Complexity across scales in the work of Le Corbusier

Using box-counting as a method for analysing facades

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Abstract: *Since Benoît Mandelbrot raised the question about the length of Britain's coastline in 1967, it has become obvious that fractal geometry is appropriate for describing irregular forms. In 1996 Carl Bovill applied box-counting, a fractal analysis method, for the first time to architecture in order to quantify the characteristic visual complexity of facades. This paper presents an approach utilizing fractal analysis to provide another view on Le Corbusier's architectural composition. Altogether 17 house designs are considered, 14 of them have been built between 1916 (Schwob Villa) and 1928 (Savoye House). Throughout this paper an implementation of the box-counting method written by the author is used. Besides discussing the results, the implementation itself with its advantages and disadvantages is explored.*

Keywords: *Architectural Analysis, Fractal Analysis, Box-counting, Visual complexity*

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BACKGROUND

This paper discusses a selection of 17 house designs by Le Corbusier from the viewpoint of 'visual complexity in the early phase of classical modern architecture'. The author uses a fractal analysis method to answer the following questions: First, does the visual complexity in Le Corbusier's oeuvre change? Second, regarding a single facade, does the degree of visual complexity change over multiple scales?

Box-counting, a fractal analysis method, determines the fractal dimension of an image and provides the possibility for quantifying characteristic visual complexity [1]. Since Carl Bovill [2] applied box-counting for the first time to architecture in 1996, the method has been used in multiple studies (e.g. to measure and compare the fractal dimension of architectural order [3], the work of Frank Lloyd Wright and Le Corbusier [4], indigenous architecture [5] and Hindu temples [6]). Box-counting turned out to show promising results (especially when applied to facades) [2, 7, 8]. The author has written an implementation of the algorithm in AutoCAD, which is used throughout this paper.

Le Corbusier was an influential architect of the modern movement. From the 1920ies onward his designs are characterised by a balance between smooth closed parts, different sized openings in dependence of the function of the belonging room and the use of proportion across different scales. The latter suggests that the whole and its parts are held together. If the assumption that all parts follow the same underlying rules is true, this may be reflected in the results of box-counting. In general, the author believes in a clear difference of visual complexity between the early modern architecture and later examples of investors driven smooth perforated façades. Such a difference may as well be reflected by the results of box-counting dimension, its corresponding range of scale and various statistical values. The present paper represents a puzzle piece to confirm these assumptions.

THE METHOD

Lewis Frey Richardson [9] realised the difficulty of length measurement while examining geographical curves (including borderlines). In his study, data was received by way of polygons with equal sides and their corners located on the curve [10]. It is hence obvious that the total length of the polygon was determined by the scale of the measuring device ('side length') and by the scale of the used map ('resolution'). With irregular curves, such as borderlines or fractals (e.g. Koch curve, figure 1a) and in contrast to a straight line, the total length generally increases as the side length decreases – at least for a certain range of scales. Richardson [9] was able to establish a relationship between the total length and the side length. This relation was given by a power law:

$$L_G = M \times G^{1-D} \qquad (1)$$

where M and D (a characteristic of the curve) are constants, L_G is the total length and G is the side length.

Later, Benoît Mandelbrot [11], the 'father' of fractals, related the exponent to what he called fractal dimension [10], a characteristic value for fractals. While one-dimensional curves (in terms of topological dimension D_T) can be replaced by polygons of different side lengths to determine the fractal dimension, two dimensional images ask for another method. With box-counting, grids of different sizes are placed over a two-dimensional black and white image. This time for each grid size the smallest amount of boxes that contain a part of the image is counted (figure 2a-c). The characteristic value is given by the relationship between the number of covering boxes and the grid size. The box-counting dimension, as an index of the ratio of irregularity, is defined by:

$$D_B = \lim_{\varepsilon \to 0} \frac{\log N_\varepsilon}{log\frac{1}{\epsilon}} \qquad (2)$$

For many different grid sizes, D_B equals the slope of the regression line in a double logarithmic graph with log N (the number of covering boxes) versus log ϵ (the grid size) (figure 2d). Of particular interest is the range of scales for which a constant relationship between both values exists. Such a relationship is reflected by a straight line of data points. However, data points are seldom exactly positioned on the regression line but, at best, close to it (figure 2d). The coefficient of determination (r^2) defines accuracy, with a value close to one reflecting a clear dependence between N and ϵ. In the case of elevations from a certain scale onwards the one-dimensional lines of the composition comes into focus. In the double logarithmic graph this is reflected by a straight line with a slope of 45 degree. In this special case the box-counting dimension (D_B=1) equals the topological dimension (D_T=1).

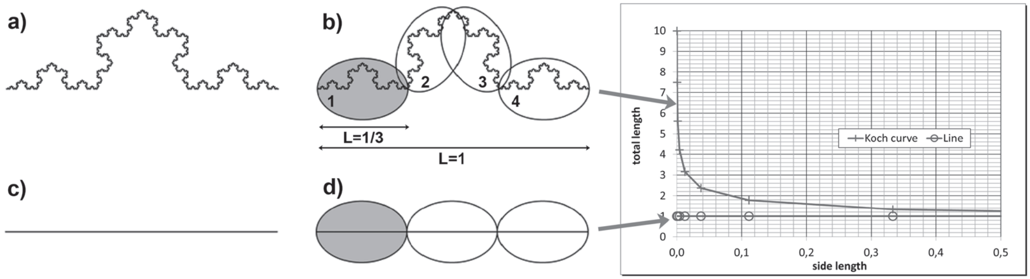

Figure 1:
Differences in length increase (right) of the Koch curve, a fractal, (a & b) and a straight line (c & d).

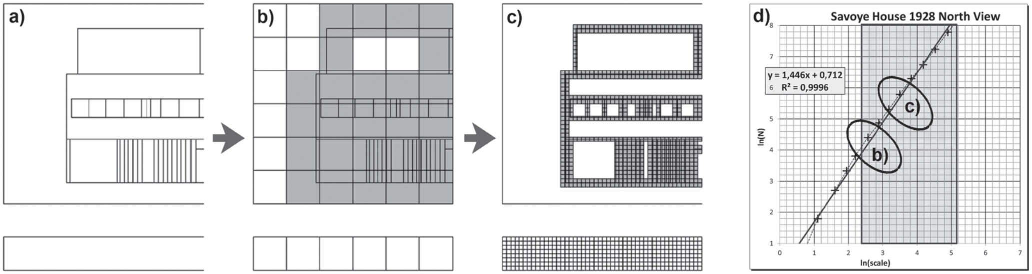

Figure 2: Box-counting grids of different sizes are placed over an image (a to c). Double logarithmic graph with log N, the number of covering boxes, versus log ϵ, the grid size (d).

SIGNIFICANT VALUES

Various parameters of the box-counting method itself have a deep impact on the result [12, 1, 13]. Since it analyses a two dimensional black and white representation of the original facade (elevation), the 'correct' selection of architectural elements is of particular importance [7]. In order to guarantee accuracy of selection and consistent preparation all examples of this study are taken from the same source [14]. Real facades are perceived in perspective using a constrained field of vision [7]. The used elevations, on the other hand, have none of these two properties; however, they do allow an insight into design intent.
Several measurements using different settings are considered for each single facade. This includes

- changing the white space around the image,
- using different smallest grid sizes and
- altering the starting positions of a grid.

The box-counting software written by the author is an VBA implementation in AutoCAD and offers maximum flexibility to adjust the parameters mentioned above. A major advantage of using AutoCAD is the use of vector graphics, which minimizes influences caused by line width. The software has successfully been tested on classical mathematical fractals, including the Koch curve (figure 1a) with a calculated difference between the self similar dimension d_s and D_B of -0.65% [12]. The self similar dimension is defined by:

$$d_s = \frac{\log N}{log\frac{1}{\epsilon}} \qquad (3)$$

where N is the number of pieces, each of which is an exact copy of the whole scaled by the reduction factor ϵ (figure 1b). To ensure comparability of analysis results all examples in this paper use similar adjustments. The final statistical evaluation in a spreadsheet software is automatically carried out using another script by the author. Important statistical values are:

- the median of the box-counting dimensions of the set of measurements,
- the interquartile range of the set of measurements that defines accuracy,
- the smallest and the average coefficient of determination that mirrors deviation and
- the range of scale that displays the range of coherence.

Accuracy is expressed by a small coefficient of correlation for each single measurement (at least 0.998) and a small interquartile range of the box plot for each set of measurements.

RESULTS

From a fractal perspective modern architecture (facades) is often called 'smooth' and expresses an affinity to 'scalebound' objects [11, 12]. The first means that a facade reveals, at best, only little more elements when the scale of observation becomes smaller and smaller. Regarding the second aspect, a 'scalebound' object offers a limited number of characteristic elements of scale that are clearly distinct in their size. Such behaviour is reflected by small box-counting dimensions and a limited range of scales (which reflects no or less elements at smaller scales). Earlier measurements of Le Corbusier's work by the author [12] indicate that in contrast to results by Bovill [2] higher visual complexity exists for a broad range of scales [12]. However, in line with Bovill, from a

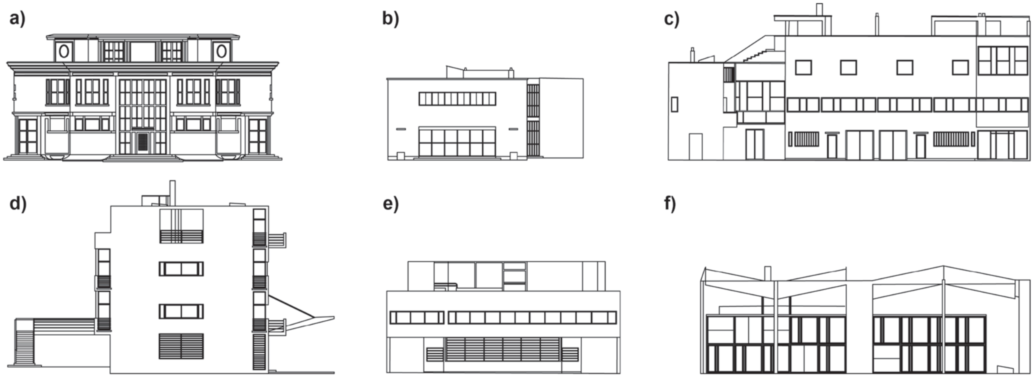

Figure 3:
Vector drawings: South-East view of Schwob Villa, 1916 (a), South view of Besnus House, 1922 (b), South view of Roche-Jeanneret House, 1923 (c), East view of Stein Villa, 1927 (d), North view of Savoye House, 1928 (e), South view of Maison de l'Homme, 1963 (f).

certain scale onwards the visual complexity decreases immediately. Such behaviour suggests the lack of (levels of) detail on smaller scales.

The study described here analyses 64 elevations of 19 house designs by Le Corbusier and comprises 64 sets of measurements, 576 single measurements and 14.400 single data sets.. For all examples new vectorised drawings (figure 3) were prepared based on the same source [14]. The first 14 house designs were built between 1916 (Schwob Villa) and 1928 (Savoye House). Schwob Villa was deliberately chosen as start point because, although its appearance is still of a conventional classical taste, it already offers some of those characteristic known as the five points of architecture formulated by Le Corbusier in the 1920ies [15].

On closer examination, some elevations display a 'sharp bend' in the data curve of single measurements. This 'sharp bend' clearly separates two ranges of data points with different regression lines and slopes (figure 4a-c). Elevations with such behaviour are of greater visual complexity at larger scales, expressed by a steeper slope at the beginning of the data curve. In most cases this first range shows a relatively high dispersion, both in a single measurement (smaller r^2) and in the set of measurements (larger interquartile range of the box plot) (figure 4a & c left). However, from a certain scale onwards visual complexity (and dispersion) changes immediately, and the elevation tends to be smoother (figure 4b & c right). Such behaviour could already be observed at previous analysis of 'modern architecture' by the author (e.g. Savoye house by Le Corbusier [12]; Villa Tugendhat by Mies van der Rohe [16]; Steiner House, Scheu House and Mandl House by A. Loos [17]). This category of elevations is called 'sharp bend in the data curve'.

Other results of the study behave differently. They show a large range of similar visual complexity. In the double logarithmic graph this becomes evident by data points that follow a straight line for many different grid sizes. This category of elevation is called 'continuous data curve' (figure 4d & e).

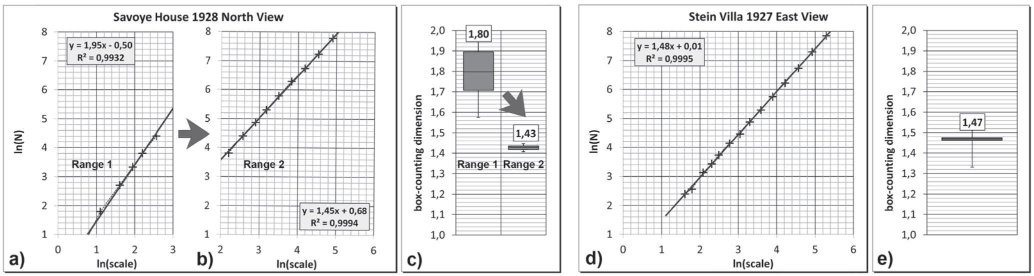

Figure 4:
'Sharp bend in the data curve' in the North view of Savoye House, 1928; one single measurement with two ranges (a-b) and box-plot diagram of the set of measurements for range 1 and 2 (c). 'Continuous data curve' in the East view of Stein Villa, 1927; one single measurement with only one range (d) and box-plot diagram of the set of measurements (e).

DISCUSSION

It can be concluded that all investigated elevations show a middle (and some even a high) level of visual complexity for at least a certain range of scales. This confirms, first of all, the assumption that the rejection of ornaments, a central topic to modernist architects, does not per se lead to 'smooth' facades – which has already been demonstrated with buildings by A. Loos [17]. The majority of the here discussed elevations have a box-counting dimension (median) between 1.3 and 1.5 (if the category 'continuous data curve' and the second range of 'sharp bend' are considered). This value, however, is below most previously examined buildings by A. Loos and F.L. Wright. In future work we plan to explore the reasons for this phenomenon.

The initial question whether the visual complexity in Le Corbusier's oeuvre changes can be answered with the aid of statistical analysis. Measurements so far demonstrate that no relation between the year of design and the degree of visual complexity exists. There is no clear tendency from the earliest to the later examples. That means that earlier examples do not differ from later ones in a clear defined way. The only exception concerns Schwob Villa (1916), which shows higher values with box-counting dimensions between 1.67 and 1.74. This is due to the fact that the appearance is still of a conventional classical taste.

Looking at the results of single measurements helps to answer the second question whether the degree of visual complexity changes over multiple scales. The majority of facades examined belong to the category 'sharp bend in the data curve'. This suggests that most of Le Corbusier's designs change visual complexity while smaller scales are examined. The turnaround occurs immediately ('sharp bend in the data curve'). In other words from a certain grid size (scale) onwards the building seems to be 'smoother'. Dependency on certain architectural components, orientation and environmental influences will be examined in future work to establish possible reasons for the 'sharp bend'.

Apart from a separate analysis of each facade, this study examined all views of one and the same building. Two different outcomes are possible: Either all elevations show 'similar' results (figure 5a) or (partly) 'differ' (figure 5b-c). The latter means that visual complexity change from one elevation to another. This likely reflects the influence of orientation (North, East, South and West), and/or privacy relation (garden view and street view) [18].

The measurement results can essentially be classified into four categories: first, continuous single measurements and similar (+/- 5%) visual complexity for all elevations (figure 5a), second, continuous single measurements and divergent visual complexity, third, 'sharp bend' in the single measurements and similar visual complexity (figure 5b) and fourth, 'sharp bend' in the single measurements and divergent visual complexity (figure 5c). However, there are some exceptions: the characteristics of the data curve (continuous versus 'sharp bend') may change within a single building (figure 5c). Most of the here analysed examples belong to the fourth category. This underlines the assumption that, apart from design conceptions, elevations depend on various 'outer' influence factors such as orientation and environment (context). Moreover, results depend on the selection of elements. Maison aux Mathes (1935), for example, has characteristic values between 1.78 and 1.89 if the material of the stone wall is

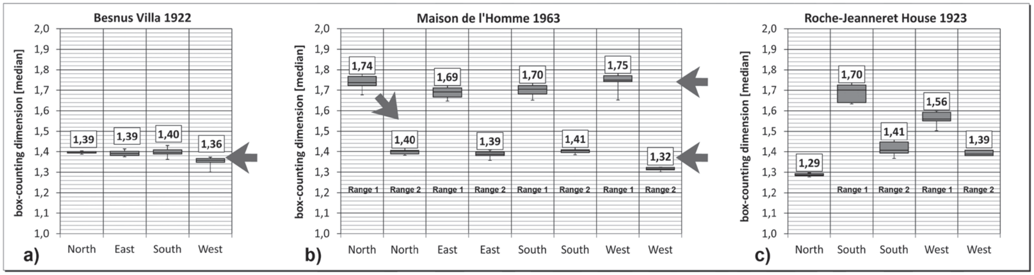

Figure 5: Category 1;' continuous' single measurements and similar visual complexity for all elevations; Besnus House, 1922 (a). Category 3; 'sharp bend' in the single measurements and similar visual complexity; Maison de l'Homme, 1963 (b). Category 4; 'sharp bend' in the single measurements and divergent visual complexity; Roche-Jeanneret House, 1923 (c).

taken into account. Thus, all elevations are of similar visual complexity. However, results vary widely if single stones are excluded; in this case characteristic values are between 1.22 and 1.66.

CONCLUSION AND OUTLOOK

The results presented in this paper confirm the suitability of the box-counting method for comparing different elevations with regard to visual complexity.

If we have assumed that buildings by Le Corbusier are – in a fractal sense – 'smooth', this is objected by the results. On the contrary, Le Corbusier's oeuvre is rather diversified, even though the majority of the analysed buildings follow largely the same basic rules, his five points of architecture, developed throughout the 1920s. Differences are reflected by either similar or divergent results for all elevations of one and the same building. In turn, results of a single elevation differ in regard to the curve characteristic, which is either continuous or displays a 'sharp bend'. Future work will analyse the dependency of the results on certain architectural components, orientation and other external influences.

The study has shown so far that visual complexity of Le Corbusier's designs is tending to be smaller than those of other pioneer modernists F. L. Wright and A. Loos. The box-counting dimensions of the analysed elevations are mostly between 1.3 and 1.5 and seldom exceed 1.5 (the latter reflects high visual complexity at least for a certain range of scales). The results of this study show that for certain values no clear period in Le Corbusier's oevre can be allocated. That means, it could be demonstrated that no relation between the year of design and the degree of visual complexity exists (figure 6).

From a methodical standpoint, it is important to note that the same settings have to be used in order to be able to reproduce the results. Furthermore, the box-counting method cannot be used to derive statements about quality. It is a method to quantify the characteristic visual complexity of facades and a tool for comparison by means of statistical analysis.

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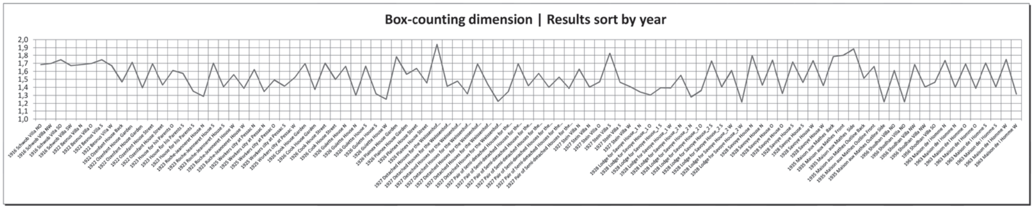

Figure 6:
Relation between the year of design and the box-counting dimension.

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