An Examination of Acoustic Propagation Effects in a Perfect Wedge

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Objectives of the Paper

- Evaluate Time Records of the Pressure at Ranges $r$ from the Source
  Which Are Small and Large in Comparison with the Water Depth $h$
  at the Source Location
  - Down-Slope-Receiver-Ranges: $r/h = 5, 10, 100, 200$
  - Cross-Slope-Receiver-Ranges: $r/h = 5, 10, 100, 200$
  - Up-Slope-Receiver-Ranges: $r/h = 5, 10$

- Examine the Effect of the Range $r$ on Received Pressure Pulses

Fig. 1. Geometry of the 3-D perfect wedge problem.
Outline of the Paper

• Overview of the Ray-Integral Solution Used

• Application to a Rigid-Bottom Wedge

• Propagation Effects in Range and Cross-Range Directions

• Conclusions

• References


Overview of the Ray-Integral Solution Used

• Analyze the Image Field into a Sum of the Effects of the Source and a Finite Set of Its Images

\[ \phi = \phi_0 + \sum_{k=1}^{N} \phi_{\pm k} \]

Wave

The \( \pm k \)-th Multi-Reflected Wave

Emitted

by the Source

\[ N = \frac{\pi}{\alpha} \]

• The Solution for \( \phi_0 \)

\[ \phi_0 = f \left( t - \frac{R}{c} \right) / 4\pi c^2 R \]

\[ R^2 = x^2 + y^2 + (z - z_0)^2 \]

• The Solution for \( p_0 \) (Pressure Pulse Emitted by the Source)

\[ p_0 = -\rho \frac{\partial \phi_0}{\partial t} = \left( \frac{p_c}{R} \right) f \left( t - \frac{R}{c} \right) \]

\[ p_c = -\rho / 4\pi c^2 \]
Overview of the Ray-Integral Solution Used (cont.)

• The Ray-Integral Solution for $\phi_{\pm k}$ in a Penetrable-Bottom Wedge

$$\phi_{\pm k} = \int_{t_A}^{t} f(t-\tau) I_{\pm k}(\tau) d\tau$$

Time History of the Source Pulse

$$I_{\pm k}(t) = \frac{1}{2\pi^2 c^2} H(t-t_A) \text{Re} \left[ \int_{0}^{q(t)} S \Pi_{\pm k} \frac{dg_{\pm k}^{-1}}{dt} dq \right]$$

Ray Integral

$S =$ Source Function

$\Pi_{\pm k} =$ Cumulative Product of Reflection Coefficients

$g_{\pm k}^{-1} =$ Inverse of the Phase Function $g_{\pm k}$

• The Pressure Response $p$ at the Receiver-Point $x(x,y,z)$

$$p = p_0 + \sum_{k=1}^{N} p_{\pm k}$$

$$p_{\pm k} = -\rho \int_{t_A}^{t} f(t-\tau) I_{\pm k}(\tau) d\tau$$

• The Ray-Integral Solution Also Evaluates Propagation Paths for Multi-Reflected Waves
• The Diffraction Field Is Zero When $\alpha$ Is Set at $\alpha = 3^\circ$

• For $\alpha = 3^\circ$, 121 Pulses Contribute to Time Records of the Pressure

• The Ray-Integral Solution for $\phi_{\pm k}$ in a Perfect Wedge Can Be Evaluated in Closed Form

$$\phi_{\pm k} = (-1)^n \frac{f(t - R_{\pm l}/c)}{4\pi c^2 R_{\pm l}}$$

$n = \text{Number of Specular Reflections from Dirichlet Boundary}$

$R_{\pm l} = \text{Equivalent Distance from the Source to the Receiver}$

• The Benchmark Pressure Response $p$ at the Receiver-Point $x(x, y, z)$

$$p = p_0 + \sum_{k=1}^{N} p_{\pm k}$$

$$p_{\pm k} = (-1)^n \left( \frac{p_c}{R'_{\pm l}} \right) f\left(t - R'_{\pm l}/c\right)$$

$N = 60$
Source Pulses

- Heaviside Unit Pulse
• Broadband Pulse with Gaussian Spectrum

  • Normalized Center Frequency: \( cf = 3.33 \)
  • Normalized Bandwidth: \( w = 0.33 \)
  • For \( h = 200 \text{ m} \) and \( c = 1500 \text{ m/s} \), \( cf = 25 \text{ Hz} \) and \( w = 2.5 \text{ Hz} \)

\[
\begin{align*}
  r/h &= 1 \\
  \text{Normalized Pressure } p/p_c &\text{ vs. Normalized Time } t/t_c
\end{align*}
\]
Down-Slope Propagation

\[
\begin{align*}
\text{Normalized Time } t/t_c & \quad \text{(Normalized Pressure } p/p_c) \\
r/h = 5 & \\
0 & \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \\
0 & \quad 0.1 \quad 0.2 \\
-0.2 & \quad -0.1 \\
\text{Normalized Time } t/t_c & \\
r/h = 10 & \\
0 & \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \\
0 & \quad 0.1 \\
-0.1 & \quad -0.05 \\
\end{align*}
\]
Up-Slope Propagation

Normalized Pressure $p/p_c$ vs. Normalized Time $t/t_c$

$r/h = 5$

$r/h = 10$
Cross-Slope Propagation

For $r/h = 5$:

- Normalized Pressure $p/p_c$ vs. Normalized Time $t/t_c$

For $r/h = 10$:

- Normalized Pressure $p/p_c$ vs. Normalized Time $t/t_c$
Down-Slope Propagation

$r/h = 5$

$r/h = 10$
Down-Slope Propagation (cont.)

$r/h = 100$

$r/h = 200$
Up-Slope Propagation

$r/h = 5$

Normalized Pressure $p/p_c$

Normalized Time $t/t_c$

$r/h = 10$

Normalized Pressure $p/p_c$

Normalized Time $t/t_c$
Cross-Slope Propagation

$\frac{r}{h} = 5$

Normalized Pressure $\frac{p}{p_c}$

$\frac{r}{h} = 10$

Normalized Pressure $\frac{p}{p_c}$
Conclusions

• Ray-Integral Solution Provides Benchmark-Quality Pressure Records in a Perfect Wedge

• Down-Slope Large-Range Propagation in a Perfect Wedge Seems to Be Non-Dispersive
Overview of the Ray-Integral Solution Used (cont.)

• The Ray-Integral Solution for \( \phi_{\pm k} \) in a Liquid Wedge over Elastic Substratum

\[
\phi_{\pm k} = \int_{t_A}^{t} f(t-\tau) I_{\pm k}(\tau) d\tau
\]

\( I_{\pm k}(t) = \frac{1}{2\pi^2 c^2} H(t-t_A) \text{Re} \int_0^{q(t)} S \Pi_{\pm k} \frac{dg_{\pm k}^{-1}}{dt} dq \)

Time History of the Source Pulse

Ray Integral

• Operational Representation of the Ray Integral Includes the Contributions From:
  
  (i) Multi-Reflected Spherical \( P \) Wave
  
  (ii) Critically Refracted \( P \) and \( S \) Waves
  
  (iii) Pseudo-Rayleigh and Stoneley Interface Waves

• The Pressure Response \( p \) at the Receiver-Point \( x(x, y, z) \)

\[
p = p_0 + \sum_{k=1}^{N} p_{\pm k}
\]

\[
p_{\pm k} = -\rho \int_{t_A}^{t} \dddot{f}(t-\tau) I_{\pm k}(\tau) d\tau
\]

• The Ray-Integral Solution Also Evaluates Propagation Paths for Multi-Reflected Waves