Finite element analysis of layered wooden shells under application of an orthotropic single-surface plasticity model

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ABSTRACT: The analysis of layered wooden shells requires a suitable constitutive model for multi-axially loaded wood. In this contribution a brief overview of the development of an orthotropic material model for the simulation of spruce wood under simultaneous biaxial in-plane stresses and transverse shear stresses is given. The model considers an initially linear elastic domain as well as hardening and softening behaviour at higher states of stress and strain, respectively. Combining the advantage of a smooth single-surface plasticity model with the identification of several modes of failure is the key to the proposed mathematical formulation. The applicability of the constitutive model will be demonstrated by means of a nonlinear finite element analysis of a layered cylindrical shell with an opening and surrounding stiffeners.

1 INTRODUCTION

Realistic finite element ultimate load analysis of layered wooden shells requires knowledge of both, suitable constitutive equations for the prediction of the deformation behaviour of biaxially loaded solid wood and the transverse shear forces inducing shear stresses in cross-sections perpendicular to the middle surface of the shell.

The model development is based on a comprehensive test series on clear spruce wood by (Eberhardsteiner 2002). By performing tests with loading-unloading-reloading cycles the use of the theory of plasticity has been detected. The respective biaxial experiments provide the information concerning the stress-strain relations in the pre-failure domain as well as the failure locations for arbitrary strain paths. The obtained failure locations reveal an elliptic shape of the failure envelope which is described by means of the orthotropic failure criterion by (Tsai & Wu 1971). It identifies failure states as a boundary of a linear elastic domain. By defining characteristic strength values, depending on the material parameters of the elliptic failure criterion, (Müllner et al. 2004a) combined these values with micromechanically motivated failure modes. The determination of the material parameters results in a system of nonlinear equations. Thus, the advantage of a single-surface model has to be paid for by the extra effort of locally solving these equations. The application of the classical return mapping algorithm by (Simo & Hughes 1998) yields an unsymmetric consistent tangent for this material model.

This paper reviews ideas first introduced with a multi-surface approach by (Mackenzie-Helnwein et al. 2003) in the attempt to apply them to the initial single-surface description suggested by (Eberhardsteiner 2002). A related study was successfully performed by (Müllner et al. 2004a) for the in-plane biaxial loading. The generalization of this formulation for stress states observed in layered shells was done by (Mackenzie-Helnwein et al. 2005).

This paper is structured as follows: Chapter 2 will introduce the definitions needed for the analysis of wooden shells. In Chapter 3 a short description of the proposed material model is given. Finally, a representative example analysis is given in Chapter 4. Conclusions will close the paper.

2 DESCRIPTION OF LAYERED SHELLS

The mechanical description of layered wooden shells requires a suitable element for layered shells and an appropriate constitutive model for wood. Moreover, the presented analysis will impose further requirements on both parts.

The present contribution deals with ultimate load analysis of wooden shells. Hence, a non-linear shell formulation will be used. Wood, on the other hand, shows inelastic behaviour and failure at small to moderate strains. Thus, the small strain assumption appears reasonable for most cases.

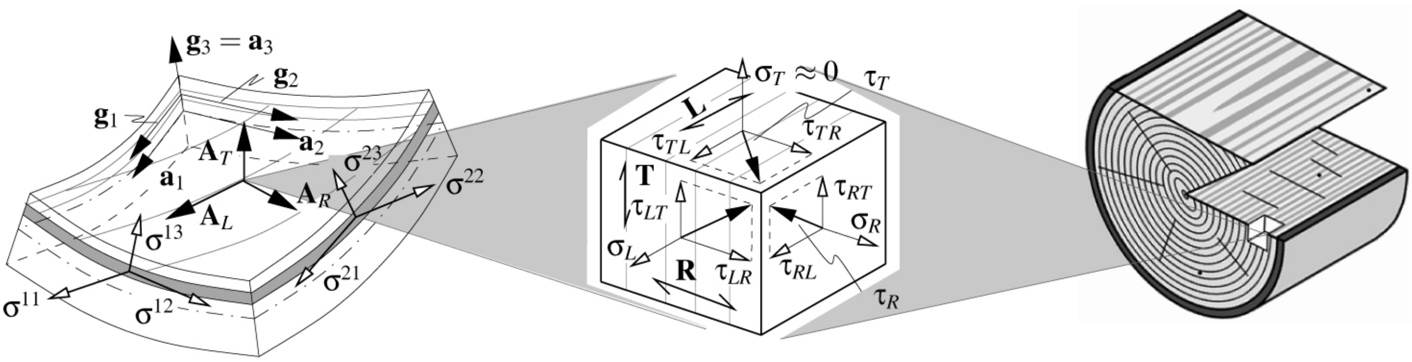

Figure 1. Shell geometry and material coordinate system for orthotropic layered wooden shells.

Since a typical wooden shell is constructed with initially straight boards, it is assumed that the wood fibres are always parallel to the middle surface of the shell. In general, this cannot be assumed for the *R*- and *T*-direction. The stress-strain behaviour in these directions, however, is reasonable similar to justify a simplified approach such that one characteristic stress-strain function may be used and scaled by the respective uniaxial stiffness and strength parameters. Throughout this paper it will be assumed that the tangential direction of the stem is aligned with the normal to the shell surface. This yields an orientation of the material as illustrated in Figure 1. This also defines the general stress state in the material, where the thickness stress σ_T is assumed to vanish for thin shells.

3 CONSTITUTIVE MATERIAL MODEL

3.1 *Material model for in-plane stress states*

The properties and the concept of the used single-surface plasticity model are summarised in (Müllner et al. 2004a). The formulation of evolution laws requires control variables. These so-called primary variables are collected in a vector α. The determination of α is subject to a non-associated hardening and softening rule.

Its formulation is required because of the brittle tensile and the ductile compressive behaviour of wood. The rule considers different modes of failure which were identified by (Mackenzie-Helnwein et al. 2003) as:

- brittle tensile failure in fibre direction,
- compressive failure in fibre direction,
- brittle tensile failure perpendicular to grain, and
- ductile compressive behaviour perpendicular to grain.

3.2 *Material model for layered wooden shells*

Concerning stress states in layered wooden shells, a fifth failure mode may become relevant. It is controlled by transverse shear stresses and cannot be observed in the tests by (Eberhardsteiner 2002).

The extension of the material model for transverse shear stresses was done by (Mackenzie-Helnwein et al. 2005). Experimental observations by (Lucena-Simon et al. 2000) show typical shear failure as cracking parallel to the fibres. Hence failure due to transverse shear has to be controlled by τ_{LR}, τ_{RT} and τ_{TL}. In order to keep the model as simple as possible, shear failure in given planes perpendicular to the *R*- and *T*-direction shall be characterized by the respective effective shear stresses τ_R and τ_T. These effective stresses are shown in Figure 1. The appropriate definitions can be found in Figure 2.

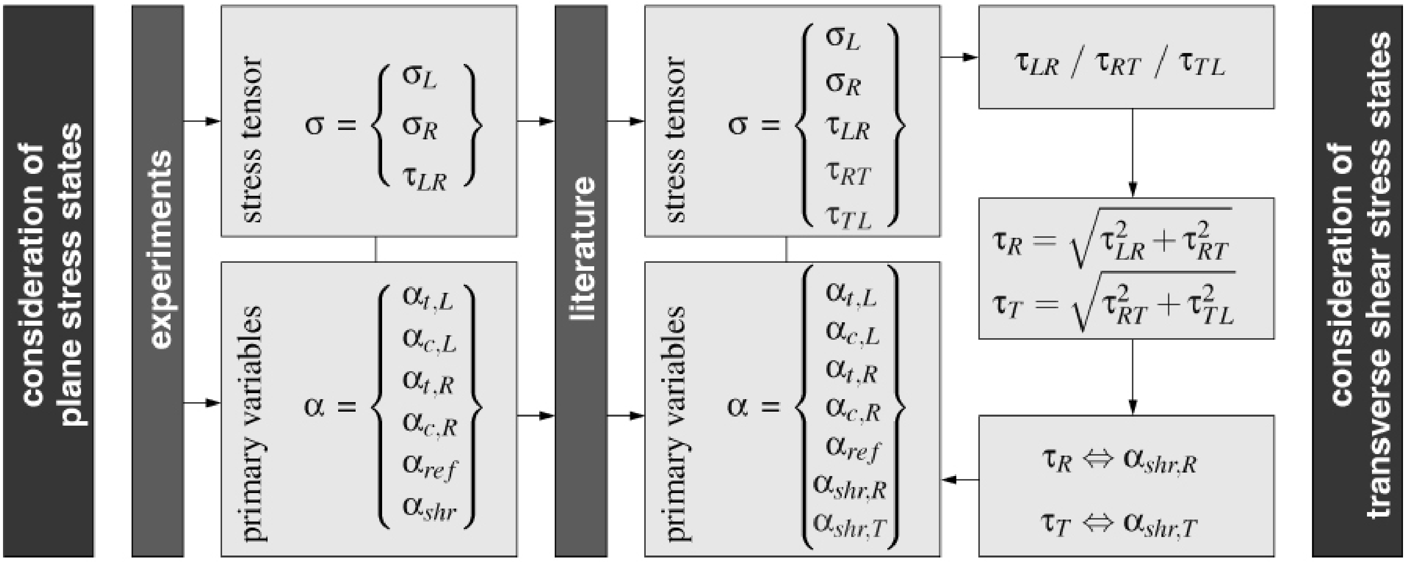

Figure 2. Types of material models – model for in-plane stress states and model for transverse shear stress states.

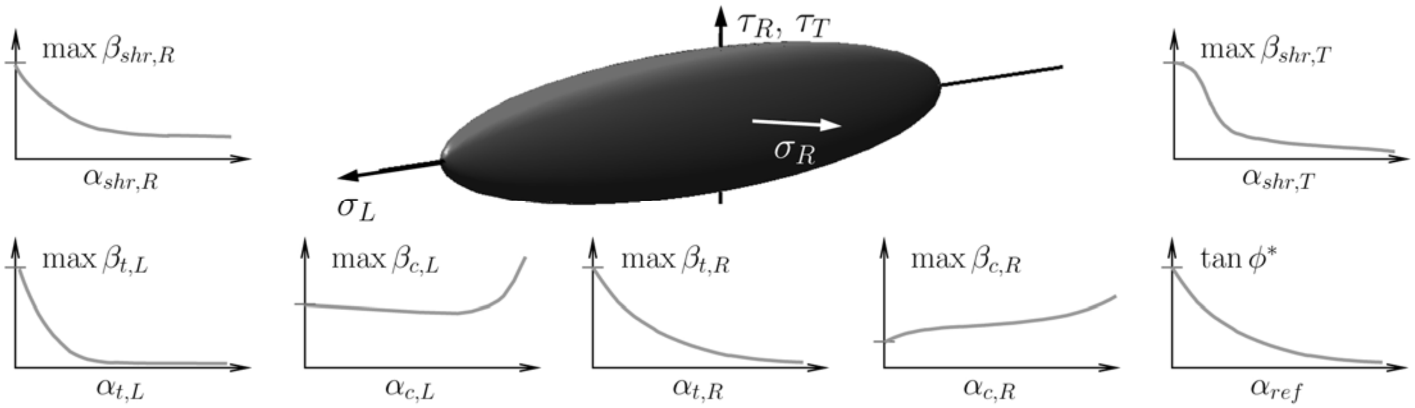

Figure 3. Yield surface of single-surface plasticity model in the orthotropic stress space and evolution laws for the strength values depending on the strain-like primary variables α_i ($i \in \{(t,L), (c,L), (t,R), (c,R), (ref), (shr,R), (shr,T)\}$) by (Müllner et al. 2004b).

The latter shows the chosen approach of the material modelling. The extension of the material model leads to an increase of the number of stress invariants $\boldsymbol{\sigma}$ and the vector of the primary variables $\boldsymbol{\alpha}$. The used elliptical yield surface and the appropriate evolution laws for the consideration of transverse shear stress states are shown in Figure 3.

4 NUMERICAL EXAMPLE

The suggested material model was used for the analysis of a cylindrical wooden shell with an opening and stiffening beams as shown in the horizontal projection and the cross section of the shell in Figure 4. The arch rise of the circular cylindrical shell is 2.0 m. With a length of 10.0 m and a width of 8.0 m, the thickness of the shell is only 48 mm. In thickness direction, the shell consists of three layers of equal thickness.

The complete shell is made out of the same material but the grain angle varies for different layers. The outer layers have a grain angle of $\varphi = -30°$ with respect to the x-y-plane. The centre boards are arranged at an angle of $\varphi = +30°$.

One additional 0.30 m wide layer of strengthening boards is attached along the outer edges at the bottom of the shell. These boards are aligned parallel to the edges. An opening of 2.50 x 1.50 m is located in the centre of the structure. The edges of this opening are strengthened by a 150 x 240 mm gluelam beam as shown in cross-section A-A of Figure 4. It consists of the shell structure in the centre and 12 strengthening boards, six on both the top and the bottom of the shell. All boards have a thickness of 16 mm which follows from a 3/4 inch sawing pattern and subsequent drying and planning of the boards.

Table 1. Material parameters for the linear elastic behaviour.

Young's modulus	Poisson's ratio	Shear modulus
E_L = 13,000 MPa	ν_{LR} = 0.5000	G_{LR} = 632 MPa
E_R = 700 MPa	ν_{RT} = 0.3800	G_{RT} = 222 MPa
E_T = 500 MPa	ν_{TL} = 0.0133	G_{TL} = 470 MPa

The material parameters for the linear elastic behaviour are summarized in Table 1. The material parameters for the plastic behaviour can be found in (Müllner et al. 2004b).

Two different load cases were considered in the analysis. First, a uniformly distributed dead load of 0.45 g/cm³ and second, a live load of up to 225 kN, equally distributed along the stiffening beam which surrounds the opening.

Figure 5 shows the used finite element mesh. The minimum element size is 150 mm, the largest one is 300 mm. Moreover, this figure shows the deformed shape at a scaling factor of 15. The maximum displacements are along the longitudinal edges of the opening. The deformation pattern shows point symmetry about the centre of the structure which corresponds to the point symmetric composition of the shell.

The shell is subject to both membrane and bending stresses. The latter is dominant near the opening and causes extreme stress values to appear near the corners of the opening. This fact is shown in Figure 6 for the radial stress σ_R in the top layer of the shell.

A significant improvement of the used material model compared to classical orthotropic stress analysis in combination with a failure envelope is its ability to identify active failure modes. This is achieved by means of the non-associative flow rule.

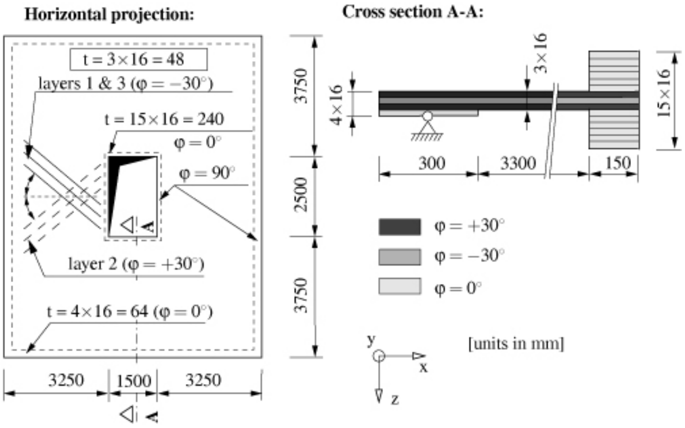

Figure 4. Horizontal projection and cross section of the shell.

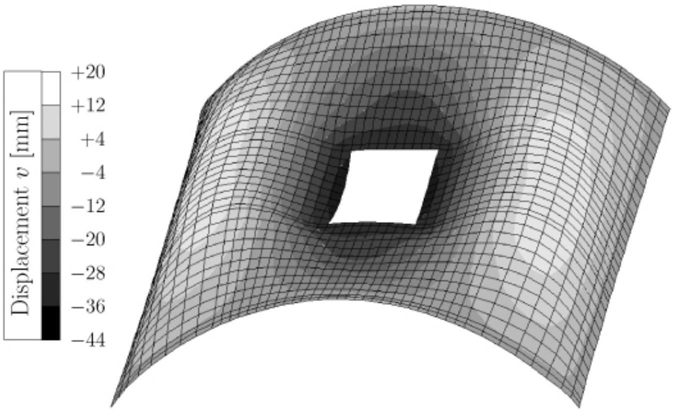

Figure 5. Vertical displacements at maximal applied load.

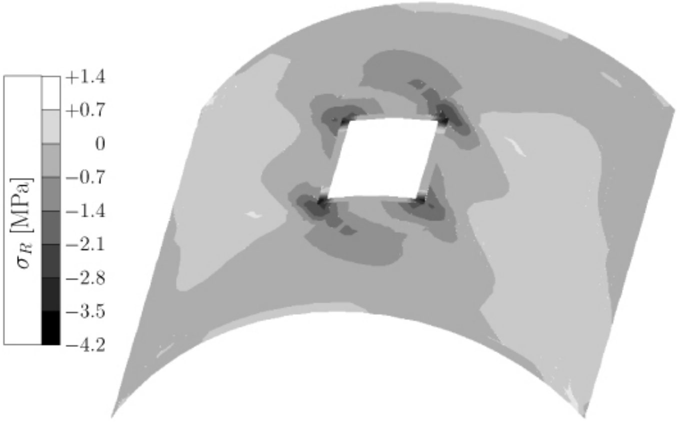

Figure 6. Perpendicular to grain stress σ_R in the top layer.

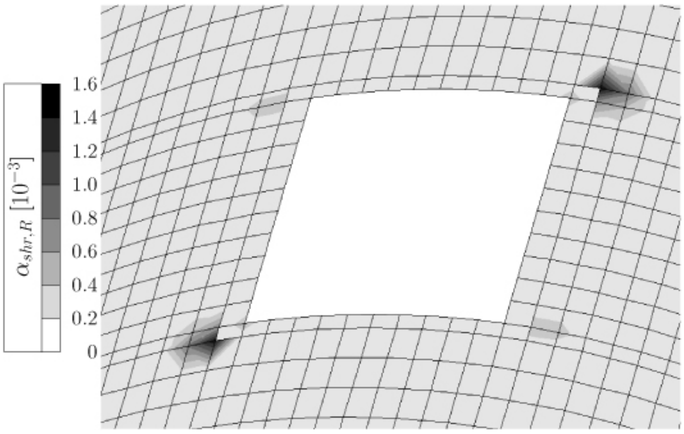

Figure 7. Equivalent shear strain parameter $\alpha_{shr,R}$.

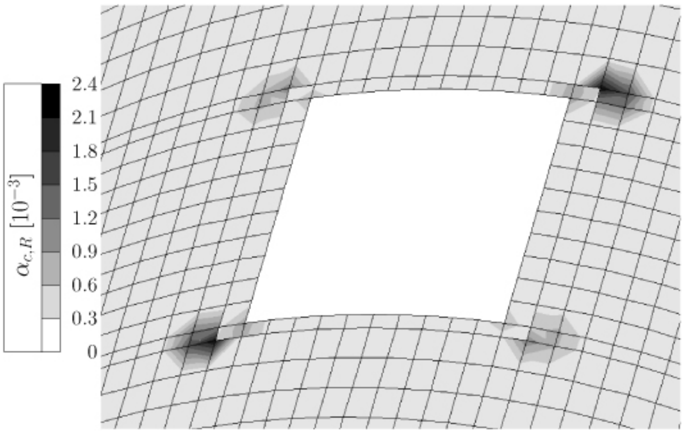

Figure 8. Equivalent compressive strain parameter $\alpha_{shr,R}$.

The equivalent strain parameters, collected in α, are directly linked to the failure modes. Figures 7, 8 show the equivalent strain parameters $\alpha_{shr,R}$ and $\alpha_{c,R}$, respectively, for the top layer. The pictures show that inelastic deformations only appear in the proximity of the opening. Both crushing perpendicular to grain and shear failure remain localized at the corners. The point symmetry is clearly visible for the development of inelastic deformations. The majority of the shell though remains elastic.

The presented analysis demonstrates that the load bearing capacity of the analyzed shell is controlled by collapse due to material failure. The proposed model, however, can be used to identify critical zones where structural modifications can aid both performance and durability of the wooden shell.

5 CONCLUSIONS

This paper contains an overview on the development of a constitutive model for the simulation of spruce wood. The applicability of the model was proven by means of a nonlinear finite element analysis of a layered cylindrical shell with an opening and surrounding stiffening beams.

The presented numerical example gave a brief demonstration of the suitability of the presented material model and its numerical implementation. Further numerical studies with experimental verification on model structures are planned for the near future.

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