Even if the energy performance model is trustworthy (although outdated today, it has been used continuously for many years by a multitude of planners in Austria to issue energy certificates), the results are somewhat disappointing from a designer's standpoint. What is missing is a densification of the building in its lower areas. In trying to get to that result, we tried exploring a different energy performance method based on the calculation of relative compactness (Mahdavi and Gurtekin 2002),

$$RC_{cube} = \frac{6 \times volume^{\frac{2}{3}}}{surface\ area} \quad (1)$$

to a cube, yielding a similarity in the range $[0, 1]$. As Pessenlehner and Mahdavi (2004) have shown, this relative compactness can be used as indicator for heating loads, and thus as energy performance model. Quite discomfotingly, our results (Figures 9 and 10) were in perfect agreement with the previous model, meaning that relative compactness also capitalizes on high vertical and low orthogonal/diagonal probability and thus produces rather similar geometries.

Instead of top/worst 1000 cases, we had top 1043 and worst 1111 (effect of applying the formula in cell space - many cell configurations have the same fitness value). The lowest cell budget ($B_c = 90$ cells, i.e. 10 Megacells; see again Figure 10) always produced a result in the top 1043 (Fitness > 62%).

Solutions in that region were furthermore characterized by a high vertical probability ($P_v=95\%$ in 45% of the cases, 75% in 31% and 50% in 18% of the cases) and a low orthogonal ($P_o=5\%$ in 41% and 25% in 27% of the cases) and diagonal probability ($P_d=5\%$ in 57% and $P_d=25\%$ in 22% of the cases).

DISCUSSION

"Interesting" designs, it seems, cannot evolve solely from an energy-based fitness test. However, adding a "desirable volume" (see left in Figure 11) and calculating

$$ratio_{inside} = \frac{built\ cells\ inside\ volume}{total\ built\ cells} \quad (2)$$

a measure which states how many of the built cells are within that desirable volume (range [0, 1]), we can get to a fitness evaluation that rewards solutions which fall into the area. Multiplying the performance-based fitness with this ratio, e.g.

$$F_{overall} = ratio_{inside} \times RC_{cube} \quad (3)$$

it is possible to obtain morphologies that are both performance-optimized and driven by a prescriptive form. As the right part of Figure 11 shows, results obtained via that approach are non-trivial, since the building shape does not simply follow the desirable volume.

In our experiment using the above fitness, we employed an arbitrary form for the "desirable volume" (left in Figure 11) and ran our generation (parameter sweep without repetitions, 1000 cell arrangements total) just to prove that getting a more "interesting" form is possible. Furthermore, we have not weighted $ratio_{inside}$ and RC_{cube} , which would be an obvious addition for future work.

Further elaboration

The introduction of a "desirable volume" could be seen as a way of shifting responsibility to design. This is certainly true to some extent - it is after all a target function which the optimization wants to satisfy. On the other hand, that does not mean that it must be chosen arbitrarily:

- Its form may be determined by many factors such as the form of the building spot, land use and zoning regulations (inclusion/exclusion of cells giving allowed spots and building plot boundary; stacking of cells to determine allowed height), design intent (cells resembling three-dimensional block diagrams) and so forth.
- In generalization, we may also several prescriptive volumes to store several layers of information either for (a.) driving the generation (cf. Vidmar 2013; e.g. by basing the three probabilities P_v , P_o and P_d on a certain prescriptive layer), or (b.) making them part of the overall fitness function $F_{overall}$.

Even though that addition certainly goes into the right direction, it cannot be used to account for all factors involved in the design of a complex building - thinking e.g. statics or work processes in the case of hospitals. Furthermore, it must be noted that such factors are not independent - they may influence one another to a great extent. In reality, a finished building is a trade-off between its influencing factors rather than a "fully optimized structure". *Early stage planning* has the advantage that it reduces this complexity to some extent. Even then, it is hard to imagine a single tool answering all questions, which is why we have additionally explored *coupling* between morphology generation and evaluation.

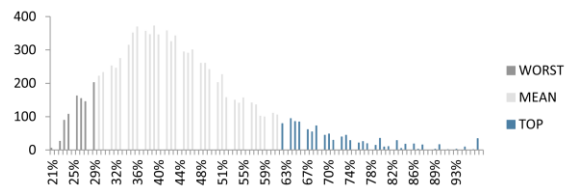


Figure 9: Fitness distribution of results when using relative compactness as fitness function.

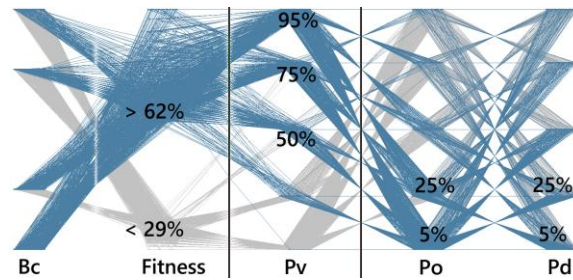


Figure 10: Results when using relative compactness, as parallel coordinates diagram. Top 1043 results shown blue, worst 1111 dark gray.

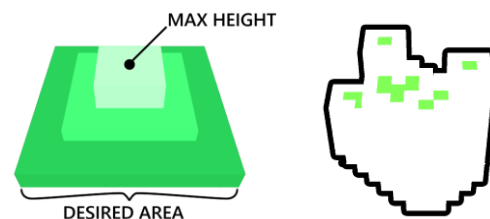


Figure 11: (left) desired volume (right) best solution ($P_v=95\%$, $P_o=5\%$, $P_d=5\%$, passive; $F_{overall}=0.75$)

SUMMARY

We have presented an approach that couples building morphology optimization to energy efficiency computation in the context of early stage hospital planning. In doing that, it was observed that energy performance alone is not sufficient for generating "interesting" designs. We have thus added a possibility for expressing "designerly intent", which also becomes part of the fitness function used for selecting preferable solutions.

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APPENDIX

Why Parallel Coordinates and not pure Correlation Analysis?

Correlation Analysis can uncover dependence between two variables, for example between variable "passive" stating the use of highly efficient building constructions and the variable "fitness" (see highlighted cell in Table 3).

Table 3:

Correlation between fitness values shown in Figure 6

	<i>Bc</i>	<i>passive</i>	<i>Fitness</i>	<i>Po</i>	<i>Pd</i>	<i>Pv</i>
<i>Bc</i>	1,00					
<i>passive</i>	0,00	1,00				
<i>Fitness</i>	-0,29	0,83	1,00			
<i>Po</i>	0,00	0,00	0,07	1,00		
<i>Pd</i>	0,00	0,00	0,15	0,00	1,00	
<i>Pv</i>	0,00	0,00	-0,31	0,00	0,00	1,00

During analysis, correlation is a good starting point for determining dependence but cannot account for the *causal relationships* which need to be mined from within the data. This is where parallel coordinates truly excel, thanks to two concepts:

Brushing. "Interesting" data is firstly isolated, the method acts only on selected data points.

Transformation. Every data point is depicted as line between the parallel axes. Every coordinate of a data point is transformed into a point on an axis. When selecting a certain value or value range, one can easily spot the relationship to the preceding and succeeding axes because of the (incoming, outgoing) lines (see Figure 12). In consequence, this also means that the order of the axes is of vital importance.

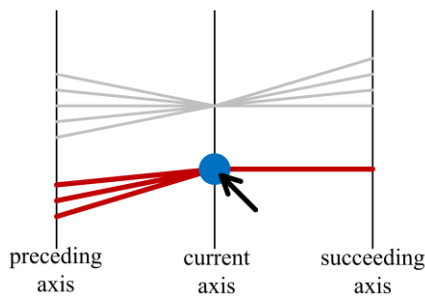


Figure 12: Selecting a value reveals a relationship with the preceding and succeeding axis