LOW-COMPLEXITY AND FULL-DIVERSITY MIMO DETECTION BASED ON CONDITION NUMBER THRESHOLDING

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ABSTRACT

In this paper, we consider MIMO spatial multiplexing systems and elaborate the impact of the channel condition number on the performance of ML and ZF detection. In particular, we show that for channels with bounded condition number ZF detection achieves the same diversity as ML detection. Motivated by this, we propose a novel threshold receiver that uses simple ZF detection for well-conditioned channels and ML detection for poorly condtitioned channels. We show that this reveiver achieves full diversity and we provide an upper bound on its SNR gap to ML detection. We further investigate cost-reduced versions of the threshold receiver and examine their performance in terms of simulation results.

Index Terms— MIMO Systems, data detection, diversity, spatial multiplexing.

1. INTRODUCTION

Two of the most important benefits of multiple-input multiple-output (MIMO) wireless systems are boosted capacity (multiplexing gain) and improved reliability (diversity gain) [1]. Here we consider $M_\mathrm{R} \times M_\mathrm{T}$ spatial multiplexing systems that offer full multiplexing gain of M_T and a (receive) diversity gain of M_R. Exploiting the latter requires appropriate data detection algorithms. Maximum-likelihood (ML) detection minimizes the error probability and exploits all available diversity but in general has a high computational complexity, even if it is implemented using the sphere-decoding algorithm [2]. On the other hand, suboptimum detection schemes like linear equalization, nulling and canceling (NC), and decision-feedback techniques [3–5] are much less complex but have a significantly poorer performance. In particular, they do not achieve a diversity order of M_R. This is no longer true if these sub-optimum detectors are preceded by LLL lattice reduction (LR) [6] in which case full diversity is retained [7]. A major problem with sub-optimum detectors is their poor performance for channels with large condition number [8].

In this paper, we study the impact of the channel condition number on suboptimum MIMO detection (Section 2). Specifically, we show that simple zero forcing (ZF) detection has full diversity for MIMO channels with bounded condition number. Motivated by this insight, we propose a novel threshold receiver (TR) that exploits all available diversity and has a very small complexity (Section 3). The TR uses ZF for channels with condition number below a prescribed threshold and ML detection otherwise; the threshold enables a continuous trade-off between average complexity (percentage of ML calls) and the SNR gap to ML detection. Finally, some more efficient variants of the TR are presented and simulation results are provided in Section 4.

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System Model. We consider a MIMO spatial multiplexing communication system with M_T transmit and $M_\mathrm{R} \geq M_\mathrm{T}$ receive antennas. The transmit vector $\mathbf{x} \in \mathcal{A}^{M_\mathrm{T}}$ (with $\mathcal{A}$ denoting the symbol alphabet) is normalized such that $\mathrm{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}$. Assuming a flat-fading channel[1], the length-M_R receive vector equals

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

Here, $\mathbf{H}$ is the $M_\mathrm{R} \times M_\mathrm{T}$ channel matrix, whose elements $[\mathbf{H}]_{n,m}$ are the complex fading coefficients between the the mth transmit and the nth receive antenna. Furthermore, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I})$ denotes spatially white complex Gaussian noise. The channel $\mathbf{H}$ is assumed perfectly known at the receiver.

Review of ML and ZF Detection. For further reference, we next briefly review ML and ZF detection.

ML Detection. ML detection [2, 9] yields minimum vector error probability. For our system model (1), it amounts to solving

$$\hat{\mathbf{x}}_\mathrm{ML} = \arg\min_{\mathbf{x} \in \mathcal{A}^{M_\mathrm{T}}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2.$$

The computational complexity of ML detection in general grows exponentially with M_T, even if it is implemented using sphere-decoding algorithm [10].

For given channel $\mathbf{H}$, the pair-wise error probability (PEP) of the ML detector is given by [1],

$$P_\mathrm{ML}(\mathbf{H}, \sigma_n^2) = Q\left(\frac{\|\mathbf{H}\boldsymbol{\delta}\|}{\sqrt{2}\sigma_n}\right) = Q\left(\sqrt{\mathrm{SNR}_\mathrm{ML}}\right), \tag{2}$$

where $\boldsymbol{\delta}$ is the error vector, $Q(\cdot)$ denotes the Q-function, and the effective SNR of the ML detector is given by $\mathrm{SNR}_\mathrm{ML} \triangleq \frac{\|\mathbf{H}\boldsymbol{\delta}\|^2}{2\sigma_n^2}$. Assuming i.i.d. Rayleigh fading, the mean PEP is upper bounded by [1]

$$\bar{P}_\mathrm{ML}(\sigma_n^2) = \mathrm{E}_\mathbf{H}\{P_\mathrm{ML}(\mathbf{H}, \sigma_n^2)\} \leq C_\mathrm{ML}\left(4 + \frac{\|\boldsymbol{\delta}\|^2}{\sigma_n^2}\right)^{-M_\mathrm{R}}, \tag{3}$$

where C_ML is a positive constant. Defining the diversity order as

$$\mathcal{D} \triangleq \lim_{1/\sigma_n^2 \to \infty} \frac{\log(\bar{P})}{\log(\sigma_n^2)}, \tag{4}$$

it is easily seen from (3) that $\mathcal{D}_\mathrm{ML} = M_\mathrm{R}$.

ZF Detection. ZF detection is based on quantizing the ZF equalized receive vector, i.e.,

$$\hat{\mathbf{x}}_\mathrm{ZF} = \mathcal{Q}\{\mathbf{y}_\mathrm{ZF}\}, \qquad \mathbf{y}_\mathrm{ZF} = \mathbf{H}^\# \mathbf{r} = \mathbf{x} + \tilde{\mathbf{n}}, \tag{5}$$

[1]The restriction to flat-fading is not serious since frequency-selective channels can be converted into parallel flat-fading channels using OFDM.

where $\mathcal{Q}\{\cdot\}$ denotes component-wise quantization with respect to $\mathcal{A}$ and $\mathbf{H}^{\#} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ denotes the pseudo-inverse of $\mathbf{H}$.

For given $\mathbf{H}$, the PEP of the ZF detector is given by [11]

$$P_{\text{ZF}}(\mathbf{H}, \sigma_n^2) = Q\Big(\sqrt{\text{SNR}_{\text{ZF}}}\Big)\,, \quad \text{SNR}_{\text{ZF}} \triangleq \frac{\|\boldsymbol{\delta}\|^4}{2\sigma_n^2 \boldsymbol{\delta}^H (\mathbf{H}^H\mathbf{H})^{-1}\boldsymbol{\delta}}\,. \tag{6}$$

For i.i.d. Rayleigh fading, the mean of (6) is bounded as [12]

$$\bar{P}_{\text{ZF}}(\sigma_n^2) \,\le\, C_{\text{ZF}}\left(1 + \frac{\|\boldsymbol{\delta}\|^2}{4\sigma_n^2}\right)^{-M_{\text{R}}+M_{\text{T}}-1}, \tag{7}$$

where C_{ZF} is a positive constant. The ZF detector thus yields a diversity gain of $\mathcal{D}_{\text{ZF}} = M_{\text{R}} - M_{\text{T}} + 1$.

2. IMPACT OF CONDITION NUMBER

In this section, we analyze the impact of the channel condition number on the performance of ML and ZF detection.

Preliminaries. The condition number of the channel matrix $\mathbf{H}$ is defined as

$$c_{\mathbf{H}} \triangleq \frac{\sigma_{\max}}{\sigma_{\min}}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the largest and the smallest singular value of $\mathbf{H}$, respectively. For $M_{\text{T}} = M_{\text{R}}$ and $\mathbf{H} \sim \mathcal{CN}(0, \mathbf{I})$, the probability density function (pdf) of the normalized condition number $\tilde{c}_{\mathbf{H}} = c_{\mathbf{H}}/M_{\text{T}}$ is given by [13]

$$f_{\tilde{c}_{\mathbf{H}}}(\gamma) = \frac{8}{\gamma^3}\exp\biggl(-\frac{4}{\gamma^2}\biggr) \tag{8}$$

in the system limit $M_{\text{T}} \to \infty$. For $M_{\text{R}} = 2$ and $M_{\text{T}} = M$, the pdf of $c_{\mathbf{H}}$ reads

$$f_{c_{\mathbf{H}}}(\gamma) = \frac{2\,\Gamma(2M)}{\Gamma(M)\,\Gamma(M-1)}\frac{\gamma^{2M-3}\,(\gamma^2-1)^2}{(\gamma^2+1)^{2M}}, \qquad \gamma \ge 1\,, \tag{9}$$

where $\Gamma(\cdot)$ is the complete Gamma function. Both (8) and (9) decay only polynomially. From this we can conclude that the occurence of channel matrices with large condition number is quite likely.

Impact on Detection SNR. The condition number was previously observed to have a major impact on detector performance in spatial multiplexing system [8, 14]. We next analytically assess the performance degradation of ZF detection for large $c_{\mathbf{H}}$ by comparing SNR_{ZF} to SNR_{ML}. Since the ML detector minimizes error probability, evidently $\text{SNR}_{\text{ML}} \ge \text{SNR}_{\text{ZF}}$ or equivalently $\text{SNR}_{\text{ML}}/\text{SNR}_{\text{ZF}} \ge 1$ (this can also be proved directly via Schur complement techniques [15] using only the respective SNR definitions). However, we show in the following that the SNR gap $\frac{\text{SNR}_{\text{ML}}}{\text{SNR}_{\text{ZF}}}$ between ML detection and ZF detection is bounded from above in terms of the condition number. To this end, we rewrite the ratio of the two SNRs as

$$\frac{\text{SNR}_{\text{ML}}}{\text{SNR}_{\text{ZF}}} = \frac{\boldsymbol{\delta}^H\mathbf{H}^H\mathbf{H}\boldsymbol{\delta}}{\boldsymbol{\delta}^H\boldsymbol{\delta}}\,\frac{\boldsymbol{\delta}^H(\mathbf{H}^H\mathbf{H})^{-1}\boldsymbol{\delta}}{\boldsymbol{\delta}^H\boldsymbol{\delta}}. \tag{10}$$

This is the product of two Rayleigh quotients induced by the matrix $\mathbf{H}^H\mathbf{H}$ and its inverse $(\mathbf{H}^H\mathbf{H})^{-1}$, respectively. Due to the Rayleigh-Ritz theorem [15] there is $\frac{\boldsymbol{\delta}^H\mathbf{H}^H\mathbf{H}\boldsymbol{\delta}}{\boldsymbol{\delta}^H\boldsymbol{\delta}} \le \sigma_{\max}^2(\mathbf{H})$, $\frac{\boldsymbol{\delta}^H(\mathbf{H}^H\mathbf{H})^{-1}\boldsymbol{\delta}}{\boldsymbol{\delta}^H\boldsymbol{\delta}} \le \frac{1}{\sigma_{\min}^2(\mathbf{H})}$, which implies that

$$\frac{\text{SNR}_{\text{ML}}}{\text{SNR}_{\text{ZF}}} \,\le\, c_{\mathbf{H}}^2. \tag{11}$$

However, in general there is no $\boldsymbol{\delta}$ that can achieve the two Rayleigh-Ritz bounds simultaneously, i.e., (11) is not tight. A tight upper bound can be obtained by maximizing (10) directly with respect to $\boldsymbol{\delta}$. It turns out that the worst case error vector is $\boldsymbol{\delta} = \mathbf{v}_{\max} + \mathbf{v}_{\min}$ with $\mathbf{v}_{\max}$ and $\mathbf{v}_{\min}$ denoting the right singular vectors of $\mathbf{H}$ associated with $\sigma_{\max}$ and $\sigma_{\min}$, respectively. The resulting tight upper bound equals

$$\frac{\text{SNR}_{\text{ML}}}{\text{SNR}_{\text{ZF}}} \,\le\, \left(\frac{c_{\mathbf{H}} + \frac{1}{c_{\mathbf{H}}}}{2}\right)^2 \,=\, \frac{1}{4}\left(c_{\mathbf{H}}^2 + \frac{1}{c_{\mathbf{H}}^2} + 2\right). \tag{12}$$

It is seen that the SNR penalty of the ZF detector is upper bounded in terms of the channel's condition number, i.e., large performance penalties will only be incurred for poorly conditioned channels.

Impact on Diversity. In this subsection, we discuss the impact of the channel condition number on diversity order. To this end, we split the pdf of $\mathbf{H}$ into two parts that correspond to channels with small and large condition number, i.e.,

$$f(\mathbf{H}) = f(\mathbf{H}|c_{\mathbf{H}}\!\le\!\kappa)\,p_\kappa + f(\mathbf{H}|c_{\mathbf{H}}\!>\!\kappa)\,(1\!-\!p_\kappa)\,. \tag{13}$$

Here, $p_\kappa = \Pr\{c_{\mathbf{H}} \le \kappa\} = \int f(\mathbf{H})\,\chi_\kappa(\mathbf{H})\,d\mathbf{H}$ and

$$f(\mathbf{H}|c_{\mathbf{H}}\!\le\!\kappa) = \frac{f(\mathbf{H})\,\chi_\kappa(\mathbf{H})}{\int f(\mathbf{H})\,\chi_\kappa(\mathbf{H})\,d\mathbf{H}}, \tag{14}$$

where we used the indicator function

$$\chi_\kappa(\mathbf{H}) = \begin{cases} 1 & c_{\mathbf{H}} \le \kappa \\ 0 & c_{\mathbf{H}} > \kappa\,. \end{cases}$$

The mean PEP of the ML detector can now be written as

$$\bar{P}_{\text{ML}}(\sigma_n^2) = \int P_{\text{ML}}(\mathbf{H}, \sigma_n^2) f(\mathbf{H}) d\mathbf{H} = \bar{P}_{\text{ML}}^{(1)}(\sigma_n^2) + \bar{P}_{\text{ML}}^{(2)}(\sigma_n^2)\,, \tag{15}$$

where $\bar{P}_{\text{ML}}^{(1)}(\sigma_n^2) = p_\kappa \int P_{\text{ML}}(\mathbf{H}, \sigma_n^2)\, f(\mathbf{H}|c_{\mathbf{H}} \le \kappa)\,d\mathbf{H}$ and $\bar{P}_{\text{ML}}^{(2)}(\sigma_n^2) = (1-p_\kappa)\int P_{\text{ML}}(\mathbf{H}, \sigma_n^2)\, f(\mathbf{H}|c_{\mathbf{H}}\!>\!\kappa)\,d\mathbf{H}$ are the mean PEPs for well-conditioned and poorly conditioned channels, respectively. We have $\bar{P}_{\text{ML}}^{(i)}(\sigma_n^2) \ge 0$ and thus $\bar{P}_{\text{ML}}^{(i)}(\sigma_n^2) \le \bar{P}_{\text{ML}}(\sigma_n^2)$, $i = 1, 2$. For $\mathbf{H} \sim \mathcal{CN}(0, \mathbf{I})$ the PEP $\bar{P}_{\text{ML}}(\sigma_n^2)$ amounts to diversity order M_{R}. We can conclude that ML detection achieves diversity order at least M_{R} irrespective of the channel condition number. While for well-conditioned channels this behavior is expected, we conjecture that in this case the diversity order is actually infinite, i.e., the finite diversity order of ML detection is due to poorly conditioned channels.

Well-Conditioned Channels. We next show that ZF detection exploits all available diversity in the case of channels whose condition number is bounded from above, i.e., $\Pr\{c_{\mathbf{H}} > \kappa\} = 0$. We use a split-up of the mean PEP of ZF detection similar to (15), $\bar{P}_{\text{ZF}}(\sigma_n^2) = \bar{P}_{\text{ZF}}^{(1)}(\sigma_n^2) + \bar{P}_{\text{ZF}}^{(2)}(\sigma_n^2)$. Under the assumption $\Pr\{c_{\mathbf{H}} > \kappa\} = 0$, we have $\bar{P}_{\text{ML}}(\sigma_n^2) = \bar{P}_{\text{ML}}^{(1)}(\sigma_n^2)$ and $\bar{P}_{\text{ZF}}(\sigma_n^2) = \bar{P}_{\text{ZF}}^{(1)}(\sigma_n^2)$. Using (12) and the fact that the Q-function is a monotonic decreasing function, we further obtain

$$\begin{aligned} \bar{P}_{\text{ML}}^{(1)}(\sigma_n^2) &= p_\kappa \int Q\Big(\sqrt{\text{SNR}_{\text{ML}}}\Big)\, f(\mathbf{H}|c_{\mathbf{H}}\!\le\!\kappa)\,d\mathbf{H} \\ &\ge p_\kappa \int Q\left(\frac{c_{\mathbf{H}} + \frac{1}{c_{\mathbf{H}}}}{2}\sqrt{\text{SNR}_{\text{ZF}}}\right) f(\mathbf{H}|c_{\mathbf{H}}\!\le\!\kappa)\,d\mathbf{H} \\ &\ge \bar{P}_{\text{ZF}}^{(1)}\left(\frac{4\sigma_n^2}{(\kappa + \frac{1}{\kappa})^2}\right). \end{aligned} \tag{16}$$

Since $\bar{P}_{\text{ML}}^{(1)}(\sigma_n^2)$ amounts to all achievable diversity, (16) implies that the same is true for $\bar{P}_{\text{ZF}}^{(1)}(\sigma_n^2)$, i.e., for well-conditioned channels ZF detection achieves full diversity. The only difference between ML and ZF detection in this case is a shift of the mean PEP curve to larger SNRs by at most (in dB)

$$\Delta\text{SNR} \leq 20 \log_{10}\left(\frac{\kappa + \frac{1}{\kappa}}{2}\right) \approx 20 \log_{10}(\kappa) - 6. \quad (17)$$

It is seen that the worst-case SNR gap ΔSNR is essentially directly proportional to the condition number bound κ in dB. We finally note that the above discussion implies that the low diversity order of ZF detection must be attributed to channel realizations with large condition number.

3. THRESHOLD RECEIVER

In Section 2, we analyzed the impact of the channel condition number $c_{\mathbf{H}}$ on the performance of MIMO ML and ZF detection schemes. Motivated by these results, we propose a novel receiver that combines the advantages of ML and ZF detection, namely full diversity and low complexity, respectively. In particular, we saw that ZF detection exploits all available diversity if the condition number happens to be bounded from above. The main idea now is to let the ZF detector only "see" well-conditioned channels and let the ML detector take care of poorly conditioned channel realizations, i.e.,

$$\hat{\mathbf{x}}_{\text{TR}} = \begin{cases} \hat{\mathbf{x}}_{\text{ZF}} & \text{if } c_{\mathbf{H}} \leq \kappa \\ \hat{\mathbf{x}}_{\text{ML}} & \text{if } c_{\mathbf{H}} > \kappa, \end{cases} \quad (18)$$

where $\hat{\mathbf{x}}_{\text{ML}}$ and $\hat{\mathbf{x}}_{\text{ZF}}$ are defined in (2) and (5), respectively, and the threshold κ here represents a design parameter. We will refer to (18) as *threshold receiver* (TR) since it involves a thresholding with respect to the channel condition number. We note that ML and ZF detection are special cases of our TR obtained with $\kappa = 1$ and $\kappa \to \infty$, respectively.

TR Complexity. The complexity of the TR receiver can be continuously adjusted via the threshold κ. For $\kappa = 1$, there is $\hat{\mathbf{x}}_{\text{TR}} = \hat{\mathbf{x}}_{\text{ML}}$ and complexity equals that of ML detection. On the other hand letting κ tend to infinity yields ZF complexity arbitrarily close. In between, κ determines the percentage of ML calls, $\Pr\{c_{\mathbf{H}} > \kappa\} = 1 - p_\kappa$ which depends on the distribution of $c_{\mathbf{H}}$ (equivalently, of $\mathbf{H}$).

Using the limiting distribution (8) as an approximation for finite $M_{\text{T}} = M_{\text{R}}$, we obtain for the i.i.d. Rayleigh fading case

$$p_\kappa \approx \exp\left(-\frac{4M_{\text{T}}^2}{\kappa^2}\right), \qquad \kappa \approx \frac{2M_{\text{T}}}{\sqrt{\log(\frac{1}{p_\kappa})}}.$$

For $M_{\text{T}} = M_{\text{R}} = 6$, the choice $\kappa = 25$ leads to $p_\kappa \approx 0.79$, i.e., ML detection is performed only for about 21% of the channel realizations (the exact value is slightly lower). We note that for the 2×2 case, (9) can be used to obtain p_κ exactly.

TR Performance. Since the ZF part of the TR "sees" only well-conditioned channels, it achieves full diversity according to the discussion in Section 2. The ML part as well achieves full diversity for the poorly conditioned channel realizations that it "sees". Explicitly, the mean PEP of the TR equals

$$\bar{P}_{\text{TR}}(\sigma_n^2) = \bar{P}_{\text{ZF}}^{(1)}(\sigma_n^2) + \bar{P}_{\text{ML}}^{(2)}(\sigma_n^2), \quad (19)$$

with $\bar{P}_{\text{ZF}}^{(1)}(\sigma_n^2)$ and $\bar{P}_{\text{ML}}^{(2)}(\sigma_n^2)$ as defined before. Since both terms in (19) amount to full diversity we conclude that the TR achieves full diversity order as well. Comparing (19) to (15), it is seen that

$$\begin{aligned}\bar{P}_{\text{TR}}(\sigma_n^2) - \bar{P}_{\text{ML}}(\sigma_n^2) &= \bar{P}_{\text{ZF}}^{(1)}(\sigma_n^2) - \bar{P}_{\text{ML}}^{(1)}(\sigma_n^2) \\ &\leq \bar{P}_{\text{ML}}^{(1)}\left(\frac{(\kappa + \frac{1}{\kappa})^2}{4}\sigma_n^2\right) - \bar{P}_{\text{ML}}^{(1)}(\sigma_n^2),\end{aligned}$$

where in the last step we used (12). From this we see that with respect to ML detection the TR incurs an SNR penalty that is upper bounded according to (17). The threshold parameter thus allows a continuous adjustment of the TR's SNR loss and hence can be used to trade complexity against performance.

Condition Number Estimator. The TR presupposes knowledge of the condition number $c_{\mathbf{H}}$ for each channel realization. We propose an efficient estimator of $c_{\mathbf{H}}$ that is based on the power method (PM), a method to iteratively calculate the largest eigenvalue of a matrix [15]. The PM is first applied to $\mathbf{W} = \mathbf{H}^H\mathbf{H}$ to obtain $\sigma_{\max}^2$ and then to $\mathbf{W}^{-1}$ to obtain $1/\sigma_{\min}^2$. The product of the two PM passes yields an estimate of $c_{\mathbf{H}}^2$ which can be compared to κ^2. The computational overhead of this procedure is small: on the one hand, for many channel realizations $\mathbf{W}^{-1}$ and $\mathbf{W}$ are required for ZF detection anyway; on the other hand, we observed that a few PM iterations suffice to achieve almost ideal performance.

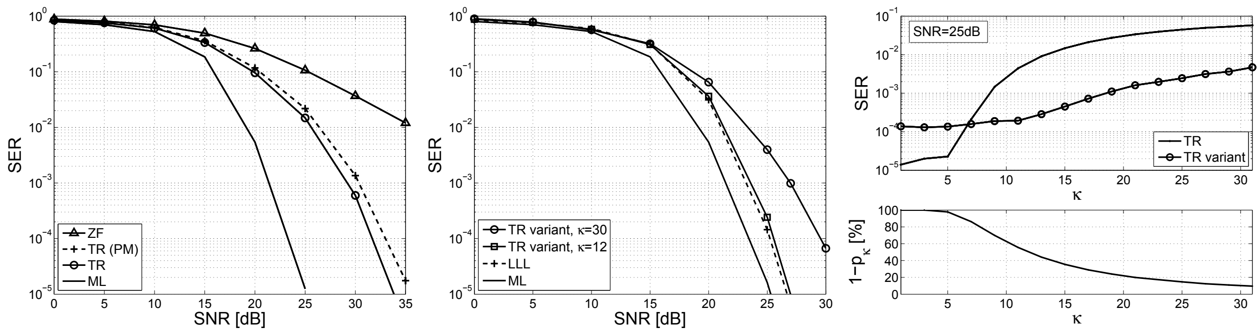
TR Variants. In order to either further reduce computational complexity or to improve error rate, many variants of the TR can be defined by replacing ML/ZF with some other detection schemes. When replacing the ML part, the goal is to reduce complexity. One possibility that maintains diversity order is to use LLL lattice reduction in conjunction with ZF, MMSE, or nulling and canceling (NC). This will result in an additional SNR loss as compared to ML detection. Replacing the ZF part targets to close the SNR gap to ML detection. In fact, the full diversity result for well-conditioned channels carries over to MMSE detection, NC, etc. However, alternatives to ZF may slightly increase computational complexity.

4. SIMULATIONS

In this section, we assess the performance of the proposed TR by means of simulations. We considered an uncoded 6×6 MIMO spatial multiplexing system with 16-QAM symbol alphabet. All results were obtained using between $4 \cdot 10^3$ and $4 \cdot 10^6$ realizations of an i.i.d. Rayleigh fading channel.

Fig. 1(a) compares the TR with threshold $\kappa = 15$ to ML and ZF detection in terms of symbol-error rate (SER) versus SNR. With this threshold, the percentage of ML calls is reduced down to 35 %. There are two curves for the TR, one corresponding to perfect knowledge of $c_{\mathbf{H}}$ (labeled "TR") and one where the condition number is estimated using a single PM iteration (labeled "TR (PM)"). It is seen that TR indeed achieves full diversity and features an SNR gap of about 8.5 dB to ML. Furthermore, estimation of the condition number degrades TR performance only slightly (in fact, with two PM iterations there is no visible difference to the ideal case) .

In Fig. 1(b), the performance of a TR variant for various κ is compared to ML. Here, ZF detection is replaced with MMSE-based NC [16] and ML detection is replaced with MMSE based NC preceded by LLL lattice reduction [16]. For $\kappa = 1$, LLL-assisted MMSE based NC is performed for all channel realizations and achieves close-to ML performance within a SNR gap of 2 dB. Increasing the threshold to $\kappa = 12$ incurs virtually no performance loss but allows to avoid LLL lattice reduction for about 50 % of the channel realiza-



(a) (b) (c)

Fig. 1. SER versus SNR comparison of (a) TR, ML, and ZF, and (b) modified TR and ML; (c) SER and percentage of channels with $c_{\mathbf{H}} \leq \kappa$ versus threshold κ for TR and TR variant for an SNR of 25dB.

tions. With $\kappa = 30$ this percentage is further decreased to $10\,\%$ at the cost of another $4\,$dB SNR penalty.

Finally, Fig. 1(c) depicts the SER (top) and the percentage of channel realizations with $c_{\mathbf{H}} \geq \kappa$ (bottom) versus the TR threshold κ for the ML/ZF-based TR and the previously described TR variant. The SNR was $25\,$dB. We see that channels with $c_{\mathbf{H}} \leq 5$ occur almost never and hence ML/ZF-based TR remains at ML performance for $\kappa = 1 \ldots 5$. Increasing κ further leads to a gradual complexity-performance trade-off that ends up in ZF performance with almost no ML calls for $\kappa > 30$. For the TR variant, we see that SER performance remains almost constant up to $\kappa \approx 11$, in which case complexity (percentage of LLL calls) is reduced to $60\,\%$. Beyond $\kappa = 11$, we again observe a continuous trade-off of complexity and performance.

5. CONCLUSIONS

We analyzed the impact of the channel condition number on the performance of detectors for MIMO spatial multiplexing. In particular, we demonstrated that channels with large condition numbers dominate the performance of ZF and ML detection. For channels with bounded condition number, even ZF was seen to exploit all available performance. Motivated by this, we introduced a novel threshold receiver (TR) that combines the advantages of ML (full diversity) and ZF (low complexity) detection. Replacing the ML and/or ZF stage with alternative MIMO detectors leads to variants of TR with similar properties and increased flexibility. Finally, the performance of the TR and a TR variant were assessed using numerical simulations.

We conclude that the flexibility and continuous complexity-performance trade-off of TR render it attractive for hardware implementations where ML detection or LLL cannot always be performed to achieve the required throughput.

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