tened pulsed wavefronts contribute more magnitude to diagonal elements of R than to others and thus in the extreme simply appear as noise. The analysis and results apply equally to wide-band processes in that their correlation times effectively time-gate their wavefronts with respect to their contribution to the covariance matrix.

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Improved Signal Synthesis from Pseudo-Wigner Distribution

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***Abstract*—A recent paper presents the "overlapping method" (OM) for signal synthesis from modified pseudo-Wigner distributions. In this correspondence, we show that the OM yields improper results even for simple time-frequency models, and we present a new synthesis method with significantly improved results.**

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I. Introduction

Time-frequency representations (TFR's), like Wigner distribution (WD) or pseudo-Wigner distribution (PWD), display signals as surfaces over a time-frequency plane. Apart from signal *analysis*, an application of TFR's is signal *processing*. In TFR-based signal processing schemes, TFR surfaces are modified (e.g., by masking or smoothing) which generally results in a "nonvalid" surface which is no longer the TFR of any signal. A *signal synthesis* step is then used to produce a signal which best fits the nonvalid surface [1].

A general theory and solution of the signal synthesis problem exists for those TFR's with bilinear structure which satisfy a unitarity property for all signals (e.g., continuous-time WD) or for signals of a predefined signal subspace (e.g., discrete-time WD) [2]. In practice, however, a short-time WD version known as *pseudo-Wigner distribution* (PWD), rather than WD itself, is often used; unfortunately, the windowing employed in PWD destroys the unitary structure of WD.

While a closed-form solution of the PWD signal synthesis problem is not at present available, some heuristic algorithms for PWD signal synthesis exist. The *pseudopower method* (PPM) [3], [4] is an iterative synthesis method which calculates the theoretically optimal signal but is computationally expensive when longer signals are to be synthesized. This problem is overcome by a recursive version of the PPM termed *partial sum method* (PSM) [4]. The PSM is capable of synthesizing signals with arbitrary length. It is suboptimal but has been observed to yield good results.

Two other suboptimal synthesis algorithms have been presented in [5]. The "outer product approximation" synthesizes signals in segments in order to remove any length restriction; however, the adequate combination of the synthesized segments seems to be a problem. Finally, the overlapping method (OM) features a recursive structure similar to the PSM and thus appears to be attractive for the synthesis of signals with arbitrary length.

In the following, we present a new suboptimal PWD signal synthesis algorithm which, just as the OM, has recursive structure and thus allows signal synthesis without an inherent restriction of signal length. This algorithm is in fact a simplified version of the PSM of [4] and will accordingly be called the "simplified partial sum method" (SPSM). While the SPSM appears to be similar to the OM in its formulation, its results are generally quite different. We compare OM and SPSM in a simple synthesis experiment and show that, while OM results are clearly improper, SPSM yields satisfactory performance.

II. PWD Signal Synthesis

Using the notation of [5], the discrete-time PWD of a signal $f(n)$ is defined as [6]

$$\mathrm{PWD}_f(n, \theta) = 2 \sum_{k=-L}^{L} f(n+k) f^*(n-k) h(k) h^*(-k) e^{-j2k\theta} \tag{2.1}$$

where $h(k)$ is a window function which is zero outside $[-L, L]$; the window length is thus $2L + 1$. PWD is always real valued and π periodic with respect to the angular-frequency variable θ. The *PWD signal synthesis problem* is formulated as follows:

Given a real-valued time-frequency function $Y(n, \theta)$ defined for $-\pi/2 \leq \theta \leq \pi/2$, find the signal $f(n)$ whose PWD best approximates the "model" $Y(n, \theta)$, i.e., minimizes the error norm

$$J(f) = \sum_n \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \left| Y(n, \theta) - \mathrm{PWD}_f(n, \theta) \right|^2 d\theta. \tag{2.2}$$

In practical applications, also *suboptimal* synthesis methods may be helpful which do not yield the exact solution of the above syn-

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thesis problem but yield a "plausible" signal with "small" error norm $J(f)$.

Using Parseval's theorem and separating the even-numbered signal samples $f_e(n) = f(2n)$ and the odd-numbered signal samples $f_o(n) = f(2n+1)$, the error norm $J(f)$ can be rewritten as $J(f) = J_e(f_e) + J_o(f_o)$, where, e.g.,

$$J_e(f_e) = 4 \sum_i \sum_{m=i-L}^{i+L} \left| y(i+m, i-m) - f_e(i) f_e^*(m) h(i-m) h^*(m-i) \right|^2 \quad (2.3)$$

with

$$y(n, k) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} Y(n, \theta) e^{j2k\theta} \, d\theta. \quad (2.4)$$

A similar expression involving $f_o(n)$ holds for $J_o(f_o)$. The minimization problem $J(f) \rightarrow \min$ therefore comprises two independent minimization procedures $J_e(f_e) \rightarrow \min$ and $J_o(f_o) \rightarrow \min$ which produce the even-numbered and odd-numbered signal samples $f_e(n)$ and $f_o(n)$, respectively [1].

Since these two minimization procedures have identical structure, we shall here consider only the minimization for $f_e(n)$. Letting the gradient of $J_e(f_e)$ with respect to $f_e^*(n)$ be zero [1] yields the following third-order equation which is a necessary condition for the synthesis solution $f_e(n)$:

$$\sum_{m=i-L}^{i+L} \left[y_e(i, m) - f_e(i) f_e^*(m) h(i-m) h^*(m-i) \right] \cdot f_e(m) h^*(i-m) h(m-i) = 0 \quad (2.5)$$

for all i, with

$$y_e(i, m) = y(i+m, i-m). \quad (2.6)$$

III. Overlapping Method and Simplified Partial Sum Method

OM and SPSM are two heuristic signal synthesis algorithms which are suboptimal since they do not solve (2.5). Both OM and SPSM may be motivated by the special case of signal *reconstruction* where the model $Y(n, \theta)$ is *valid*, i.e., is the PWD of some signal $f(n)$. For a valid model $Y(n, \theta) = \mathrm{PWD}_f(n, \theta)$, we easily show that

$$y_e(i, m) - f_e(i) f_e^*(m) h(i-m) h^*(m-i) \equiv 0, \quad |i - m| \leq L. \quad (3.1)$$

Based on this equation, two equivalent sequential (recursive) reconstruction algorithms can be developed. We suppose that $f_e(n)$ is *known* for $M \leq n \leq M + P$ (with $P < L$) and calculate $f_e(n)$ for $M + P + 1 \leq n \leq M + L$ by means of formulae which are derived from (3.1) as follows.

OM [5]: Multiplying (3.1) by $f_e(m)/[h^*(i-m)h(m-i)]$, summing over $M \leq m \leq M + P$ and solving for $f_e(i)$ yields

$$f_e(i) = \frac{\sum_{m=M}^{M+P} y_e(i, m) f_e(m) / [h^*(i-m) h(m-i)]}{\sum_{m=M}^{M+P} |f_e(m)|^2}, \quad M + P + 1 \leq i \leq M + L. \quad (3.2)$$

SPSM: Multiplying (3.1) by $f_e(m) h^*(i-m) h(m-i)$, summing over $M \leq m \leq M + P$ and solving for $f_e(i)$ yields

$$f_e(i) = \frac{\sum_{m=M}^{M+P} y_e(i, m) f_e(m) h^*(i-m) h(m-i)}{\sum_{m=M}^{M+P} |f_e(m) h^*(i-m) h(m-i)|^2}, \quad M + P + 1 \leq i \leq M + L. \quad (3.3)$$

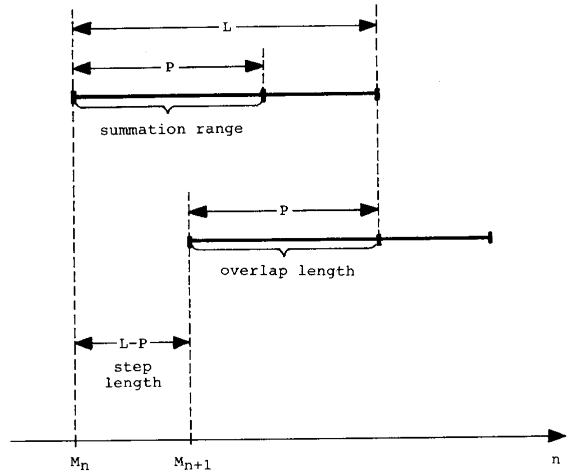

Fig. 1. Overlapping scheme for OM and SPSM.

Based on the formulae (3.2) or (3.3), a signal of arbitrary length can be reconstructed recursively from a (valid) model. The recursive reconstruction scheme is organized as follows (compare Fig. 1) [5]: in the nth step, $f_e(i)$ is calculated for $M_n + P + 1 \leq i \leq M_n + L$ using (3.2) (OM) or (3.3) (SPSM). M_n is then increased by the *step length* $L - P$ according to $M_{n+1} = M_n + (L - P)$ and the recursion starts anew. The intervals $[M_n, M_n + L]$, $[M_{n+1}, M_{n+1} + L]$ of successive steps overlap each other by the *overlap length* P (see Fig. 1). In the extreme case of maximal overlap length $P = L - 1$ (corresponding to minimal step length $L - P = 1$), only one signal sample is reconstructed in each step. Computational expense increases linearly with the overlap length P.

IV. Comparison of OM and SPSM for Signal Synthesis

OM and SPSM have been derived as (equivalent) signal *reconstruction* algorithms which will yield identical results if the model $Y(n, \theta)$ is a valid PWD. Formally, they may also be used as (suboptimal) signal *synthesis* methods when the model $Y(n, \theta)$ is not valid. It turns out, however, that the results of OM and SPSM may be dramatically different in the synthesis case. In experiments, we always observed SPSM to yield superior synthesis performance. This seems to be due to the fact that the SPSM formula (3.3)—unlike the OM formula (3.2)—is closely related to (2.5) which characterizes the exact (optimal) solution of PWD synthesis. Indeed, the SPSM formula (3.3) can be derived from the equation

$$\sum_{m=M}^{M+P} \left[y_e(i, m) - f_e(i) f_e^*(m) h(i-m) h^*(m-i) \right] \cdot f_e(m) h^*(i-m) h(m-i) = 0, \quad M + P + 1 \leq i \leq M + L \quad (4.1)$$

which equals (2.5) apart from the fact that the summation range $i - L \leq m \leq i + L$ of (2.5) is replaced by a partial range $M \leq m \leq M + P$, where $i - L \leq M$ and $M + P \leq i - 1$. In contrast, the OM formula (3.2) does not appear to be related in any sense to the "optimal" equation (2.5).

The superior synthesis performance of SPSM is demonstrated by the following simple synthesis experiment where the model $Y(n, \theta)$ was designed to simulate the PWD of a signal with sinusoidal frequency modulation (Fig. 2). The model is obviously not a valid PWD since it does not contain any interference terms [7]. Figs. 3 and 4 show the synthesis results (signal plus PWD) of SPSM and OM, respectively. The parameters of synthesis were identical for SPSM and OM (model length: 512 time points; PWD window

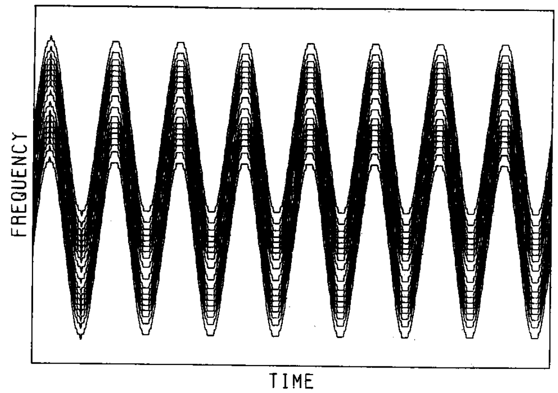

Fig. 2. Model $Y(n, \theta)$ simulating PWD of sinusoidal FM signal.

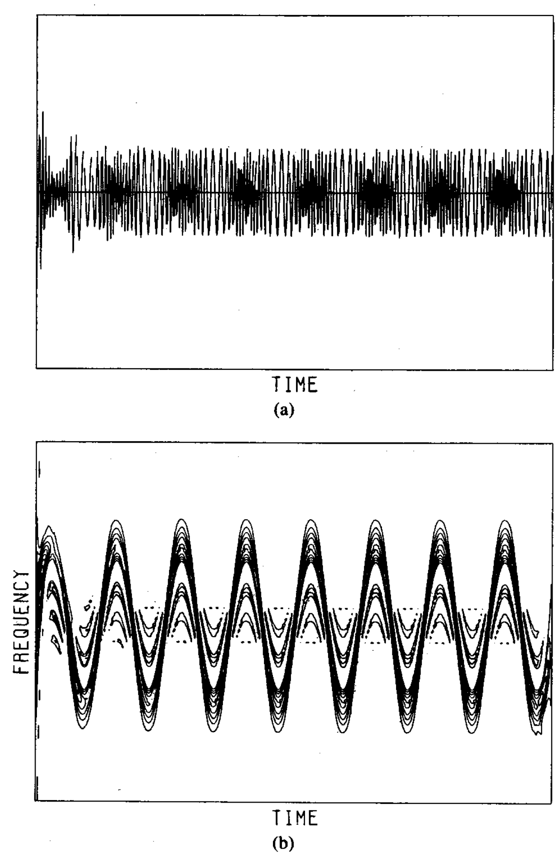

Fig. 3. SPSM synthesis result: (a) synthesized signal, (b) PWD of synthesized signal.

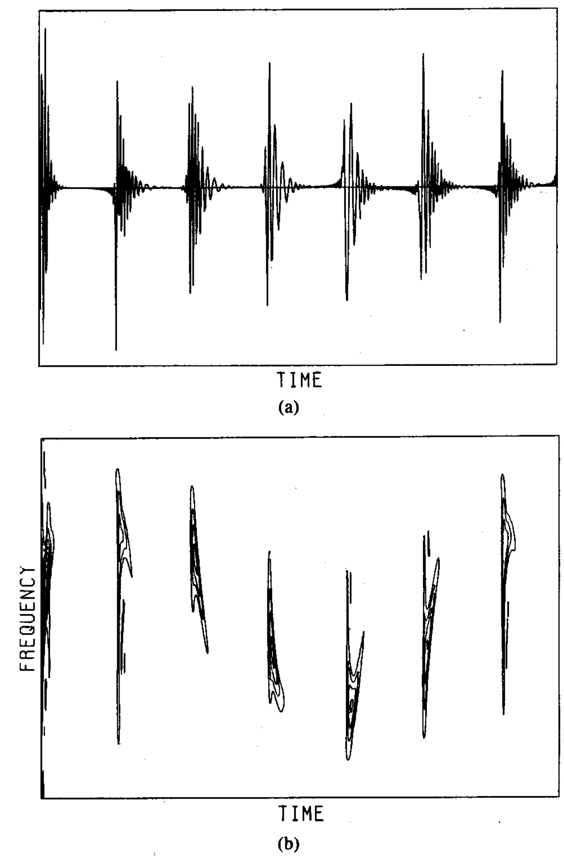

Fig. 4. OM synthesis result: (a) synthesized signal, (b) PWD of synthesized signal.

$h(n)$: Hamming with length $2L + 1 = 63$; step length $L - P = 1$). We see from Fig. 3 that the SPSM result is, after a brief convergence phase due to the naturally erroneous initial signal samples at the start of the recursion, indeed a sinusoidal FM signal; the OM result shown in Fig. 4, on the other hand, does not seem to match the model in any sense and does not converge to a steady-state behavior.

We finally discuss and compare some general properties of SPSM and OM. It should be noted that the computational complexity of SPSM is slightly greater than that of OM since the denominator of (3.3) must be computed separately for each i while it is constant in (3.2). Computational complexity of both OM and SPSM increases with growing PWD window length L.

Both methods will fail if the model contains a zero interval (gap) whose length exceeds $2L - 1$ since this will produce a division by zero in both (3.2) and (3.3). Depending on the parameters L, P and the exact position of the gap, also gaps with smaller lengths may produce this situation. In the worst case, both methods fail if the gap length exceeds $2P - 1$. Of course, gaps in the model can easily be detected, and the problem may then be resolved by starting the recursion anew after the gap.

The quality of SPSM results depends on the overlap length P. If the overlap length (summation range) P is increased or, equivalently, if the step length $L - P$ is decreased, then the residual synthesis error $J(f)$ becomes smaller. This means that the synthesis result is closer to the optimal result as defined by (2.5). Indeed, the equations (4.1) (defining the SPSM result) and (2.5) (defining the optimal result) become more similar if the summation range P is increased. We stress, however, that SPSM's (as well as OM's) computational expense grows linearly with increasing summation range P.

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Exact Equivalence of the Steiglitz-McBride Iteration and IQML

J. H. McClellan and D. Lee

Abstract—**The exact equivalence of the Steiglitz-McBride mode-1 algorithm and the iterative quadratic maximum likelihood (IQML) algorithm is demonstrated. Other methods, published earlier by Kumaresan, Scharf, and Shaw, and by Evans and Fischl, are also known to give the same equations and answer as IQML. Hence, all these methods are equivalent to the Steiglitz-McBride iteration. Since the IQML method was originally formulated to solve a multiple measurement case, an extended version of the Steiglitz-McBride mode-1 algorithm, applicable to the multiple measurement case, is also stated and shown to be equivalent to the general form of IQML.**

I. Introduction

The purpose of this correspondence is to prove exact equivalence of the Steiglitz-McBride mode-1 algorithm [1] and the iterative quadratic maximum likelihood (IQML) method [5]. Other algorithms, which predate IQML and which are also equivalent, can be found in [2]-[4]. Therefore, these methods are also equivalent to the Steiglitz-McBride mode-1 algorithm. The equivalence to be demonstrated is that all these methods compute exactly the same "poles" at each step of their respective iterations. Since the Steiglitz-McBride algorithm iteratively estimates both the numerator and denominator coefficients of a pole-zero model, while IQML estimates only the denominator coefficients, the equivalence need only apply to the poles of the model. In this correspondence, we will show that the error expressions (in terms of the poles) for the Steiglitz-McBride method and IQML can be reduced to the same form using projection operators. Finally, we will show that this equivalence applies to a multiple measurement version of the Steiglitz-McBride algorithm, even though that algorithm was originally formulated for the single measurement case only.

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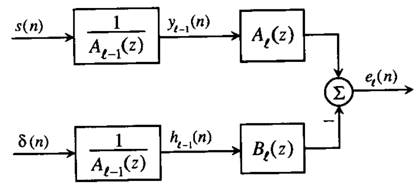

Fig. 1. Steiglitz-McBride block diagram. $A_l(z)$ and $B_l(z)$ are FIR filters whose coefficients are determined by minimizing the total energy in the output error signal $e_l(n)$. $A_{l-1}(z)$ is the estimate of the denominator polynomial from the previous iteration. Since $s(n)$ is known, the impulse response signal $h(n)$ and the output signal $y(n)$ can be calculated, and used as inputs to the FIR filters.

II. Equivalence of Steiglitz-McBride and IQML

In the modeling problem under consideration, N samples of a signal $s(n)$ are given, and the objective is to derive an exponential model for $s(n)$, i.e., $s(n) \approx \Sigma\, c_i(\lambda_i)^n$. This is the same as approximating $s(n)$ as the impulse response of a pole-zero system, if the number of poles equals the number of exponentials. The model system will be called $H(z)$, and its z transform has the following rational form:

$$H(z) = \frac{B(z)}{A(z)} \tag{1}$$

where

$$A(z) = \sum_{k=0}^{P} a(k) z^{-k} \quad \text{and} \quad B(z) = \sum_{k=0}^{Q} b(k) z^{-k}.$$

The Steiglitz-McBride method, shown as a block diagram in Fig. 1, is an iterative algorithm for computing a pole-zero model. This method was originally formulated as a system identification technique based on input and output measurements. However, the impulse-input case is more important for signal modeling. The iteration can be summarized as follows: On the lth iteration, an estimate of the unknown parameters is obtained by minimizing the following error equation over the set of $P + Q + 1$ coefficients $\{a_l(1), a_l(2), \cdots, a_l(P), b_l(0), b_l(1), \cdots, b_l(Q)\}$:

$$E_{\text{StMcB}}^{(l)} = \sum_{n=0}^{N-1} \left| A_l\left[\frac{1}{A_{l-1}}[s(n)]\right] - B_l\left[\frac{1}{A_{l-1}}[\delta(n)]\right]\right|^2. \tag{2}$$

$A_l(z)$ and $B_l(z)$ are the estimates of $A(z)$ and $B(z)$ at the lth iteration. A constraint such as $a_l(0) = 1$ must be imposed to guarantee that the solution will never be identically zero. The notation $A_l[\cdot]$ and $B_l[\cdot]$ denotes filtering, in this case with the FIR filters $A_l(z)$ and $B_l(z)$, respectively. Likewise, the operator $1/A_{l-1}[s(n)]$ represents the all-pole inverse filter $1/A_{l-1}(z)$ applied to the signal $s(n)$.

It is extremely useful to express the error equation (2) in matrix form. This can be done by using convolution matrices, so that we obtain

$$E_{\text{StMcB}}^{(l)} = \| \boldsymbol{Y}_{l-1}\boldsymbol{a}_l - \boldsymbol{H}_{l-1}\boldsymbol{b}_l \|^2 \tag{3}$$

where

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