On The Practical Use of Analytical MIMO Channel Models

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The practical use of so-called analytical models for representing measured and simulated narrowband MIMO channels is discussed with respect to several metrics. Four analytical models are compared (the Kronecker model, the Weichselberger model, the virtual channel representation, and the diagonal-decorrelation model) using several performance metrics. The investigation is based upon indoor experimental results at 5.2 GHz as well as geometry-based statistical propagation models.

Introduction

A well-known result of space-time signal processing is that the average channel capacity grows linearly with the number of antennas if the fades between pairs of transmit (Tx) and receive (Rx) antenna elements are independent and identically Rayleigh-distributed (Rayleigh i.i.d.). In practice, however, the MIMO channel can deviate significantly from this assumption, owing to a variety of reasons. A general representation of an $n_r \times n_t$ Rayleigh correlated channel is obtained as:

$$\mathrm{vec}(\mathbf{H}) = \mathbf{R}^{1/2}\,\mathrm{vec}(\mathbf{H}_w) \qquad (1)$$

where[1] $\mathbf{H}_w$ is an $n_r \times n_t$ i.i.d. channel matrix and $\mathbf{R} = E\left\{\mathrm{vec}(\mathbf{H})\,\mathrm{vec}(\mathbf{H})^H\right\}$ is the covariance. Elements in $\mathbf{R}$ can be classified as antenna correlations (often denoted as r and t) and diagonal or cross correlations (denoted as s_1, s_2, s_3, etc.). The complexity of the above representation grows rapidly with the array size, so a number of simplified analytical models have been published in the last years, e.g. [1]–[4]. The goal of this communication is to assess the adequacy of analytical representations with respect to several statistical metrics extracted from experimental results on one hand and geometry-based propagation models on the other hand. Since there is no ultimate metric covering all aspects of a MIMO channel, the following metrics are used:

- the channel correlations, as complex metrics can be expressed as a function of these,
- the mutual information with equal power allocation (often simply called *capacity*), related to the capacity increase offered by the spatial multiplexing gain,
- the angular power spectrum (APS), giving insight to the multipath structure and therefore the potential beamforming gain of a MIMO channel,
- the Diversity Measure introduced in [5],
- the symbol error rate for QPSK modulation in 2×2 spatial multiplexing schemes.

Review of Some Analytical Channel Models

Kronecker Model: This model approximates the correlation matrix $\mathbf{R}$ as the Kronecker product of the (marginal) correlation matrices at Rx and Tx, denoted as $\mathbf{R}_r$ and $\mathbf{R}_t$:

$$\mathbf{R} = \mathbf{R}_t^T \otimes \mathbf{R}_r \Leftrightarrow \mathbf{H} = \mathbf{R}_r^{1/2}\,\mathbf{H}_w\,\mathbf{R}_t^{1/2} \qquad (2)$$

[1] Throughout this communication, we use the following notations : $\mathrm{vec}\{\cdot\}$ stacks the elements of a matrix into a vector columnwise; $\{\cdot\}^H$ denotes Hermitian transposition; $\{\cdot\}^T$ denotes Hermitian transposition; $\otimes$ stands for the Kronecker product; $\circ$ denotes the Hadamard element-wise multiplication.

Physically, the Kronecker model means that the angular spectrum at Rx is assumed to be independent of the Tx direction (and vice versa). Equivalently, all cross correlations s_j are the products of corresponding Rx and Tx antenna correlations (separability assumption).

Weichselberger Model: This separability assumption is lifted in a model introduced by [2]. It models the *joint correlation properties* of both link ends; the dependencies between directions-of-arrival (DoAs) and directions-of-departure (DoDs) are preserved. The model consists of three components: the spatial eigenbasis of the Rx (and Tx) correlation matrix, denoted by $\mathbf{U}_r$ and $\mathbf{U}_t$, and a power-coupling matrix $\mathbf{\Omega}_{weichsel}$ whose structure is strongly linked to the radio environment. Each pair of receive and transmit eigenmodes spans a SISO channel, its average energy defining one element of $\mathbf{\Omega}_{weichsel}$. The important property of these eigenmode SISO channels is that they are mutually completely uncorrelated. Hence, the channel matrix is modelled as:

$$\mathbf{H} = \mathbf{U}_r\left(\tilde{\mathbf{\Omega}}_{weichsel} \circ \mathbf{H}_w\right)\mathbf{U}_t^T \tag{3}$$

where $\tilde{\mathbf{\Omega}}_{weichsel}$ is the element-wise square root of $\mathbf{\Omega}_{weichsel}$.

Virtual Channel Representation: The virtual channel representation (VCR) models the MIMO channel in the beamspace with predefined steering vectors [3]:

$$\mathbf{H} = \mathbf{A}_r\left(\tilde{\mathbf{\Omega}}_{virt} \circ \mathbf{H}_w\right)\mathbf{A}_t^T \tag{4}$$

with unitary steering matrices $\mathbf{A}_r$ and $\mathbf{A}_t$ and $\tilde{\mathbf{\Omega}}_{virt}$ being defined as the element-wise square root of the power coupling matrix $\mathbf{\Omega}_{virt}$ in the virtual angular domain. The angular resolution of the VCR, and hence its accuracy, depends on the number of virtual angles. These cannot be chosen arbitrarily but are given by the antenna array configurations.

Diagonal-Decorrelation Model: A fourth model used in [4] assumes (though without any physical justification) that all cross correlations are equal to zero, independently from the values of antenna correlations (but **R** should remain semi-positive definite).

Experimental Results and Geometry-Based Statistical Propagation Models

Measurements: A variety of MIMO channels were measured at the *Institut für Nachrichtentechnik und Hochfrequenztechnik* (TU Wien) at 5.2 GHz, in an indoor office environment (Tx inter-element spacing of $\lambda/2$; directional 8-element uniform linear array array at Rx with 0.4 λ spacing). More details can be found in [6]. Note that only MIMO channel measurements that are Rayleigh distributed are used in the following.

Geometry-Based Statistical Propagation Models:
1) One-Ring Model: This widely used model has been used for MIMO systems in [7], [8]. The original approach only accounts for local scattering arising from obstacles located near the subscriber unit by means of effective scatterers placed on a ring. It should be mentioned that when the antenna spacing are chosen as to force the Tx and Rx correlations to zero, the cross correlations in a 2 × 2 broadside configuration are equal to –0.24 and 1, i.e. far from the Kronecker assumption.
2) Combined Elliptical-Ring Model: The model developed in [9] combines the previous approach with iso-delay ellipses, in order to match specified tap-delay profile and angle-spreads. An important parameter of the model is the local scattering ratio (LSR), which can be directly related to the angular spreading properties: a low LSR denotes a directional distribution of scatterers, while a LSR of 1 is equivalent to an omnidirectional distribution of these scatterers on the local ring.

3) One-Disk Model: A third geometrical single-bounce model, related to the model of [10] consists in a large single disk, centered at mid-distance between Rx and Tx, and filled with a large number of uniformly distributed scatterers. In contrast to the ring-based models discussed above, correlation properties highly depend upon the path-loss exponent. Again, the magnitude of the cross correlations can be high, while the antenna correlations are small.

Validation of Analytical Models

Measured Channels: For 8×8 channels, DoDs and DoAs are linked such that the joint APS is not separable into a product of the marginal DoD and DoA spectra. The Kronecker factorization thus introduces artifact paths lying at the intersections of DoD and DoA spectral peaks [11]. Although the Weichselberger model lifts this assumption, it does not render the multipaths structure completely correctly either, nor does the VCR (because of its fixed and predefined steering directions). The performance of the Kronecker and the Weichselberger models improves when decreasing antenna numbers owing to the reduced spatial resolution, while the mismatch of the VCR increases. Regarding the mutual information, simulations at 20 dB SNR (see Figure 1) show that the Kronecker model generally underestimates the *true* mutual information, up to more than 10 % for large arrays (8×8) and low capacity values (i.e. at high correlation levels). By contrast, the VCR overestimates the *true* mutual information significantly, since it tends to model the MIMO channel with more multipaths than the underlying channel. The Weichselberger model fits the measurements best with relative errors within the range of a few percents.

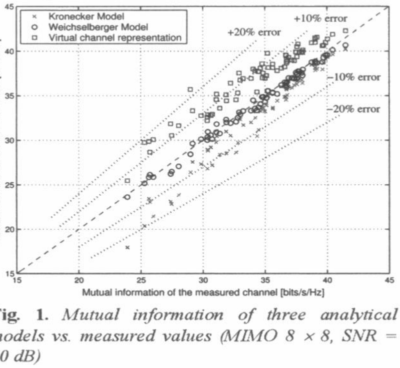

Fig. 1. *Mutual information of three analytical models vs. measured values (MIMO 8×8, SNR = 20 dB)*

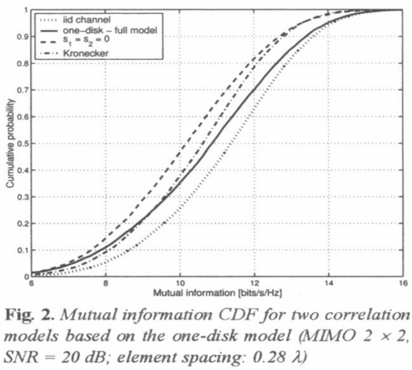

Fig. 2. *Mutual information CDF for two correlation models based on the one-disk model (MIMO 2×2, SNR = 20 dB; element spacing: 0.28 λ)*

As far as the Diversity Measure is concerned, comparison results show decreasing relative errors with decreasing array size for all tested models, with the Weichselberger model performing best. For MIMO 2×2, it shows almost perfect match, but the match of the Kronecker model is also tolerable (less than 10 %). The VCR significantly overestimates the Diversity Measure, even in the 2×2 case.

Simulated Channels: For 2×2 systems, the diagonal-decorrelation model may yield significant errors on the mutual information and SER (more than 10 to %), even at SNRs around 20 dB. For all tested geometry-based propagation models (at 20 dB SNR), the Kronecker model yields errors of 10 to 30 % on the SER; relative errors are small on ergodic mutual information (~ 4 %), but more significant on outage values. Let us consider two examples (MIMO 2 x 2, SNR of 20 dB). For the one-ring model, the Kronecker model underestimates the mutual information by up to 4 % (note that for some correlation levels, it is overestimated by 2 %). The SER is underestimated by 10 to 30 %. For the one-disk model, the error of the Kronecker approximation on the 90 % outage mutual information is about −10 %, while the error becomes slightly positive at 10 % outage (see Figure 2). Also, both the Kronecker and the diagonal-decorrelation models underestimate the SER by 15 %. The complete simulation results are detailed in [12].

Conclusions

As far as measured channels are concerned, the validation shows that, especially for large antenna numbers, the Weichselberger model outperforms the Kronecker model as well as the VCR with regards to the mutual information and the Diversity Measure. Note that for large arrays, none of the models is able to reproduce the experimental Diversity Measures. Concerning the joint angular spectrum, the Weichselberger model can cope with systems up to 4 × 4, whereas the Kronecker model should be limited to 2 × 2. By contrast, the VCR improves its performance for very large antenna numbers. In the case of simulated channels, neither the Kronecker nor the diagonal-decorrelation models are exact representations of popular geometry-based propagation models, especially for large arrays. The diagonal-decorrelation model may yield significant errors on both considered metrics. The Kronecker model achieves satisfactory performance for 2 × 2 systems as far as the ergodic mutual information is concerned, but can lead to higher mismatch when simulating outage mutual information or SER, even for 2 × 2 systems.

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