What Makes a Good MIMO Channel Model?

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***Abstract*— Using different meaningful measures of quality, this paper investigates the accuracy of analytical MIMO channel models. Different metrics should be applied if the underlying MIMO channel supports predominantly beamforming, spatial multiplexing or diversity. The number of envisaged antennas plays an important role. By comparing the results of an extensive indoor measurement campaign at 5.2 GHz, we find the following main conclusions: (i) The recently developed Weichselberger model predicts capacity for any antenna number and represents diversity best of all three models, but still not satisfactorily. (ii) Except for 2×2 MIMO systems the Kronecker model fails to predict capacity, joint angular power spectrum, and diversity. (iii) The virtual channel representation should only be used for modeling the joint angular power spectrum for very large antenna numbers.**

The answer to the question given in the title: The appropriate model has to be chosen according to the considered application.

MIMO; analytical channel models; quality metrics

I. Introduction

Multiple-Input Multiple-Output (MIMO) systems are a promising candidate for future wireless communications systems. It is the radio propagation channel that determines crucially the characteristics of the entire MIMO system. Therefore, accurate modeling of MIMO channels is an important prerequisite for MIMO system design, simulation, and deployment. Especially analytical MIMO channel models that describe the impulse response (or equivalently the transfer function) of the channel between the elements of the antenna arrays at both link ends by providing analytical expressions for the channel matrix are very popular for developing MIMO algorithms in general. Most popular examples include the Kronecker model [1]–[3], the Weichselberger model [4], [5, Ch. 6.4.3] and the virtual channel representation [6]. In order to judge on the goodness of such models, metrics or performance measures are needed. Since the application of a specific metric implies a reduction of reality to some specific aspects, a single metric alone is not capable of capturing all properties of a MIMO channel.

As a consequence, we will use three different metrics covering different aspects of MIMO systems to verify the suitability of the narrowband Kronecker model, Weichselberger model and virtual channel representation (VCR) in this paper. These metrics will be (i) the *double-directional angular power spectrum (APS)*, (ii) the *mutual information with equal power allocation*, and (iii) a *diversity metric* recently introduced by Ivrlac and Nossek [7].

Based on the results we will try to answer the question what makes a good MIMO channel model.

II. Review of Considered Channel Models

In the following we consider frequency-flat fading MIMO channels with m transmit and n receive antennas where each single realization of the channel can be described by the $n \times m$ channel matrix $\mathbf{H}$.

A. Kronecker model

The Kronecker model [1]–[3] can be expressed as[1]

$$\mathbf{H}_{\mathrm{kron}} = \frac{1}{\sqrt{\mathrm{tr}\{\mathbf{R}_{\mathrm{Rx}}\}}} \mathbf{R}_{\mathrm{Rx}}^{1/2} \mathbf{G} (\mathbf{R}_{\mathrm{Tx}}^{1/2})^T, \tag{1}$$

where $\mathbf{R}_{\mathrm{Tx}} = \mathrm{E}\{\mathbf{H}^T \mathbf{H}^*\}$ and $\mathbf{R}_{\mathrm{Rx}} = \mathrm{E}\{\mathbf{H}\mathbf{H}^H\}$ denote the transmit and receive correlation matrices. Further, $\mathbf{G}$ is an i.i.d. random fading matrix with unity-variance, circ. symmetric complex Gaussian entries. The parameters for the Kronecker model are the transmit and receive correlation matrices.

The Kronecker model became popular because of its simple analytic treatment. However, the main drawback of this model is that it forces both link ends to be separable [8], irrespective of wheter the channel supports this or not.

B. Weichselberger model

The idea of Weichselberger was to relax the separability restriction of the Kronecker model and to allow for any arbitrary coupling between the transmit and receive eigenbase, i.e. to model the correlation properties at the receiver and transmitter *jointly*.

Introducing the eigenvalue decomposition of the receive and transmit correlation matrices

$$\begin{aligned} \mathbf{R}_{\mathrm{Rx}} &= \mathbf{U}_{\mathrm{Rx}} \mathbf{\Lambda}_{\mathrm{Rx}} \mathbf{U}_{\mathrm{Rx}}^H, \\ \mathbf{R}_{\mathrm{Tx}} &= \mathbf{U}_{\mathrm{Tx}} \mathbf{\Lambda}_{\mathrm{Tx}} \mathbf{U}_{\mathrm{Tx}}^H, \end{aligned} \tag{2}$$

Weichselberger [4], [5, Ch. 6.4.3] proposed

$$\mathbf{H}_{\mathrm{weichsel}} = \mathbf{U}_{\mathrm{Rx}} \left(\tilde{\Omega}_{\mathrm{weichsel}} \odot \mathbf{G} \right) \mathbf{U}_{\mathrm{Tx}}^T, \tag{3}$$

where $\mathbf{G}$, again, is an i.i.d. complex Gaussian random fading matrix, and $\tilde{\Omega}_{\mathrm{weichsel}}$ is defined as the element-wise square

[1] The following notation will be used throughout this paper: $(\cdot)^{1/2}$ denotes the matrix square root; $(\cdot)^T$ stands for matrix transposition; $(\cdot)^*$ stands for complex conjugation; $(\cdot)^H$ stands for matrix Hermitian; $\odot$ denotes the element-wise Schur-Hadamard multiplication; $\otimes$ denotes the Kronecker multiplication; $\mathrm{E}\{\cdot\}$ denotes the expectation operator; $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix; $\mathrm{vec}(\cdot)$ stacks a matrix into a vector, columnwise; $\|\cdot\|_F$ stands for the Frobenius norm.

	number of real-valued parameters
Kronecker	$m^2 + n^2$
Weichselberger	$mn + m(m-1) + n(n-1)$
VCR	mn

TABLE I

NUMBER OF MODEL PARAMETERS OF CONSIDERED CHANNEL MODELS.

root of the power coupling matrix Ω_{weichsel}. The positive and real-valued elements $\omega_{\text{weichsel},ij}$ of the coupling matrix determine the average power-coupling between the i-th transmit eigenmode and the j-th receive eigenmode.

The Weichselberger model parameters are the eigenbasis of receive and transmit correlation matrices and a coupling matrix.

C. Virtual channel representation

In contrast to the two prior models, the virtual channel representation (VCR) models the MIMO channel in the beamspace instead of the eigenspace. In particular, the eigenvectors are replaced by fixed and predefined steering vectors [6].

The VCR can be expressed as

$$\mathbf{H}_{\text{virtual}} = \mathbf{A}_{\text{Rx}} \left(\tilde{\Omega}_{\text{virtual}} \odot \mathbf{G} \right) \mathbf{A}_{\text{Tx}}^{T}, \tag{4}$$

where orthonormal response and steering vectors constitute the columns of the unitary response and steering matrices $\mathbf{A}_{\text{Rx}}$ and $\mathbf{A}_{\text{Tx}}$. Further, $\tilde{\Omega}_{\text{virtual}}$ is defined as the element-wise square root of the power coupling matrix Ω_{virtual}, whose positive and real-valued elements $\omega_{\text{virtual},ij}$ determine - this time - the average power-coupling between the i-th transmit and the j-th receive *direction*.

The VCR can be easily interpreted. Its angular resolution, and hence 'accuracy', depends on the actual antenna configuration. Its accuracy increases with the number of antennas, as angular bins become smaller.

The model is fully specified by the coupling matrix. Note that there still exists one degree of freedom in choosing the first direction of the unitary transmit/receive matrices $\mathbf{A}_{\text{Tx/Rx}}$.

D. Number of parameters

Table I summarizes the number of real-valued parameters that have to be specified for modeling an $n \times m$ MIMO channel using the models previously reviewed. However, mind the following exception: When only mutual information (or channel capacity) is of interest, the number of necessary parameters of the Kronecker model and the Weichselberger model reduce to $m + n$ and mn, respectively.

III. MEASUREMENTS

The model validation was based on a comprehensive indoor office environment measurement campaign our institute, at 5.2 GHz. The transmitter (Tx) consisted of a positionable sleeve antenna on a 20×10 grid with an inter-element spacing of $\lambda/2$, where only a sub-set of 12×6 Tx antenna positions was used to avoid large-scale fading effects [9, Ch. 4.3.4]. The receiver (Rx) was a directional 8-element uniform linear array of printed dipoles with 0.4λ inter-element spacing and $120°$ 3dB field-of-view. The channel was probed at 193 equispaced frequencies over 120 MHz of bandwidth. The (virtual) transmit array was positioned in a hallway and the receiver assumed 24 different positions each looking into 3 different directions (rotated by $120°$) in several offices connected to this hallway without line-of-sight (except one position/direction), leading to 72 different 'scenarios'. A detailed description of the measurement campaign can be found in [9, Ch. 4].

For each scenario, we generated spatial realizations of 2×2, 4×4 and 8×8 MIMO channels [9, Ch. 4.3.3]. This paper shows results for $0.5\lambda/0.4\lambda$ Tx/Rx interelement spacing; additional results for $1.0\lambda/0.8\lambda$ and $3.5\lambda/2.8\lambda$ can be found in [9, Ch. 5].

IV. MODEL VALIDATION

The investigated models assume that the channel is sufficiently described by its second order moments, hence by the full channel correlation matrix $\mathbf{R}_{\mathbf{H}}$, only. As a consequence, measurements used for the evaluations have to fulfil this requirement, too. Only a restricted set of 58 scenarios (out of 72) met this condition; the others were excluded.

This is how we validate the models: For each scenario we will (i) extract model parameters from measurement; (ii) generate synthesized channel matrices with these parameters by Monte-Carlo simulations of the three models; (iii) compare different metrics calculated from the modeled channels with those extracted directly from the respective measurement.

A. Extraction of Model Parameters

To extract model parameters from the measurements, different realizations of the MIMO channel matrix are necessary for each scenario. Besides the spatial realizations, different frequencies were used as fading realizations.

The model parameters of the Kronecker model, i.e. the single-sided receive and transmit correlation matrix are estimated by[2]

$$\hat{\mathbf{R}}_{\text{Rx}} = \frac{1}{N} \textstyle\sum_{r=1}^{N} \mathbf{H}(r)\mathbf{H}(r)^{H}, \tag{5}$$

$$\hat{\mathbf{R}}_{\text{Tx}} = \frac{1}{N} \textstyle\sum_{r=1}^{N} \mathbf{H}(r)^{T}\mathbf{H}(r)^{*}, \tag{6}$$

where N is the number of channel realizations, while $\mathbf{H}(r)$ denotes the r-th channel realization.

Applying the eigenvalue decomposition to the estimated correlation matrices,

$$\hat{\mathbf{R}}_{\text{Rx}} = \hat{\mathbf{U}}_{\text{Rx}}\hat{\mathbf{\Lambda}}_{\text{Rx}}\hat{\mathbf{U}}_{\text{Rx}}^{H}, \text{ and} \tag{7}$$

$$\hat{\mathbf{R}}_{\text{Tx}} = \hat{\mathbf{U}}_{\text{Tx}}\hat{\mathbf{\Lambda}}_{\text{Rx}}\hat{\mathbf{U}}_{\text{Tx}}^{H}, \tag{8}$$

the estimated power coupling matrix $\hat{\mathbf{\Omega}}_{\text{weichsel}}$ of the Weichselberger model can be obtained by

$$\hat{\mathbf{\Omega}}_{\text{weichsel}} = \frac{1}{N} \sum_{r=1}^{N} \left(\hat{\mathbf{U}}_{\text{Rx}}^{H}\mathbf{H}(r)\hat{\mathbf{U}}_{\text{Tx}}^{*} \right) \odot \left(\hat{\mathbf{U}}_{\text{Rx}}^{T}\mathbf{H}(r)\hat{\mathbf{U}}_{\text{Tx}} \right). \tag{9}$$

[2]Note that *estimated* model parameters are denoted by $\hat{(\cdot)}$.

Analogously, by taking unitary steering/response matrices $\mathbf{A}_{\mathrm{Tx}}$ and $\mathbf{A}_{\mathrm{Rx}}$, the estimated coupling matrix of the VCR $\hat{\mathbf{\Omega}}_{\mathrm{virtual}}$ can be calculated by

$$\hat{\mathbf{\Omega}}_{\mathrm{virtual}} = \frac{1}{N}\sum_{r=1}^{N}\left(\mathbf{A}_{\mathrm{Rx}}^{H}\mathbf{H}(r)\mathbf{A}_{\mathrm{Tx}}^{*}\right)\odot\left(\mathbf{A}_{\mathrm{Rx}}^{T}\mathbf{H}(r)\mathbf{A}_{\mathrm{Tx}}\right). \quad (10)$$

For $\mathbf{A}_{\mathrm{Tx}}$ and $\mathbf{A}_{\mathrm{Rx}}$ one steering/response direction was selected towards the broadside direction of the antenna array.

B. Monte-Carlo simulations

Using the extracted model parameters from the measurements, channel matrix realizations according to the Kronecker model (1) the Weichselberger model (3) and the VCR (4) are synthesized by introducing different fading realizations of the i.i.d. complex Gaussian, unity-variance random fading matrix $\mathbf{G}$. For the different MIMO systems, the number of realizations was chosen to be equal to the respective number of measured realizations.

C. Metrics

If we want to judge the goodness of a MIMO channel model, we first have to specify 'good' in which sense. The quality of a model has to be defined with a view toward a specific channel property or aspect which we are interested in. For this we need performance figures that cover the desired channel aspects and apply these metrics to measured and modeled channels, enabling a comparison of the models investigated.

Of course, it would be very helpful and advantageous to have a single metric that is capable of capturing all properties of a MIMO channel. However, this is not possible since the application of a specific metric implies a reduction of reality to some selected aspects, as modeling always does.

Mind that different metrics can yield different quality rankings of channel models as both, models and metrics, cover different channel aspects. The suitability of a metric strongly depends on its relevance to the MIMO system to be deployed.

1) Double-directional (or joint) angular power spectrum: For the directional evaluations, the joint direction-of-departure/direction-of-arrival (DoD/DoA) angular power spectrum (APS) is calculated using *Capon's beamformer*, also known as *Minimum Variance Method (MVM)* [5],

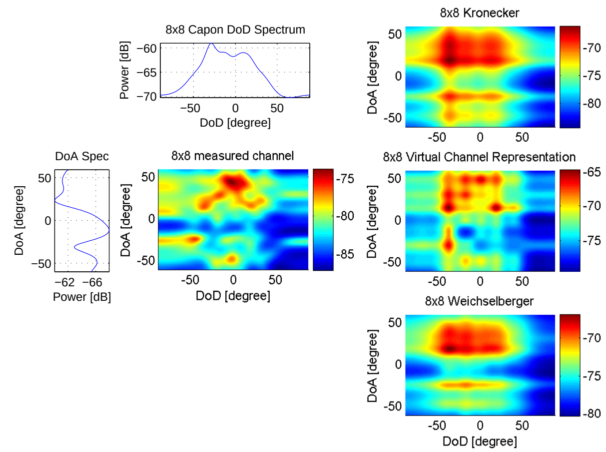
$$\mathbf{P}_{Capon}(\varphi_{\mathrm{Rx}},\varphi_{\mathrm{Tx}}) = \frac{1}{\tilde{\mathbf{a}}^{H}\mathbf{R}_{\mathbf{H}}^{-1}\tilde{\mathbf{a}}}, \quad (11)$$

with

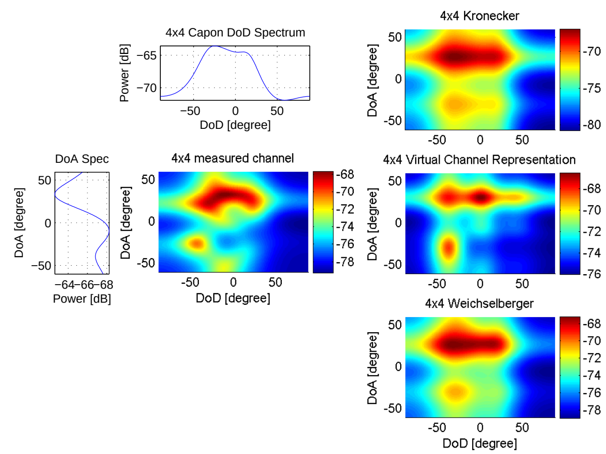
$$\tilde{\mathbf{a}} = \mathbf{a}_{\mathrm{Tx}}(\varphi_{\mathrm{Tx}})\otimes\mathbf{a}_{\mathrm{Rx}}(\varphi_{\mathrm{Rx}}), \quad (12)$$

using the normalized steering vector $\mathbf{a}_{\mathrm{Tx}}(\varphi_{\mathrm{Tx}})$ into direction φ_{Tx} and response vector $\mathbf{a}_{\mathrm{Rx}}(\varphi_{\mathrm{Rx}})$ from direction φ_{Rx}. Here, $\mathbf{R}_{\mathbf{H}} = \mathrm{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{H}\}$ denotes the full MIMO channel correlation matrix.

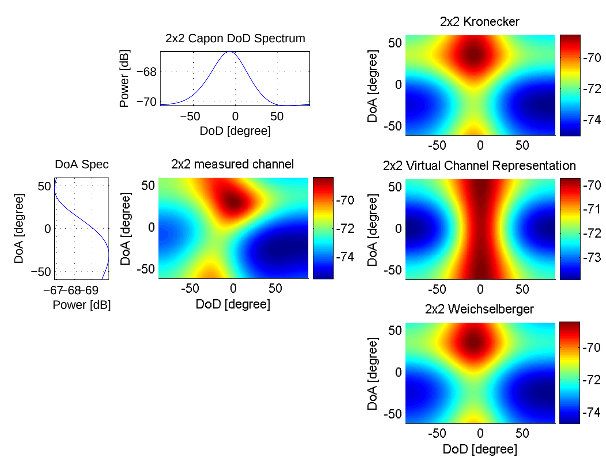
Figure 1 compares the APS of the measured and modeled 8×8 (a), 4×4 (b), and 2×2 (c) MIMO channel for an exemplary scenario. For each sub-plot, the measured APS



(a)

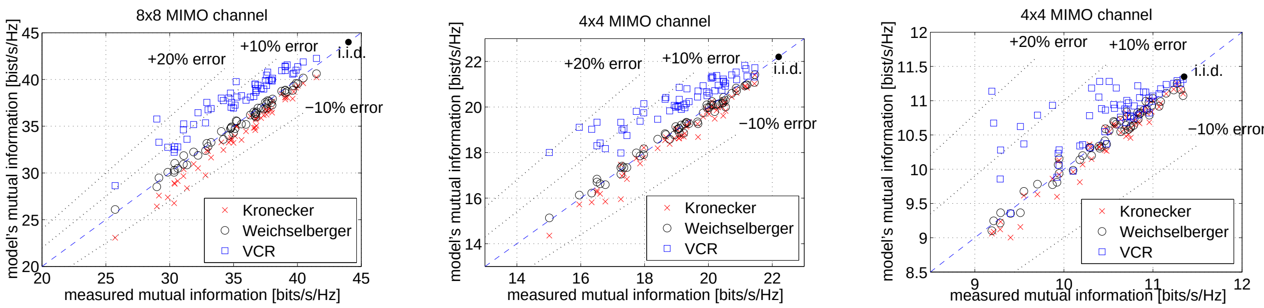


(b)



(c)

Fig. 1. Angular power spectra of measured and modeled (a) 8×8, (b) 4×4, and (c) 2×2 MIMO channels for an example scenario.



(a) (b) (c)

Fig. 2. Average mutual information of measured vs. modeled (a) 8×8, (b) 4×4 and (c) 2×2 MIMO channels at a receive SNR of 20dB.

(joint and marginal APS) are given on the left side, whereas the right side shows the models' joint APS.

Let us first investigate 8×8 (Fig. 1a). In the measured channel, specific DoDs are clearly linked to specific DoAs, such that the joint APS is *not* separable. In contrast, the Kronecker model introduces artefact paths lying at the intersections of the DoA and DoD spectral peaks. The Weichselberger model exposes this assumption to be too restrictive. Nevertheless, it does not render the multipath structure completely correct either. The VCR should be able to cope with any arbitrary DoD/DoA coupling. The joint APS shows that it does not because of the fixed and predefined steering vectors. It is not able to reproduce multipath components *between* two fixed steering vector directions properly.

Decreasing antenna numbers (4×4, 2×2) reduce the spatial resolution. The performance of both the Kronecker and the Weichselberger model improve with smaller number of antennas but the APS is still not reproduced correctly. The VCR collapses for smaller antenna numbers.

2) Average mutual information: Considering a channel unknown at Tx, and disregarding bandwidth, the mutual information of the MIMO channel with equally allocated transmit powers was calculated for each realization using [10], [11]

$$I = \log_2 \det(\mathbf{I}_n + \frac{\rho}{m}\mathbf{H}\mathbf{H}^H), \qquad (13)$$

where $\mathbf{I}_n$ denotes the $n \times n$ identity matrix, ρ the average receive SNR, and $\mathbf{H}$ the normalized $n \times m$ MIMO channel matrix.

The normalization was done such that for each scenario the power of the channel matrix elements h_{ij} averaged over all realizations was set to unity [9, Ch. 5.3.1]. In the subsequent evaluations, the average receive SNR for each scenario was always fixed at 20dB.

Figure 2 shows the results of this evaluation: Scatter plots of the average mutual information of the measured channel versus the average mutual information of the modeled channels for 8×8, 4×4, and 2×2 MIMO are depicted. For each model, a specific marker corresponds to one of the 58 scenarios.

In case of 8×8 MIMO channels (Fig. 2a), the Kronecker model (red crosses) underestimates the 'measured' mutual information[3]. Moreover, the mismatch increases up to more than 10% with decreasing mutual information. The VCR (blue squares) overestimates the 'measured' mutual information significantly. The reason again is due to its fixed steering/response directions. Thus, it tends to model the MIMO channel with more multipath components than the underlying channel actually has, thereby reducing channel correlation and increasing the mutual information. The Weichselberger model (black circles) fits the measurements best with relative errors within a few percents.

The relative model error of the Kronecker model decreases with decreasing antenna number (c.f. Fig. 2b,c). Although for 2×2 channels there exist some exceptional scenarios where the Kronecker model also overestimates the mutual information, a clear trend goes with underestimation of the mutual information. The VCR overestimates mutual information of the measured channel systematically up to 20%. The performance of the Weichselberger model does not change significantly, either. It still reflects the multiplexing gain of the measured channel best.

3) Diversity Measure: The eigenvalues λ_i of the full MIMO channel correlation matrix, $\mathbf{R_H}$, describe the average powers of the independently fading matrix-valued eigenmodes of a MIMO channel [5, Ch. 5.3.8]. Its offered degree of diversity is determined only by the *complete* eigenvalue profile.

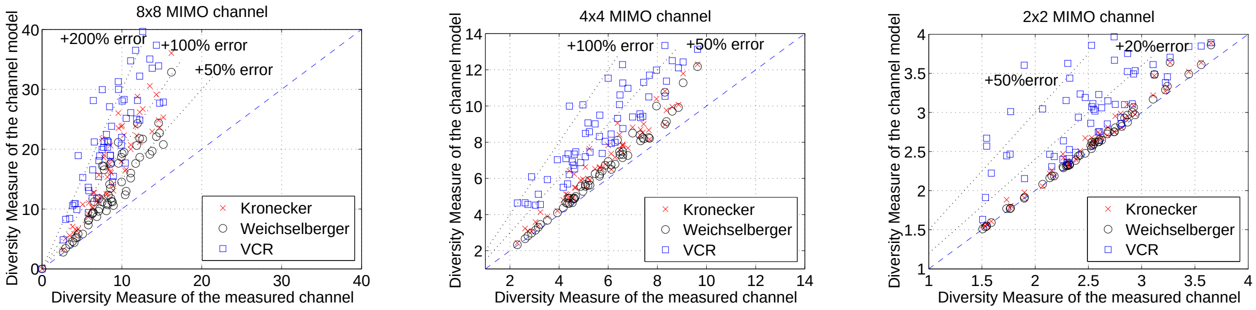
For the sake of comparison and classification of different channels, however, a single-number metric is highly advantageous, even if it can not reflect the whole information of the complete eigenvalue profile. A useful metric for Rayleigh fading MIMO systems, the so-called *Diversity Measure* $\Psi(\mathbf{R_H})$,

$$\Psi(\mathbf{R_H}) = \left(\frac{\mathrm{tr}\{\mathbf{R_H}\}}{||\mathbf{R_H}||_F}\right)^2 = \frac{\left(\sum_{i=1}^{K}\lambda_i\right)^2}{\sum_{i=1}^{K}\lambda_i^2}, \qquad (14)$$

was recently introduced by Ivrlac and Nossek [7].

Figure 3 shows the scatter plots of the models' Diversity Measures versus the Diversity Measures of the measured channels for 8×8, 4×4, and 2×2 MIMO. As can be

[3]Monte-Carlo simulations that we have performed with completely synthetic MIMO channels showed that, although very seldom, the Kronecker model might also overestimate the 'measured' mutual information. The probability of overestimation decreases with increasing antenna number.



(a) (b) (c)

Fig. 3. Diversity Measure of the measured vs. modeled (a) 8×8, (b) 4×4 and (c) 2×2 MIMO channels.

seen, the modeled channels either match or overestimate the Diversity Measures of the corresponding measured channels, independently of the number of antennas.

For 8×8 MIMO channels (Fig. 3a) it can be observed that all models overestimate the Diversity Measure, although the Weichselberger model (black circles) outperforms both the Kronecker model (red crosses) and the VCR (blue squares) clearly.

The Diversity Measures for 4×4 and 2×2 MIMO channels (Fig. 3b and c) show the same qualitative behavior as 8×8 channels, but decreasing relative errors with decreasing antenna numbers for all three models. Again, the Weichselberger model performs best. For the 2×2 channel, it shows almost perfect match except for some negligible errors for higher diversity values. Also, the match of the Kronecker model is quite tolerable, showing 10% relative errors in this case. In contrast, the VCR again fails completely, even in the 2×2 case. It still overestimates the Diversity Measure significantly. The reason for the poor performance of the VCR is, again, due to its fixed, predefined steering directions.

At this stage, we stress that the validation approach just discussed is the proper one to arrive at models that re-construct realistic MIMO channels, e.g. channels that are measured. This approach, though, is not the only one possible. Should one be interested in a *single* aspect of MIMO only, then models that contain proper parameters (that can be specified more or less freely) might perform better. For instance, the VCR allows for modeling channels with arbitrary multiplexing orders by choosing appropriate coupling matrices. Similarly, an appropriate choice of the Weichselberger coupling matrix enables the setting of arbitrary multiplexing *and* diversity orders.

V. Conclusions

If we want to judge the goodness of a MIMO channel model, we first have to specify ’good’ in which sense. The quality of a model has to be defined with a view toward a specific channel property or aspect which we are interested in and which is relevant for the MIMO system to be deployed.

A good channel model is a model that renders correctly the relevant aspects of the MIMO system to be deployed.

If no specific channel property is in focus, a good MIMO channel model reflects the spatial structure of the channel in general, as this determines the benefits of MIMO.

In an indoor environment, we assessed three analytical MIMO models by three different metrics, viz. double-directional angular power spectrum, average mutual information, and the Diversity Measure. From experimental validation we conclude that (i) the Weichselberger model performs best with respect to the analyzed metrics, even though it is inaccurate for joint APS and Diversity Measure in case of large antenna numbers, (ii) the Kronecker model should only be used for limited antenna numbers, such as 2x2, (iii) the virtual channel representation can only be used for modeling the joint APS for very large antenna numbers.

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